



ABOUT VULNERABILITY OF MASONRY STRUCTURES

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ABSTRACT

Due to the wide diffusion in Italy of masonry buildings and their old age and deterioration, the vulnerability of such structural systems, in order to determine the size of the risk connected to seismic events, is of interest. Moreover, this interest has become important considering that it involves the Italian cultural and artistic heritage (ancient churches, monuments, etc.). The paper describes a method to determine the level of vulnerability of ancient masonry structures to earthquakes, in order to organise only the right structural repairs, by studying the relationship between damage and seismic intensity. The level of general damage produced is linked to the modal parameters of structures so that it can be evaluated when the latter quantities have been determined. The dynamic characteristics, together with all the other parameters gathered in situ such as mechanical characteristics, geometry, etc., are used to formulate an f.e.m. for dynamic time history analyses and subsequent modal analyses to determine the damage.

KEYWORDS

Monumental masonry structures; material deterioration; structure deterioration; damage detection; vulnerability evaluation; modal analysis; degree of vulnerability; analytical expression of damage.

INTRODUCTION

Most of the Italian building heritage is composed by masonry buildings. During the last few years, due to the fact that many of them are historical monumental buildings and due to greater propensity to seismic damage, the evaluation of their seismic vulnerability has become more and more important. Seismic risk has become greater because of the ageing of the materials and the structures. Over the years the former have been losing their original stiffness and strength because of previous loads and chemical and atmospheric agents (Knight 1993). The structures have been losing the unity of their parts because of deterioration of the materials (connections between vertical structural elements and between vertical and horizontal ones). The present methodologies to evaluate seismic vulnerability are based on empirical data gathered in situ by means of forms. The data obtained are subjected to personal judgements in order to determine the degree of vulnerability. The data obtained in situ refer to structural kind, to the degree of constraint among structural elements, to the actual distribution of cracks and to anything that could

be useful to create a judgement that expresses the propensity of a structure to seismic effects by numbers or by words. This is an economical way to evaluate vulnerability. Nevertheless it is very sensitive to subjectiveness of judgement (semiquantitative method) This methodology to evaluate vulnerability is used most of all for common masonry buildings in order to create a priority scale upon which to organise work for the reduction of the risk level. The formulation of an accurate analytical model of the structures, able to simulate the behavior of materials and to subject them to seismic inputs with the characteristics of real seismic events, is a more rigorous approach to the problem of evaluating seismic vulnerability (quantitative method). Actually it is possible to predict the real response of the structure and the location of the most damaged areas so as to design only local interventions that do not disturb the architecture of the structure. This is very important for historical monumental buildings.

SEISMIC VULNERABILITY AND REPRESENTATION

Some ways to represent the seismic vulnerability of a structure are described widely by Corsanego (1984), Benedetti and Petrini (1984), Gavarini and Nisticò (1991), Hwang and Jaw (1990). One of these consists in creating an expected damage (D) versus seismic intensity (H) curve (Corsanego, 1984) by means of a function whose kind is:

$$D=V(H) \quad (1)$$

In a Cartesian plane the function (1) draws a curve whose name is vulnerability curve (Fig. 1-a).

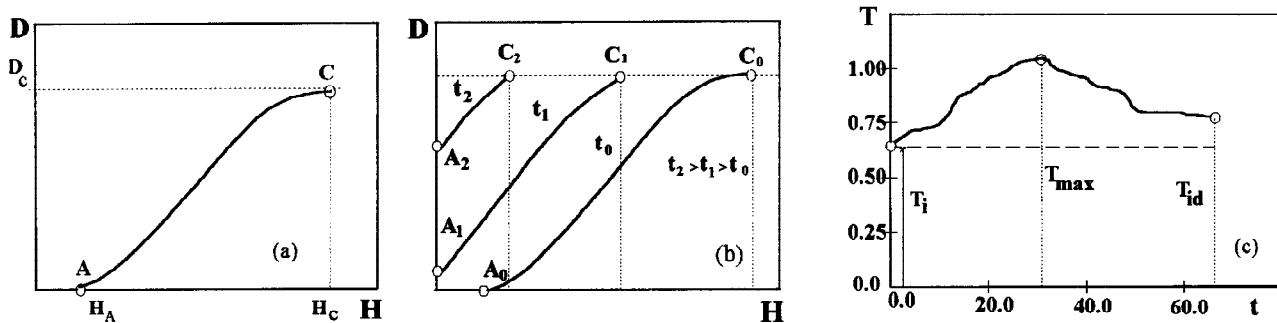


Fig.1: a) Vulnerability curve; b) Vulnerability curves for different time instants; c) Evolution of i-th equivalent natural period during earthquake.

The vulnerability curve has different locations on the Cartesian plane because of different inclination of structures to be damaged. In Fig.1-a H_A characterises the seismic intensity which produces the first damage, while H_C characterises the seismic intensity which produces the collapse of the structure. The difference between H_C and H_A depends on structural ductility. The damage causes the vulnerability curve to translate in the direction of abscissa axis towards the origin (Fig.1-b), so it is possible to have $D \neq 0$ when $H=0$ with a reduction of difference H_C-H_A . This means that with time the collapse of structures is associated with weaker and weaker earthquakes. Notice that seismic intensity in this paper is identified conventionally with the peak ground acceleration of the earthquake (PGA).

DAMAGING AND BEHAVIOR OF DAMAGED STRUCTURE

A structure becomes damaged if with time the constitutive materials change their physical state because of different causes. First porosity grows, then micro cracks develop and become big cracks. This damaging evolution is very common in masonry structures. When the structural damage increases, the parameters that define the structural response to every kind of load change. These parameters are global

stiffness and then natural periods that are proportional to eigenvalues, damping factor and mode shape (eigenvectors) (Bertero and Bresler, 1977). By monitoring a structure during an earthquake it is possible to note that the natural periods of a fictitious structure instantaneously equivalent to the real one change qualitatively as is shown in Fig.1-c (DiPasquale et al. 1990). Unlike what happens during strong motion, during the initial and final stages of the earthquake, the real structure and the fictitious one are the same because the amplitude of earthquake oscillations is not so great as to cause the non-linear behavior of the structure. So the initial and final recorded periods are real natural periods of the structure. The initial natural periods could be different from final one and the difference $(T_{id}-T_i)$ is a measure of structural damage, which shows itself with loss of stiffness. During strong motion the i-th natural period of the structure, instantaneously equivalent to the real one, increases more than T_{id} because non-linear behaviour and soil-structure interactions are superimposed on stiffness loss. Some in situ experiments on masonry structures (Foraboschi, 1993) have shown that for seismic intensity having the same order of greatness provided by EC8, a 50% increase of initial fundamental period during strong motion was followed by a recovery of stiffness in the final stage with a final difference between initial period and final period of 10%. Other authors have been interested in the dynamic behaviour of masonry structures to know their characteristics by means of experimental tests (Gulkan et al.,1990; Moghaddam et al., 1990; Benedetti et al., 1987; Vestroni et al.,1990).

The amplitude of changes in eigenvectors and eigenvalues depends on the severity and location of damage, so every cause of damage will affect every natural mode in a different way, having strong effects on some modes and weak effects on others (Hearn and Testa, 1991). The difference in the effects on different modes is the basis for locating the damage. If damage is the variation in structural stiffness with respect to the initial one it is possible to find simple quantitative relations to represent it. Variations in structural stiffness (ΔK) produce variations in the eigenvectors ϕ_i , which become $\phi_i+\Delta\phi_i$, and in the squared natural frequencies ω_i^2 , which become $\omega_i^2+\Delta\omega_i^2$. So, in damaged conditions, the eigenvalues problem for free vibrations of a structure is governed by the relation (Hearn and Testa, 1991):

$$[(K+\Delta K)-(\omega^2+\Delta\omega^2)(M)](\phi+\Delta\phi)=0 \quad (2)$$

Premultiplying the first and second members by ϕ^T relation (2) becomes:

$$\phi^T\Delta K\phi+\phi^T\Delta K\Delta\phi-\Delta\omega^2\phi^T M\phi-\Delta\omega^2\phi^T M\Delta\phi=0 \quad (3)$$

If $\Delta\phi$ is small enough it is possible to neglect $\phi^T\Delta K\Delta\phi$ with respect to $\phi^T\Delta K\phi$ and $\phi^T M\Delta\phi$ with respect to $\phi^T M\phi$ so for i-th eigenvalue we obtain:

$$\Delta\omega_i^2 = \frac{\phi_i^T \Delta K \phi_i}{\phi_i^T M \phi_i} = \frac{\sum_N \varepsilon_N^T(\phi_i) \Delta k_{N \varepsilon_N}(\phi_i)}{\phi_i^T M \phi_i} \quad (4)$$

where Δk_N and $\varepsilon_N(\phi_i)$ are respectively the variation in the matrix stiffness and the deformation vector computed from the natural mode shape ϕ_i of the n-th individual member that composes the structure. Equation (4) shows that it is possible to compute the variation in the frequency of the i-th eigenvector taking in account only the variation in stiffness if the i-th eigenvector is small enough. The last member of eq. (4) shows that the variation in ω_i^2 depends on the location of damage so that a modal shape which has a great $\varepsilon_N(\phi_i)$ where there is a great Δk_N will be more subjected to change in its frequency than other modal shapes whose deformation energy is concentrated where Δk_N is small. Making use of (4) it is possible to express the structural damage as:

$$D = \frac{\phi^T \Delta K \phi}{\phi^T K \phi} = \frac{\Delta\omega^2}{\omega^2} = \frac{\omega^2 - \omega_d^2}{\omega^2} = 1 - \frac{T^2}{T_d^2} \quad (5)$$

Referring to the i -th mode (5) is written:

$$D_i = 1 - \frac{T_i^2}{T_{id}^2} \quad (6)$$

The index D changes with the direction of seismic event for the same intensity so, fixed the direction of earthquake, it is possible to evaluate as values of D as modes that we want take in account. The index D is limited superiorly by 1, for which a mechanism of local or global collapse is reached. To the reduction of structural stiffness is always associated a reduction of stiffness and strength of materials, so the use of correct constitutive law make us able, by means numerical analysis, to predict the damage for assigned seismic input and the behaviour of structure after damage.

METHODOLOGY FOR DETERMINING THE VULNERABILITY LEVEL

From the previous considerations, an analysis method for the construction of a vulnerability curve is proposed. The method comprises a theoretical stage and an experimental one. The first stage is focused on obtaining the real modal parameters of the structure and mechanical parameters of materials in situ. In the second stage, after formulation of an f.e.m. whose theoretical modal parameters are the same as the real ones gathered in situ, for every seismic input with different peak ground acceleration and fixed direction, time history dynamic analysis is executed, after which modal analysis is executed to determine the level of damage produced D . Modal analysis, executed for determination of eigenvalues from the first to the i -th, allows one to measure structural damage in a concise way. Comparison before and after damage between eigenvalues of eigenvectors which remain qualitatively similar before and after deterioration, by means of (6), is a measure of the distance of the structure from collapse and of localisation of collapse mechanism. After modal analysis it is possible to evaluate the residual strength of the structure. The steps described are repeated when a new seismic input direction is fixed. By means of this procedure a family of curves is obtained, whose envelope curve can be used to express the vulnerability of a structure.

Material model

The finite element model for time history dynamic analysis needs a correct material model that takes into account as much as possible the real mechanical characteristics. The latter are very variable and depend on construction standards, the quality of mortar and the quality and the degree of manufacture of the stone. Masonry sometimes has the behaviour of homogeneous orthotropic material and sometimes that of homogeneous isotropic material. In the first case masonry shows regular stratification of component materials (stone squared masonry); in the second case masonry is made of mortar and non-squared stones that are jointed in a casual way. The degree of homogeneity is definable every time statistically. In the first and in the second case too the material has a behaviour that is well reproduced by the model of Bathe (1995), which is developed in the ADINA analysis code.

Tests on structure and materials

The experimental tests are executed by means of different techniques (Chiostrini and Vignoli, 1994; Chiostrini et al., 1992). For characterisation of the masonry constitutive law it is necessary to make tests on specimens of the material gathered in situ and on structural elements made of the same materials as the real structure. The latter tests can be executed in a laboratory. Moreover it is necessary to execute in-situ testing with the flat-jack technique to get further information. The experimental program comprises recording of the dynamic response of structure to low intensity forcing (environmental noise) in order to know the modal parameters of the structure without producing further damage. The modal

measurement has the purpose of defining the mechanical behaviour of the structure and materials by the identification approach. Moreover, it is possible to have the initial modal parameters of the structure for evaluation of vulnerability in the way later described. Usually, even ancient masonry buildings and very damaged masonry structures present a range of linear response that allow measurements. Otherwise the modal parameters would change because of a continuous deterioration. Anyway, the linearity of the response can be easily verified, if the input is known, by the coincidence of transfer function obtained from different width vibrations. For modal parameter measurements, vehicular traffic has proved suitable for exciting both slender and squat buildings; in particular, if the source of excitement is nearby, complete buildings can be excited even if definitely squat.

Seismic input

It can be obtained from records of real earthquakes. However, this procedure neither grasps the uncertainty in future earthquakes nor reflects the local site conditions. These concerns give rise to the use of synthetic earthquake time histories to represent ground motion. In this study, synthetic earthquakes are generated from an appropriate power spectrum density (PSD). The stationary acceleration time history $a_s(t)$ is simulated by the expression of Shinozuka (1974):

$$a_s(t) = \sum_{k=1}^n \sqrt{2S(\omega_k)\Delta\omega} \cos(\omega_k t + \phi_k^r) \quad (7)$$

where $S(\omega_k)$ = is one-sided earthquake power spectrum ; n = number of frequency intervals; $\Delta\omega = \omega_{\max}/n$ with ω_{\max} as cut-off frequency; $\omega_k = k\Delta\omega$; and ϕ_k^r = k -th random phase angle, which is uniformly distributed between 0 and 2π . For numeric analysis the PSD can be chosen to generate accelerograms compatible with the EC8 elastic spectrum. The time history $a_s(t)$ can be modulated in the first 0.1 t ($t=25$ sec) by means of the following function (Premount, 1984):

$$\xi(t) = 0.5[1 - \cos(\pi t/0.1t)], \quad (8)$$

and normalised by the absolute maximum of time history a_{\max} to obtain, finally, the non-stationary normalised time history $a_{nm}(t)$. The time history of every analysis is obtained from the product of a specific peak ground acceleration H_i and $a_{nm}(t)$:

$$a_i(t) = H_i \times a_{nm}(t) \quad (8)$$

MODAL ANALYSIS OF A F.E.M. WITH DIFFERENT DEGREES OF DAMAGE

A modal analysis was executed which was extended to the first fifteen modes of a typical aisleless church structure. The finite element model is drawn in Fig. 2. Plate-shell finite elements was used for the vertical wall while the roof was neglected because it is generally made of timber with much smaller stiffness than the other parts of the structure. Nevertheless, a mass equivalent to that of the timber roof was added to the other masses for correct computation of the mass matrix of the model. Modal analysis was executed first on the undamaged structure and then on the damaged one with different degrees of damage (Model D1, Model D2, Model D3 of Fig.2) localised where it is foreseen to bring about the collapse mechanism of longer wall out of its plane. Modal analysis gave the results in Fig.3. The damage was simulated in f.e.m. by means of proper reduction of Young's modulus E of material. Young's modulus of undamaged material was fixed at 20.000 Kg/cm² as is most frequently found in situ for masonry structures. The values of D (table 1) were computed for different hypotheses of damage. These values of D were located on the D-H plane for fictitious values of seismic intensity to visualise better the variation in each mode with respect to the others (Fig.4). The analyses show that, for different degree of

damage, the variation of every mode shape is contained in values small enough even for the most damaged model. It renders valid the hypothesis of writing the relation (5).

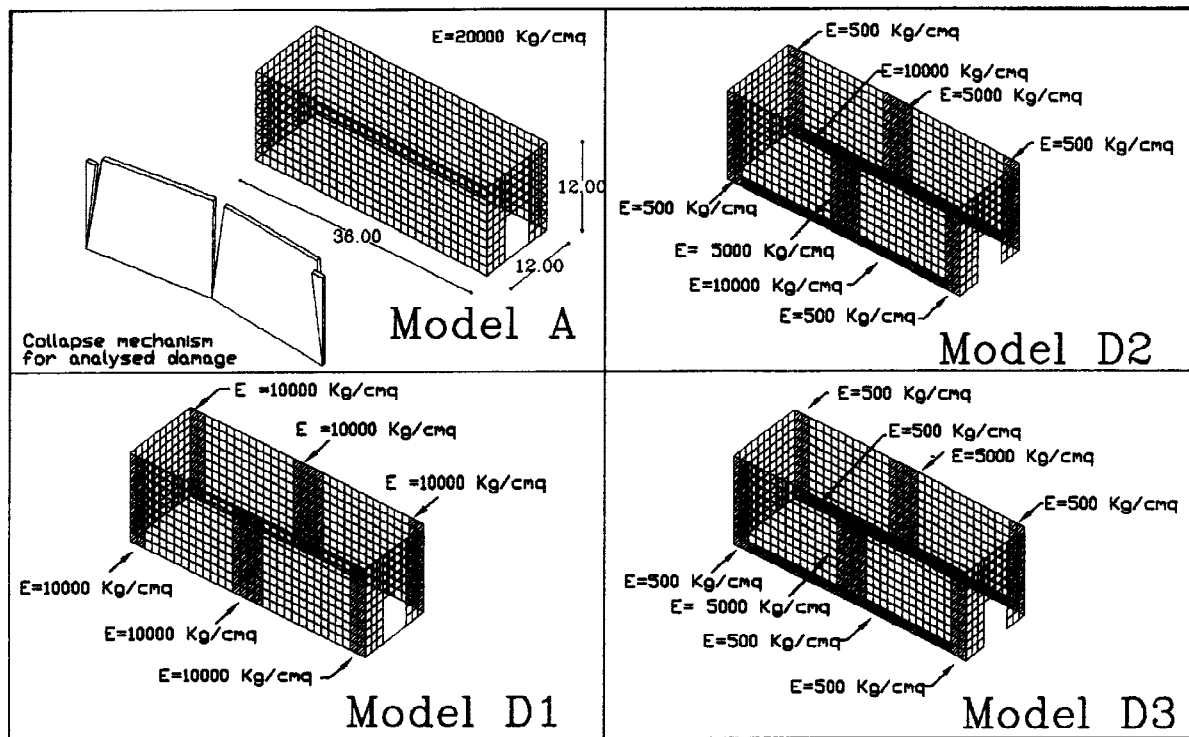


Fig.2. F.E.M. of aisleless church structure with different levels of damage

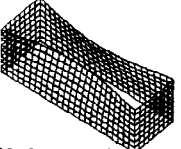
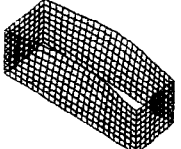
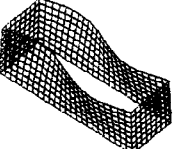


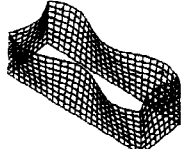


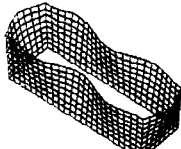

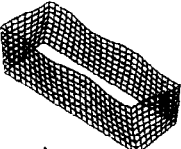

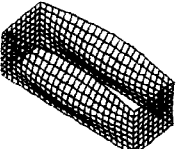
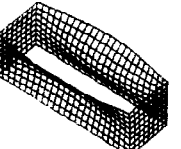
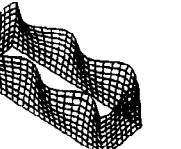
MODE 1	MODE 2	MODE 3	MODE 4	MODE 5
 Model A - $\omega_a = 2.08$ Model D1 - $\omega_{d1} = 1.84$ Model D2 - $\omega_{d2} = 1.52$ Model D3 - $\omega_{d3} = 0.93$	 Model A - $\omega_a = 2.11$ Model D1 - $\omega_{d1} = 1.85$ Model D2 - $\omega_{d2} = 1.51$ Model D3 - $\omega_{d3} = 0.92$	 Model A - $\omega_a = 3.61$ Model D1 - $\omega_{d1} = 3.22$ Model D2 - $\omega_{d2} = 2.41$ Model D3 - $\omega_{d3} = 1.95$	 Model A - $\omega_a = 3.62$ Model D1 - $\omega_{d1} = 3.25$ Model D2 - $\omega_{d2} = 2.45$ Model D3 - $\omega_{d3} = 2.01$	 Model A - $\omega_a = 5.49$ Model D1 - $\omega_{d1} = 4.98$ Model D2 - $\omega_{d2} = 3.70$ Model D3 - $\omega_{d3} = 3.34$
MODE 6	MODE 7	MODE 8	MODE 9	MODE 10
 Model A - $\omega_a = 5.67$ Model D1 - $\omega_{d1} = 5.13$ Model D2 - $\omega_{d2} = 3.96$ Model D3 - $\omega_{d3} = 3.69$	 Model A - $\omega_a = 6.51$ Model D1 - $\omega_{d1} = 5.83$ Model D2 - $\omega_{d2} = 3.55$ Model D3 - $\omega_{d3} = 2.50$	 Model A - $\omega_a = 6.76$ Model D1 - $\omega_{d1} = 6.14$ Model D2 - $\omega_{d2} = 5.03$ Model D3 - $\omega_{d3} = 2.80$	 Model A - $\omega_a = 7.89$ Model D1 - $\omega_{d1} = 6.78$ Model D2 - $\omega_{d2} = 3.56$ Model D3 - $\omega_{d3} = 3.38$	 Model A - $\omega_a = 8.35$ Model D1 - $\omega_{d1} = 7.55$ Model D2 - $\omega_{d2} = 6.11$ Model D3 - $\omega_{d3} = 4.05$
MODE 11	MODE 12	MODE 13	MODE 14	MODE 15
 Model A - $\omega_a = 9.07$ Model D1 - $\omega_{d1} = 8.04$ Model D2 - $\omega_{d2} = 5.66$ Model D3 - $\omega_{d3} = 4.72$	 Model A - $\omega_a = 9.25$ Model D1 - $\omega_{d1} = 8.17$ Model D2 - $\omega_{d2} = 5.60$ Model D3 - $\omega_{d3} = 4.3$	 Model A - $\omega_a = 9.75$ Model D1 - $\omega_{d1} = 8.37$ Model D2 - $\omega_{d2} =$ Model D3 - $\omega_{d3} = 4.55$	 Model A - $\omega_a = 9.75$ Model D1 - $\omega_{d1} = 8.35$ Model D2 - $\omega_{d2} =$ Model D3 - $\omega_{d3} =$	 Model A - $\omega_a = 11.17$ Model D1 - $\omega_{d1} = 9.93$ Model D2 - $\omega_{d2} = 6.80$ Model D3 - $\omega_{d3} = 5.75$

Fig.3. Modal shape and frequencies for the different degrees of damage

Table 1 and Fig.4 show that for model D1 the values of D referring to the first fifteen modal forms are contained in a short interval, while they are contained in bigger intervals for models D2 and D3. This means that light damage is not easy to locate while big localised damage is easier to detect because it produces bigger variation in the eingenvvalue, whose eingenvector concentrates its elastic strain energy in the damaged parts of the structure. So, the light concentrated damage is detected like uniformly distributed damage. From table 1 it can be noted that shapes 13 and 14 in Fig.3 have shifted beyond the fifteenth natural mode for models D2 and D3 so it cannot be computed D for that damage condition.

Table 1. Values of D obtained

	Model D1	Model D2	Model D3
Mode 1	0.21	0.47	0.80
Mode 2	0.22	0.48	0.80
Mode 3	0.20	0.55	0.70
Mode 4	0.19	0.54	0.69
Mode 5	0.18	0.54	0.62
Mode 6	0.18	0.51	0.57
Mode 7	0.19	0.70	0.85
Mode 8	0.17	0.44	0.82
Mode 9	0.22	0.78	0.80
Mode 10	0.18	0.46	0.70
Mode 11	0.21	0.61	0.73
Mode 12	0.22	0.63	0.78
Mode 13	0.26		0.78
Mode 14	0.26		
Mode 15	0.21	0.63	0.78

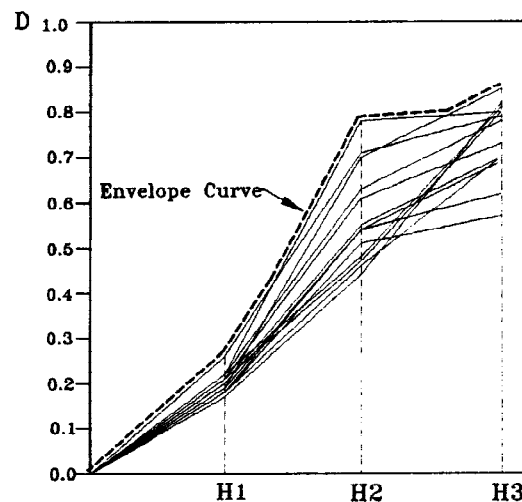


Fig.4. Fictitious vulnerability curves

The values of D obtained with model D1 (light damage) are not very different from their average so a mean value is representative of that damage and a single value can be used to represent the vulnerability of a structure for fixed direction and intensity of synthetic earthquake. On the other hand, for more marked damage, the vulnerability curves become more and more distant from each other. In particular for model D2 it is possible to note that D varies from 0.45 (modal shape 8) to 0.8 (modal shape 1) and for model D3 it is noteworthy that D varies from 0.55 (modal shape 6) to 0.82 (modal shape 7). It is clear that for model D3 the natural frequencies tend to be contained in smaller intervals than for model D2. To represent the vulnerability of a structure for fixed direction of seismic event with respect to the structure the envelope curve can be chosen. Nevertheless, to locate the damage it is necessary to compare every time the envelope curve with the vulnerability curve obtained for every single mode.

CONCLUSIONS

The method described in this abstract makes it possible to estimate the vulnerability of masonry structures by means of theoretical analysis and experimental tests. The experimental tests are necessary to obtain a f.e.m. with the real characteristics of the structure without producing further damage. This is suitable for ancient monumental masonry structures. The determination of vulnerability, based on the variation in modal frequencies produced by damage, seems suited to predicting the rise of collapse mechanism and its location, as has been shown by theoretic analysis of a damaged masonry structure. The structure vulnerability is expressed by the envelope curve of the H versus D functions obtained for every mode shape up to the i-th.

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