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FIBER-ELEMENT MODELING OF THE CYCLIC BIAXIAL BEHAVIOR OF R/C COLUMNS

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ABSTRACT

The computational aspects of simulating the moment-curvature and force-displacement behavior of reinforced concrete columns subjected to cyclic biaxial bending and axial load are examined. Starting from first principles the basic equations of biaxial behavior are derived. Advanced constitutive models for normal and high strength concrete, and for the cyclic and low cycle fatigue behavior of reinforcing and prestressing steel bars, are integrated in a Fiber-Element procedure for the simulation of the cyclic and fatigue behavior of columns subjected to biaxial loading. This approach allows the damage assessment of columns when subjected to earthquake type loading in all directions.

Two different implementations of the Fiber-Element modeling procedure are presented. The first implementation uses a five-node rectangular element using a quadratic interpolation function. The second implementation uses a five-node circular-trapezoidal element more appropriate for circular columns. The use of quadratic interpolation functions in both elements improves convergence and thus fewer elements are needed in the discretization process. When compared with actual experimental data, the agreement between the model and the experiment is remarkable. Thus the applicability of the program both as a computational experiment simulator and as a damage assessment tool is justified.

KEYWORDS

Cyclic Model, Hysteretic Model, Fiber Model, Biaxial Column Analysis, Biaxial Column Behavior, Fatigue Modeling, Constitutive Model, Reinforced Concrete Modeling, Reinforced Concrete Earthquake Analysis.

INTRODUCTION

The complex nature of the cyclic biaxial behavior of columns makes difficult the interpretation of test results. This is due to the interaction between orthogonal bending deformations and axial load, in addition to the moment-curvature relationship. The number of biaxial experiments in the technical literature is very limited, this may be due to both the relative complexity of the test and the limited usefulness of the results. A Fiber-Element analysis program may be useful as a pre-processor, to simulate an experiment before hand. If the program proves to be accurate it may in some cases replace the lack of experiment by providing a simulated experiment. The program may also be used as a post-processor, to assess the fatigue damage in a column for a given biaxial deformation history.

It is the purpose of this investigation to lay the foundation for the implementation of a comprehensive analytical tool that may be used to simulate experiments and to evaluate the damage in columns subjected to earthquake type biaxial loading.

CONSTITUTIVE MODELS

The ability of a Fiber-Element implementation to accurately simulate the actual behavior of a reinforced concrete member depends on the adequacy of the constitutive models on which it is based. Advanced constitutive models has been developed by Chang and Mander (1994a) and were used in this investigation to implement a biaxial Fiber-Element analysis program.

Steel Constitutive Model

The constitutive steel model developed is capable of simulating the cyclic and fatigue behavior of both reinforcing and prestressing steels. The degradation characteristic of steel was identified and modeled. The model is also suitable for the assessment of random fatigue damage. Strain rate effects are also taken into account. This permits the accurate simulation of steels normally used in columns.

The model uses a relocatable envelope curve base on the equation:

$$f_s = \frac{E_s \varepsilon_{ss}}{\left[1 + \left(\frac{E_s \varepsilon_{ss}}{f_y}\right)^{10}\right]^{0.1}} + \frac{\operatorname{sign}(\varepsilon_{ss} - \varepsilon_{sh}^{+}) + 1}{2}\left(f_{su}^{+} - f_{y}^{+}\right)\left[1 - \left|\frac{\varepsilon_{su}^{+} - \varepsilon_{ss}}{\varepsilon_{su}^{+} - \varepsilon_{sh}^{+}}\right|^{p^{+}}\right] \tag{1}$$

where:

$$\varepsilon_{ss} = \varepsilon_s - \varepsilon_{om}^{+} \tag{2}$$

and,

$$p^{+} = E_{sh}^{+}\frac{\varepsilon_{su}^{+} - \varepsilon_{sh}^{+}}{f_{su}^{+} - f_{y}^{+}} \tag{3}$$

in which f_s = steel stress, ε_s = steel strain, E_s = elastic Young modulus, f_y^{+} = yield stress, ε_{sh}^{+} = strain hardening strain, E_{sh}^{+} = strain hardening tangent modulus, ε_{su}^{+} = stress at ultimate stress, f_{su}^{+} = ultimate (maximum) stress and ε_{om}^{+} = relocated origin abscissa, as shown in Fig. 1a. The positive sign superindex is to denote the positive (tension) direction. A similar equation is necessary to describe the envelope curve in the opposite direction. A complete description of the cyclic properties of the constitutive model implemented is given by Chang and Mander (1994a).

Concrete Constitutive Model

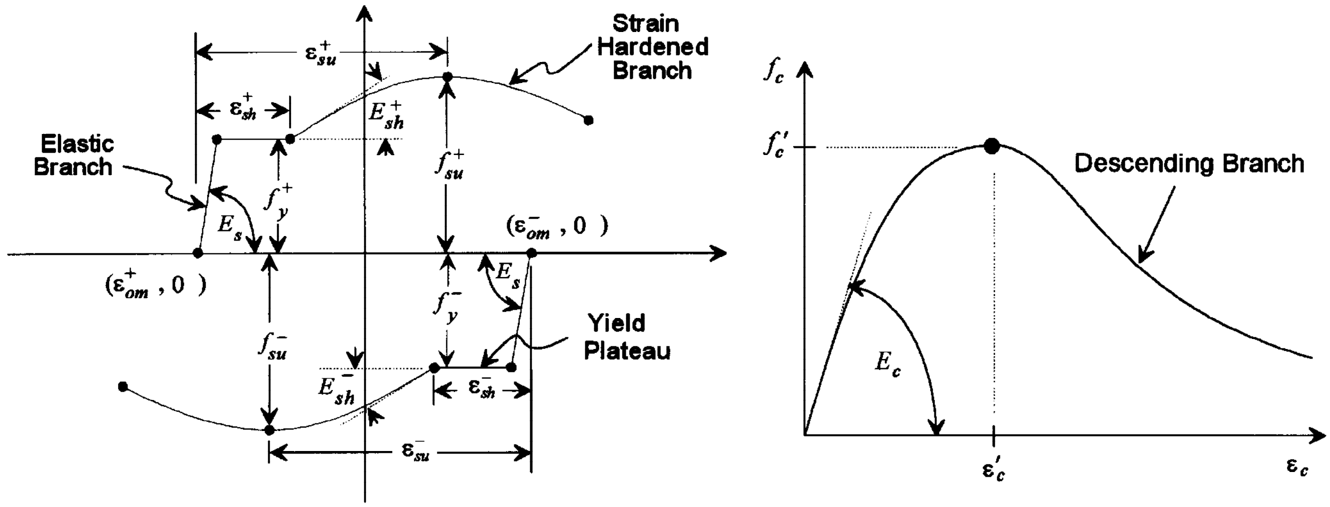
The constitutive concrete model implemented is capable of simulating the cyclic behavior of both normal and high strength concrete that may be confined or unconfined. Gradual crack closure, dynamic effects and tension cyclic behavior is also simulated. The model uses an envelope curve based on the following equation proposed by Tsai (Chang and Mander, 1994a).

$$f_c = \frac{n x f_c'}{1 + \left(n - \frac{r}{r-1}\right)x + \frac{x^r}{r-1}} \tag{4}$$

where:

$$x = \frac{\varepsilon_c}{\varepsilon_c'} \tag{5}$$

$$n = \frac{E_c \varepsilon_c'}{f_{cc}'} \tag{6}$$



(a) Steel Model (b) Concrete Model

Fig. 1. Envelope Curves used in the Steel and Concrete Constitutive Models

and, r is a parameter to control the descending branch of the curve, as shown in Fig. 1b. This parameter was found to be dependent of the concrete strength and given by the equation:

$$r = \frac{f'_{cc}}{750\,\text{psi}} - 1.9 \qquad r = \frac{f'_{cc}}{5.2\,\text{MPa}} - 1.9 \tag{7}$$

in which f_c = concrete stress, ε_c = concrete strain, f'_c = concrete strength, ε'_c = strain at the point of maximum stress capacity, E_c = initial tangent modulus. A complete description of the concrete model is given by Chang and Mander (1994a).

MOMENT-CURVATURE ANALYSIS FOR BIAXIAL BENDING

From first principles, the longitudinal strain at any point on a cross-section is given by:

$$\varepsilon = \varepsilon_o - \phi_x(y - y_o) - \phi_y(x - x_o) \tag{8}$$

where ε = strain at any coordinate (x, y), ε_o = strain at the plastic centroid, (x_o, y_o) = coordinate of the plastic centroid respect to an arbitrary origin, ϕ_x, ϕ_y = curvature in the x and y direction respectively. The curvature sign is taken positive if it produces a positive displacement in the perpendicular direction. The equations in this section are given in terms of the plastic centroid so that the moments in the presence of axial load are conventionally defined.

The axial load on a column in terms of stresses is expressed by the equation:

$$P = \iint_{A_g} f_c\, dA + \sum_{j=1}^{nb} (f_{sj} - f_{cj}) A_{sj} \tag{9}$$

where A_g = gross area, f_c = the stress in the concrete provided by a suitable constitutive model, f_s = stress in the steel computed through an appropriate constitutive model, A_s = steel area, and nb = number of bars in the column cross-section. The moments are computed by the following expressions:

$$M_x = y_o P - \iint_{A_g} y\, f_c\, dA - \sum_{j=1}^{nb} y_{sj} (f_{sj} - f_{cj}) A_{sj} \tag{10}$$

$$M_y = x_o P - \iint_{A_g} x\, f_c\, dA - \sum_{j=1}^{nb} x_{sj} (f_{sj} - f_{cj}) A_{sj} \tag{11}$$

The integrals in these equations may be computed as a summation if the cross-section is subdivided into a series of segments (elements). In this investigation two different elements are analyzed. The first element is appropriate for sections composed of rectangular subsections (rectangular, L-shape, C-shape, I-shape and hollow box cross-sections), while the second element is adequate for circular columns.

Rectangular Concrete Element Fiber Model Implementation

The equations used in the rectangular concrete element fiber model implementation were derived using a quadratic interpolation concrete stress function. The interpolation concrete stress function in terms of local axis has the form:

$$f_c = A + B\eta + C\xi + D\eta\xi + E\eta^2 + F\xi^2 \tag{12}$$

In this equation η and ξ represent the local axis in the x and y direction respectively, as illustrated in Fig. 2a. The element has five nodes as shown. By using the interpolating function the integral may be expressed by a summation in terms of the stress function at the nodes of the element, as given by the following equations:

$$P = \sum_{i=1}^{ne} \Delta P_i + \sum_{j=1}^{nb} (f_{sj} - f_{cj}) A_{sj} \tag{13}$$

$$M_x = y_o P - \sum_{i=1}^{ne} \Delta M_{xi} - \sum_{j=1}^{nb} y_{sj}(f_{sj} - f_{cj}) A_{sj} \tag{14}$$

$$M_y = x_o P - \sum_{i=1}^{ne} \Delta M_{yi} - \sum_{j=1}^{nb} x_{sj}(f_{sj} - f_{cj}) A_{sj} \tag{15}$$

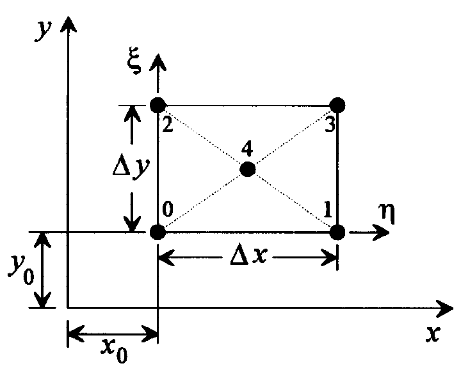
where:

$$\Delta P_i = \tfrac{1}{12} \Delta x \Delta y (f_{c0} + f_{c1} + f_{c2} + 8 f_{c3} + f_{c4}) \tag{16}$$

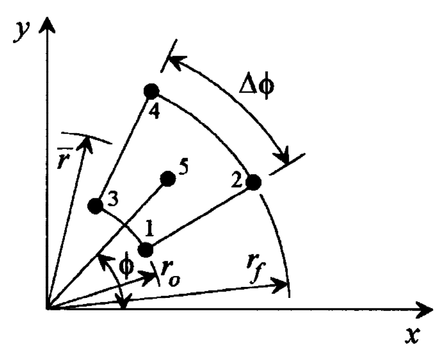
$$\Delta M_{xi} = \tfrac{1}{12} \Delta x \Delta y^2 (f_{c2} + 4 f_{c3} + f_{c4}) + y_{0i} \Delta P_i \tag{17}$$

$$\Delta M_{yi} = \tfrac{1}{12} \Delta x^2 \Delta y (f_{c1} + 4 f_{c3} + f_{c4}) + x_{0i} \Delta P_i \tag{18}$$

in which $f_{c0}, f_{c1}, f_{c2}, f_{c3}$ and f_{c4} = stresses in the concrete at nodes 0 through 4 respectively (node numbering is according to Fig. 2a), (x_{0i}, y_{0i}) = coordinate of the ith element and ne = number of concrete elements.



(a) Rectangular Element



(b) Circular Trapezium Element

Fig. 2 Fiber Elements Implemented

The discretization of a circular column by using a rectangular element is inaccurate and expensive in terms of the number of elements needed to achieve an acceptable precision. Thus an element well suited for circular cross-section columns was developed. A quadratic interpolation stress function was applied to a five node circular trapezium element. The interpolation concrete stress function used for this element is:

$$f_c = A + r(B\cos\theta + C\sin\theta) + r^2(D\cos 2\theta + E\sin 2\theta) \tag{19}$$

The longitudinal strain in polar coordinates is given by:

$$\varepsilon = \varepsilon_o - \phi_x r \sin\theta - \phi_y r \cos\theta \tag{20}$$

And the axial load and flexural moments are expressed as:

$$P = \int_0^r \int_0^{2\pi} f_c\, r\, d\theta\, dr + \sum_{j=1}^{nb} (f_{sj} - f_{cj}) A_{sj} \tag{21}$$

$$M_x = -\int_0^r \int_0^{2\pi} f_c\, r^2 \sin\theta\, d\theta\, dr - \sum_{j=1}^{nb} r_j \sin\theta_j (f_{sj} - f_{cj}) A_{sj} \tag{22}$$

$$M_y = -\int_0^r \int_0^{2\pi} f_c\, r^2 \cos\theta\, d\theta\, dr - \sum_{j=1}^{nb} r_j \cos\theta_j (f_{sj} - f_{cj}) A_{sj} \tag{23}$$

To transform the integrals in the previous equations to summations, it is necessary to discretize the cross-section. For this purpose, the circular trapezium element shown in Fig. 2b was used. In this case the equations are not derived explicitly, instead an implicit form is used. The interpolation function has five constants that may be found if the function is known at the five nodes. This leads to a system of equations:

$$\begin{Bmatrix} f_{c1} \\ f_{c2} \\ f_{c3} \\ f_{c4} \\ f_{c5} \end{Bmatrix} = \begin{bmatrix} 1 & r_o\cos\alpha & r_o\sin\alpha & r_o^2\cos 2\alpha & r_o^2\sin 2\alpha \\ 1 & r_f\cos\alpha & r_f\sin\alpha & r_o^2\cos 2\alpha & r_o^2\sin 2\alpha \\ 1 & r_o\cos\beta & r_o\sin\beta & r_o^2\cos 2\beta & r_o^2\sin 2\beta \\ 1 & r_f\cos\beta & r_f\sin\beta & r_o^2\cos 2\beta & r_o^2\sin 2\beta \\ 1 & r_m\cos\phi & r_m\sin\phi & r_m^2\cos 2\phi & r_m^2\sin 2\phi \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \\ D \\ E \end{Bmatrix} \tag{24}$$

where:

$$\alpha = \phi - \tfrac{1}{2}\Delta\phi \tag{25}$$

$$\beta = \phi + \tfrac{1}{2}\Delta\phi \tag{26}$$

$$r_m = \sqrt{\frac{r_o^2 + r_f^2}{2}}$$

in which r_o, r_f, ϕ and $\Delta\phi$ are shown in Fig. 2b.

As may be seen in equation (24), the coefficient matrix in the system of equation depends only on the geometry of the element. For computational speed, the inverse of this matrix may be computed and stored before the actual analysis is to take place. The coefficients A through E for every element are computed by multiplying the inverse of the matrix of coefficient of that element by the vector containing the node stress values. The discretized version of equations 21 to 23 are:

$$P = \sum_{i=1}^{ne} \left[\int_{r_{oi}}^{r_{fi}} \int_{\alpha_i}^{\beta_i} f_c\, r\, d\theta\, dr \right] + \sum_{j=1}^{nb} (f_{sj} - f_{cj}) A_{sj} \tag{27}$$

$$M_x = -\sum_{i=1}^{ne} \left[\int_{r_{oi}}^{r_{fi}} \int_{\alpha_i}^{\beta_i} f_c\, r^2 \sin\theta\, d\theta\, dr \right] - \sum_{j=1}^{nb} r_j \sin\theta_j (f_{sj} - f_{cj}) A_{sj} \tag{28}$$

$$M_y = -\sum_{i=1}^{ne} \left[\int_{r_{oi}}^{r_{fi}} \int_{\alpha_i}^{\beta_i} f_c\, r^2 \cos\theta\, d\theta\, dr \right] - \sum_{j=1}^{nb} r_j \cos\theta_j (f_{sj} - f_{cj}) A_{sj} \tag{29}$$

in which the integrals are given by:

$$\int_{r_o}^{r_f}\int_{\alpha}^{\beta} f\,dA = \tfrac{1}{2}A\left(r_f^2 - r_o^2\right)(\beta - \alpha) + \tfrac{1}{3}B\left(r_f^3 - r_o^3\right)(\sin\beta - \sin\alpha) - \tfrac{1}{3}C\left(r_f^3 - r_o^3\right)(\cos\beta - \cos\alpha)$$
$$+ \tfrac{1}{8}D\left(r_f^4 - r_o^4\right)[\sin(2\beta) - \sin(2\alpha)] - \tfrac{1}{8}E\left(r_f^4 - r_o^4\right)[\cos(2\beta) - \cos(2\alpha)] \tag{30}$$

$$\int_{r_o}^{r_f}\int_{\alpha}^{\beta} f\,r\sin\theta\,dA = -\tfrac{1}{3}A\left(r_f^3 - r_o^3\right)(\cos\beta - \cos\alpha) + \tfrac{1}{8}B\left(r_f^4 - r_o^4\right)\left(\sin^2\beta - \sin^2\alpha\right)$$
$$+ \tfrac{1}{16}C\left(r_f^4 - r_o^4\right)[2\beta - \sin(2\beta) - 2\alpha + \sin(2\alpha)]$$
$$+ \tfrac{1}{30}D\left(r_f^5 - r_o^5\right)[3\cos\beta - \cos(3\beta) - 3\cos\alpha + \cos(3\alpha)]$$
$$+ \tfrac{1}{15}E\left(r_f^5 - r_o^5\right)\left(2\sin^3\beta - 2\sin^3\alpha\right) \tag{31}$$

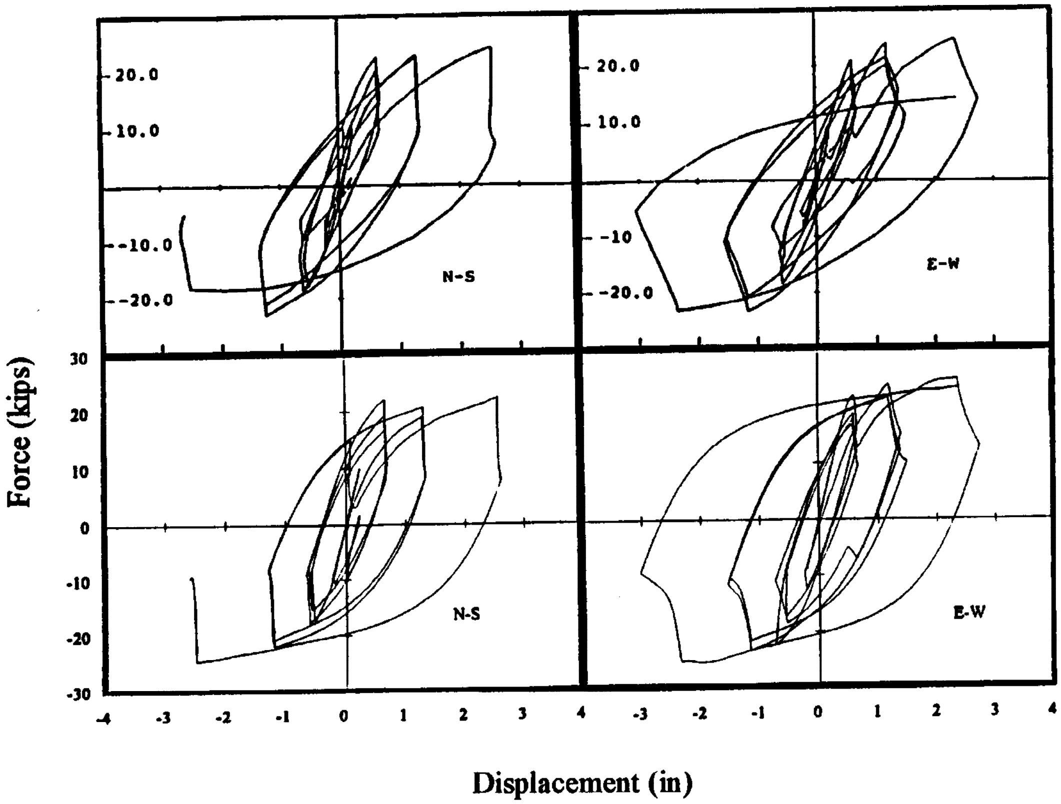
$$\int_{r_o}^{r_f}\int_{\alpha}^{\beta} f\,r\cos\theta\,dA = \tfrac{1}{3}A\left(r_f^3 - r_o^3\right)(\sin\beta - \sin\alpha) + \tfrac{1}{16}B\left(r_f^4 - r_o^4\right)[2\beta + \sin(2\beta) - 2\alpha - \sin(2\alpha)]$$
$$+ \tfrac{1}{8}C\left(r_f^4 - r_o^4\right)\left(\sin^2\beta - \sin^2\alpha\right)$$
$$+ \tfrac{1}{30}D\left(r_f^5 - r_o^5\right)[3\sin\beta + \sin(3\beta) - 3\sin\alpha - \sin(3\alpha)]$$
$$- \tfrac{1}{15}E\left(r_f^5 - r_o^5\right)(2\cos^3\beta - 2\cos^3\alpha) \tag{32}$$

FORCE-DISPLACEMENT ANALYSIS

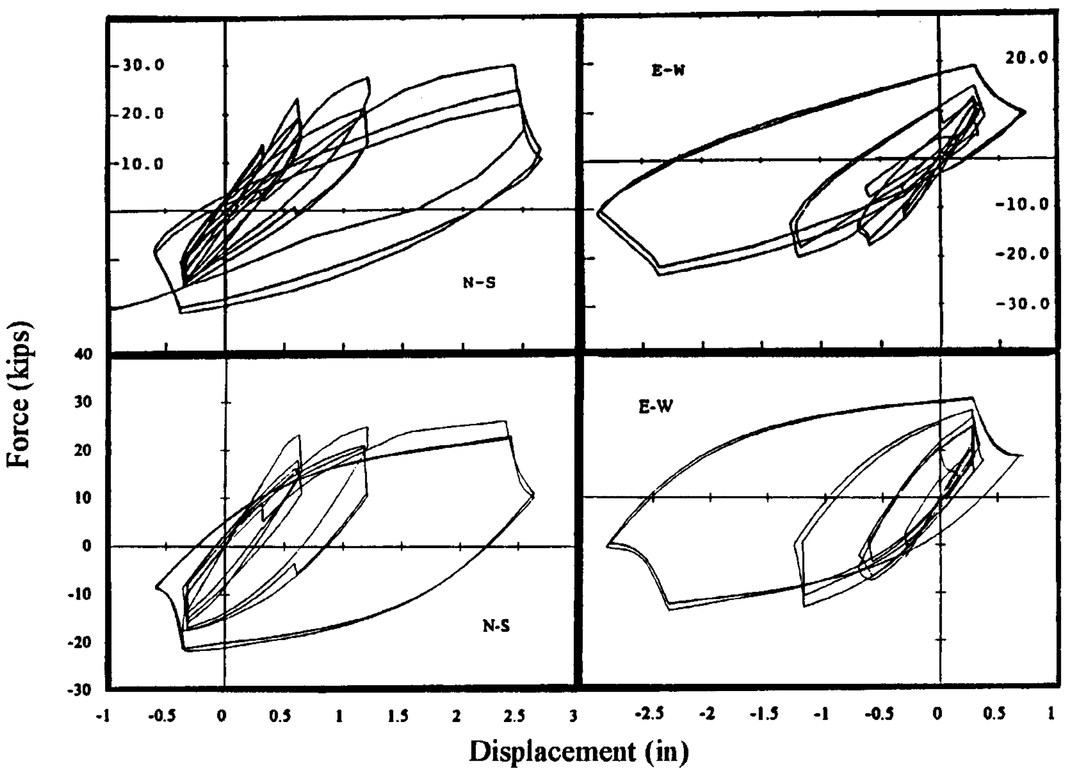
The equations shown above are used to compute the moment-curvature relationship. The force-displacement relationship may be computed by integrating the moment-curvature relationship along the column. Four displacement components may be identified: the elastic flexure deformation, the plastic flexure deformation, the elastic shear deformation and the plastic shear deformation. The flexure and shear deformation may be readily calculated. The plastic flexure deformation is computed on the assumption of a second degree parabolic inelastic curvature distribution, as proposed by Mander et al. (1984). The plastic shear deformation was not included in this investigation as no inelastic biaxial shear model has been developed yet. Plastic shear deformation are relevant, nevertheless, only for very short columns.

VALIDATION OF THE MODEL

To verify the model proposed herein, two different experiments performed by Otani and Cheung (1981) were used. The columns tested (specimens SP-7 and SP-8) by Otani and Cheung had a 305 mm square cross-section and a cantilever length of 1372 mm. The columns had 8 No. 7, grade 60, longitudinal bars. The columns were detailed so that the flexure behavior would dominate. Specimen SP-7 was subjected to an approximately square displacement pattern, in all directions, at the top of the column. Specimen SP-8 was also subjected to an approximately square displacement pattern, but the maximum displacement occurred in the east and north direction. Both columns were tested without axial load. As shown in Fig. 3 the agreement between the model and the experiment is very good. It may be necessary to note that no intent was made to get the best match, as normal average values were used in the constitutive models, because the necessary data to match the material properties is not available, so average values were adopted. It may be possible to get a better agreement fine-tuning the parameters, but the purpose of this investigation was not to match exactly some experiments, but rather to develop an analytical tool for the simulation of column behavior under axial load and biaxial flexure. No validation was made for circular columns as no experimental data was found in the literature.



(a) Otani and Cheung Specimen SP-7



(b) Otani and Cheung Specimen SP-8

Fig. 3 Force-Displacement Relationships in the NS and EW directions

CONCLUSIONS

Starting from first principles the basic equations of biaxial flexure are derived in both rectangular and polar coordinates. A rectangular fiber element was implemented to model the moment-curvature behavior of columns under biaxial loading. A circular trapezium element adequate for circular columns was also implemented. The force-displacement behavior of the column may be obtained from the moment-curvature relationship. This procedure was implemented and proved to be effective in simulating the force-displacement behavior of reinforced concrete columns subjected to biaxial lateral loads. In general terms, the model herein proposed may be used to simulate an experiment and to assess the fatigue damage of columns.

Within the context of a three-dimensional earthquake analysis procedure, the Fiber-Element model proposed, used as a pre-processor, can provide the data to calibrate biaxial macro models used in three-dimensional non-linear dynamic analysis programs. As a post-processor the model may be used to assess the fatigue damage in members for a given deformation history.

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