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THE EFFECTS OF STEEL REINFORCING IN THE STATIC ANALYSIS OF MASONRY BUILDINGS UNDER HORIZONTAL ACTIONS

D. ABRUZZESE   M. COMO   G. LANNI

Dept. of Civil Engineering , University of Rome "Tor Vergata"
Rome, ITALY

ABSTRACT

The most common reinforcement system of the historical masonry buildings consists in the insertion into the original structure of suitable horizontal devices as steel ties passing through the piers and running inside the floors with anchor plates at the ends. Aim of the paper is analyze the lateral strength of this kind of building. This study, in line with previous papers, will be performed in the framework of the limit analysis theory by using the unilateral no tension model for the masonry material. A suitable stress analysis will be developed in order to determine the axial load in the horizontal connections at the ultimate state of the structure.

KEYWORDS

Masonry structure, limit analysis, seismic behaviour, reinforcing systems

INTRODUCTION

A double order of orthogonal multistory walls with regular openings and weakly connected to wooden or iron beam floors is the main resistant structure of a typical masonry building. Such a building is extremely vulnerable to seismic action and its collapse occurs, as a rule, with out of plane failures of masonry walls. The most common system of reinforcement used to improve the seismic strength of this building consists in the insertion into the original structure of steel ties passing through the piers and running inside the floors with anchor plates at the ends. With this system of reinforcement the out of plane collapse of the walls can be avoided and the inplane strength of the walls can be fully exploited.

According to the different role played, the walls can be considered as "neutral" or "active" according to whether they are subjected to out of plane or inplane horizontal forces. The seismic horizontal forces move from the neutral to the active walls: the horizontal forces corresponding to the masses of the floors and of the neutral walls are transmitted to the active ones by means of the horizontal arches that set up inside the neutral walls at the floor levels. This load transfer is made possible by the presence of the steel ties. This reinforcement at the same time improves the inplane strength of the walls. Aim of this paper is to analyze the influence of the reinforcement in order to improve the lateral strength of the whole masonry building. We will analyze first the inplane strength of the plain or reinforced masonry wall that is the main resistant element of the masonry building. Then the lateral strength under horizontal forces of the tridimensional structure of the whole masonry building will be studied. The fundamental role played by the reinforcing system will be thoroughly examined. The masonry material is assumed tensionless and rigid in compression according to the pioneering studies of Heyman on the masonry arch (J.Heyman, 1966 and 1969).

LATERAL STRENGTH OF PLAIN OR TIE REINFORCED MASONRY WALLS

Let us consider a plane multistory wall with a regular array of openings (fig.1): the wall has N_s stories and N_p piers . The piers are linked by masonry architraves, wich are able to support only compressive forces. The wall is subjected to the action of fixed dead loads and to gradually increasing horizontal loads, representative of the seismic action, for instance acting from the left to the right. In this case we will call the left side the "up-quake" side of the wall; the right one the "lee-quake" side. The failure of the single pier occurs with a rotation mechanism at its toe; then the lateral strength of the single pier depends on the uplifting action of the weights that occurs during this rotation (Como and Grimaldi, 1984).

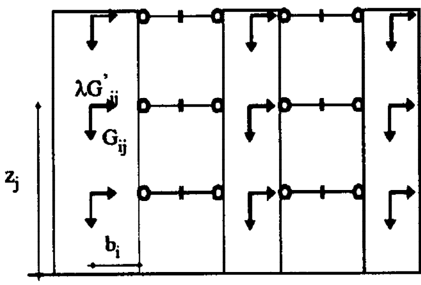

fig. 1

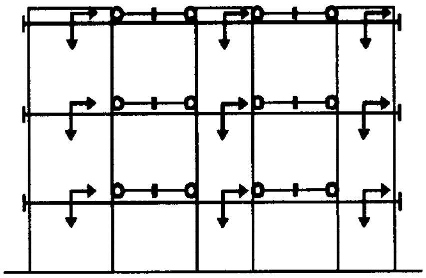

fig. 2

The inplane failure of the plain wall can occur with a sidesway mechanism involving a group of piers starting from the lee-quake side: the group of the remaining piers, to which belongs the up-quake pier, consequently will stand in vertical position. Fractures across the architraves will separate the two different groups. In fact the architraves are able to support only compressive forces and in the masonry walls, usually characterized by piers of large width, the elastic strains can be negligible. We can take therefore into account only the deformations due to the fractures. Consequently under the action of the seismic horizontal forces, the piers will remain rigid as long as the local overturning mechanism does not occur. Of course in this scheme of inplane collapse of the plain wall, it is possible the collapse of the single lee-quake pier (fig.3) or the collapse of the whole wall (fig.4).

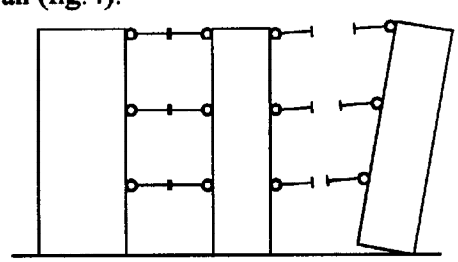

fig. 3

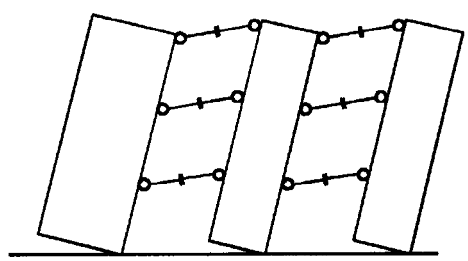

fig. 4

At a stage of the loading the following forces act on the i-th pier (fig.1):

- the vertical dead load G_{ij} applied at the storey j, representing the weight of the pier and of the floor sustained by the pier. The positions of the loads G_{ij} are defined by the arms b_{ij} evaluated with respect to the bottom right toes;
- the horizontal seismic loads $\lambda G'_{ij}$, including the rate due to the inertial forces G_{ij} and the rate due to the forces that are transferred by the neutral walls. The elevations of the stories where these forces are applied are z_j.

Each masonry pier is characterized by its proper lateral strength λ_{0i} defined by the following limit value of the multiplier λ:

$$\lambda_{0i} = \frac{M_i^S}{M_i^R} \tag{1}$$

where:

$$M_i^S = \sum_{j=1}^{N_s} G_{ij} b_{ij} \qquad M_i^R = \sum_{j=1}^{N_s} G'_{ij} z_j \tag{2}$$

if M^S_i, M^R_i , respectively represent the stabilizing and the overturning moment around the toe of all the forces acting on the i-th pier. The collapse multiplier of the plain wall is thus represented as:

$$\lambda_0 = \overline{\lambda}_{0r} = \underset{k=1,\dots,r,.N_p}{\mathrm{MIN}} \left\{ \overline{\lambda}_{0k} \right\} \tag{3}$$

if $\overline{\lambda}_{0k}$ is the failure multiplier of the "k lee-quake" group of piers:

$$\overline{\lambda}_{0k} = \frac{\sum_{k}^{N_p} M_i^S}{\sum_{k}^{N_p} M_i^R} \tag{4}$$

Thus from equation (3), when $r = N_p$, the collapse of the plain masonry wall occurs with the overturning of the last pier, i.e. the lee-quake pier; on the contrary , when $r = 1$ the collapse of the wall involves the sidesway mechanism of the whole wall.

In presence of steel ties that connect the piers the collapse occurs with a global sidesway mechanism involving the overturning of all the piers. The steel ties transfer the horizontal forces to the strongest piers, producing an equalization of the seismic loads, because the steel gives tensile strength to plain architraves. The global sidesway mechanism takes different forms according to whether the height of the architraves is or not negligible with respect to the height of the storeys. In the first case (fig.2) the failure sidesway mechanism of the wall is characterized by equal toe rotation of all the piers; the collapse multiplier is given by:

$$\lambda_0 = \frac{\sum_{1}^{N_p} M_i^S}{\sum_{1}^{N_p} M_i^R} \tag{5}$$

and the stress in the steel ties at the collapse wall can be easily evaluated with the procedure shown in (Como *et al.*, 1991), (Abruzzese *et al.*, 1992).

On the contrary, when the architraves height is not negligible with the storey height, i.e. the horizontal connections among the piers are bidimensional panels, the sidesway mechanism requires the development of plastic strains in the ties and new other fracture in the piers with a remarkable increment of the strength. In order to evaluate this effect let start to consider the elementary scheme of wall as shown in fig.5.

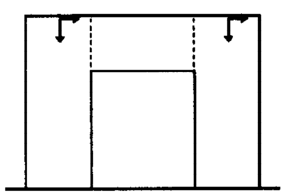

fig. 5

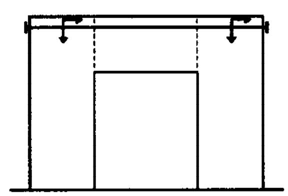

fig.6

Without steel ties the horizontal strength of the system is due only to the opposition of the weight to the overturning of the weakest pier. The corresponding collapse mechanisms are shown in fig.7, if the piers are different or in fig.8 if the piers are equal.

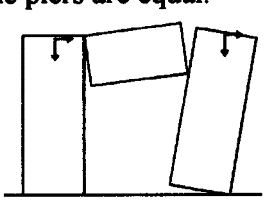

fig. 7

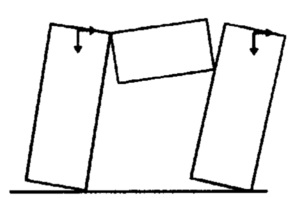

fig.8

If a reinforcing steel tie is present (fig.6) inspection of the figures 7 and 8 shows the double effect of the tie: in fact in the fig.8 the tie only opposes to the opening of the fracture between the panel and the right side pier, while in the case of the fig. 7 the tie also pulls to the overturning the left side pier. Anyway, in both cases the presence of the steel ties increases the value of the collapse multiplier respect to the values given by (3) or (5).

It is fundamental to know if the steel ties are or not in the plastic range at the collapse mechanism. Moreover, we can observe that if the tie were not yet plastic, the collapse mechanism should be that shown in fig.9. In this case the equilibrium of the right side pier over the fracture can exist only if in the tie bar there is a tension force T approximately of the same order of magnitude of the weight G. Since that is not possible, it can be concluded that the collapse multiplier can be computed with the assumption of yielding in the tie bar. In this case the collapse mechanism is that shown in the fig.10. The total strength to the horizontal actions is due to the uplifting of both weights and to the plastic work done by the yield tension T_0 in the tie bar.

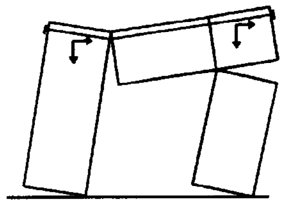
fig. 9

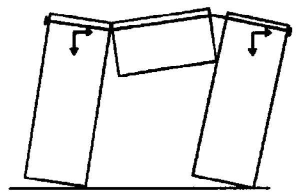
fig.10

The calculation of the collapse multiplier λ_0 can be performed by using the virtual work principle. Thus equating the acting work of the horizontal thrusts to the resisting work done by weights and tie bar forces, the governing equation can be obtained. It is easy to calculate the rotation φ_2 of the right side pier related to the rotation φ_1 of the left side one in the form:

$$\varphi_2 = k \cdot \varphi_1 \tag{6}$$

with $k \geq 1$ depending on the geometry of the structure. Then relating the value of φ_1 to the tie bar elongation Δ, the uplifting V of the weigths G and the horizontal displacements U of the thrusts, and applying the virtual work principle, we obtain, in the particular case $G_1 = G_2 = G_1' = G_2' = G$ and $b_1 = b_2 = b$:

$$\lambda_0 = \frac{b}{z} + 2\frac{T_0}{G}\frac{k-1}{1+k} \tag{7}$$

It can be pointed out the increment of the collapse multiplier given by (7), due to the tie bars, respect to that given by (5).

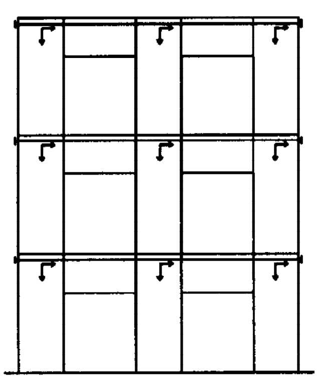
fig. 11

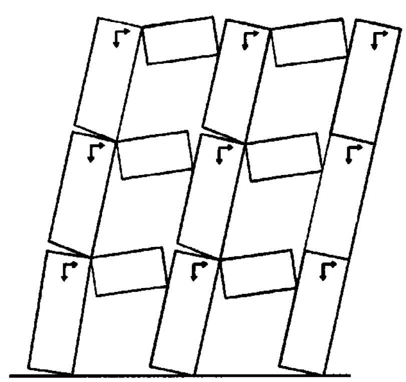
fig.12

In the most general case, the structural scheme presents more piers and stories as shown in fig.11. The collapse mechanism should have fractures at each floor and in each pier, except for the lee-quake pier having only the hinge at the toe (fig.12). If we call φ_{ij} the rotation of the i-th pier at the j-th storey, it is possible to relate all the rotations φ_{ij} to the one of the lee-quake pier adopting a step by step procedure (Abruzzese and Lanni, 1994). Then relating to the rotations φ_j the tie bar elongations Δ, the uplifting V of the weigths G and the horizontal displacements U of the thrusts, we calculate the collapse multiplier λ_0 by applying the virtual work equation.

LATERAL STRENGTH OF THE WHOLE MASONRY BUILDING

A double order of orthogonal walls, whose number is N_x and N_y, respectively along the directions x and y, is the main resistant structure of a typical masonry building. According to the various connections present between the walls the behaviour of the structure under seismic action can follow different schemes. The case of the masonry building reinforced with steel ties running parallel to the walls, with anchor plates at the heads, is very simple and common. With this system of reinforcement the out of plane collapse of the external walls can be avoided and the inplane strength of the single wall with the partecipation of all its piers can be fully exploited. In this case the global behaviour of the whole tridimensional structure of the building is strongly conditioned by the lack of connection between parallel walls. The strength of the whole building is therefore identified with the inplane strength of the weakest wall. If, on the contrary, parallel walls are suitably connected, by using diagonal steel ties, (Fig.13), or peculiar reinforced floors that behave as rigid horizontal diaphragms connected to the walls, there is full partecipation between parallel walls of the same array and the strength of the tridimensional structure of the whole building can be fully exploited. The collapse mechanism is now represented or by a single rotation of the floors around a vertical axis passing through the intersection of two orthogonal walls or by a simple translation of the floors.

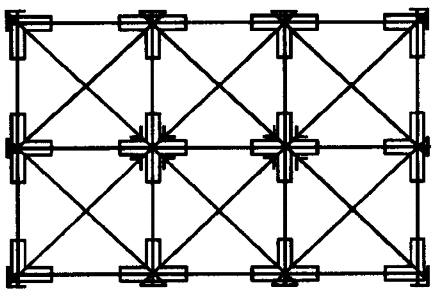

fig. 13

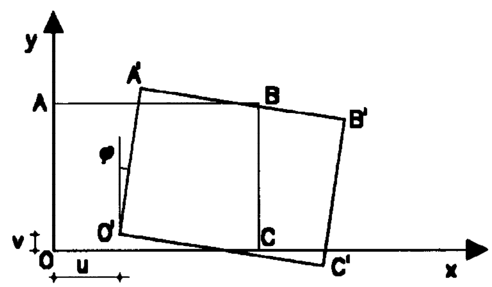

fig.14

In order to analyze the tridimensional collapse of the building, according to the formulation given by Abruzzese *et al.* (1986), with reference to fig. 14 let

$$\mathbf{D}_1^T = [u_1, v_1, \phi_1] \tag{8}$$

be the mechanism displacement row vector of the first floor whose height is z_1. The components u_1, v_1, ϕ_1 respectively represent. the translations along the orthogonal directions X and Y of the two arrays of walls and the floor torsional rotation. At the floor j, whose height is z_j, the displacement vector $\mathbf{D}_j$ can be obtained from the displacement vector of the first floor $\mathbf{D}_1$,whose height is z_1, by means of the assumption that displacements of the floors linearly increase with their floor heights,

$$\mathbf{D}_J = \frac{z_j}{z_1}\mathbf{D}_1 \tag{9}$$

Likewise, the external load acting on the building are represented by horizontal forces applied at the centers of gravity C of the various floors. Thus the load row vector applied to the building at the assigned floor is given by :

$$\mathbf{Q}^T = [F_x, F_y, M] = [F_x, F_y, F_x y_C - F_y x_C] \tag{10}$$

whose components respectively represent. the horizontal forces acting along the orthogonal directions X and Y and applied at the center O of the reference axes, and the couple M of these forces with respect to the reference center O. The loads acting to the level j will thus be given by

$$\mathbf{Q}_j = \sigma(j) \cdot \mathbf{Q} \tag{11}$$

where σ(j) is the distribution law of the forces along the height. It can be represented, for sake of simplicity, by

$$\sigma(j) = \frac{W_j z_j}{W_1 z_1} \tag{12}$$

Let the loads Q increase with the load parameter λ. The work of the external loads along the displacement D of the floors can thus be obtained as

$$\lambda \sum_j^{N_s} \mathbf{Q}_j^T \mathbf{D}_j = \lambda Q^T D \sum_j^{N_s} \sigma(j) \frac{z_j}{z_1} = \psi \lambda \mathbf{Q}^T \mathbf{D} \tag{13}$$

The deformation parameters of the walls are then defined, at the considered first floor, by the displacement d_{xj} , d_{yj} according to the wall is directed along the direction x or y. Afterwards, the displacements of the walls at the floor of height z_j are respectively

$$d_{x_j} = \frac{z_j}{z_1} d_{x_1} \qquad d_{y_j} = \frac{z_j}{z_1} d_{y_1} \tag{14}$$

The deformation of all the walls at the reference floor is thus given by the row vector

$$\mathbf{d}^T = [d_1, d_2, .., d_N] \tag{15}$$

with $N = N_x + N_y$

Once the various walls are ordered sequentially, in the vector **d** the first N_x components represent the displacement along X of the N_x walls directed along x while the remaining N_y components the displacement along Y of the N_y walls directed along y. The deformation of all the walls corresponding to the tridimensional deformation of the whole building can thus be obtained by the matrix equation

$$\mathbf{d} = \mathbf{C}\,\mathbf{D} \tag{16}$$

where **C** is the compatibility matrix that associates the inplane displacement d of the single walls to each global deformation vector **D** of the building.

The lateral strength of the single wall depends on the direction of the displacement. Thus two different values $S_1^{(0,+)}$ and $S_1^{(0,-)}$ are necessary to define the strength of the wall at the floor 1. At the floor j we will have correspondently $\sigma(j)\, S_1^{(0,+)}$ and $\sigma(j)\, S_1^{(0,-)}$. Let

$$\mathbf{r}^{oT} = [S_1^{(o,+)}, S_1^{(o,-)}; S_2^{(o,+)}, S_2^{(o,-)}; \ldots S_N^{(o,+)}, S_N^{(o,-)}] \tag{17}$$

be the row vector of the total lateral strengths of the various walls. In correspondence of a chosen collapse mechanism **D** of the building there are an internal deformation vector **d** of the walls and an internal stress vector $\mathbf{s}_o$ of the strengths effectively activated by the mechanism. This last vector can be obtained by the total strength vector by means the equation

$$\mathbf{s}_o = \Gamma(\mathbf{d}) \mathbf{r}_o \tag{18}$$

where **Γ(d)** is a matrix which choses the values of the activated limit strengths **S** depending on the direction of the inplane displacements of the single walls. To each deformation vector **D**, that represents a collapse mechanism of the building, corresponds an internal resisting work D_w :

$$D_w = \mathbf{s}_0^T(\mathbf{d})\mathbf{d} = [\Gamma(\mathbf{d})\mathbf{S}_0]^T \mathbf{d} = \mathbf{S}_0^T [\Gamma(\mathbf{d})]^T \mathbf{CD} = [\mathbf{\Phi}(\mathbf{D})]^T \mathbf{D} = \Phi_1(\mathbf{D})u + \Phi_2(\mathbf{D})v + \Phi_3(\mathbf{D})\phi \tag{19}$$

where $\Phi_1(\mathbf{D})$, $\Phi_2(\mathbf{D})$, $\Phi_3(\mathbf{D})$ represent the global limit strength component of the building, activated by the global displacement **D**, respectively corresponding to the two translations u and v and to rotation ϕ of the floors. For any tridimensional mechanism the corresponding kinematical load multiplier λ is thus given by:

$$\lambda(\mathbf{D}) = \frac{\Phi_1(\mathbf{D})u + \Phi_2(\mathbf{D})v + \Phi_3(\mathbf{D})\phi}{\psi(F_x u + F_y v + M\phi)} \tag{20}$$

The multiplier $\lambda(\mathbf{D})$ depends on the chosen global mechanism **D**. Thus the collapse multiplier is given by

$$\lambda_0(\mathbf{D}) = \inf_{M} \lambda(\mathbf{D}) \tag{21}$$

i.e. as the upper lower bound of the kinematical multipliers $\lambda(\mathbf{D})$ in the set of the global mechanisms M.

Let us consider the case of lateral loads acting only along the direction x, i.e. the load row vector

$$\mathbf{Q} = [F_x, \quad 0, \quad F_x y_c] \tag{22}$$

corresponding to the assigned floor level. At the same time a displacement row vector corresponding to the rotation of the floor around a center Ω having coordinates x_Ω and y_Ω with respect to the reference axes Oxy is given by

$$\mathbf{D}^T = \phi \cdot [-y_\Omega, x_\Omega, 1] \tag{23}$$

and the displacement floor row vector to which corresponds an unit work of the floor forces is

$$\mathbf{D}^{*T} = \frac{[-y_\Omega, x_\Omega, 1]}{F_x (y_C - y_\Omega)} \tag{24}$$

Let S* be the set of the mechanisms to which corresponds unit work of the floor loads..Thus the collapse load multiplier λ_0 is given by

$$\lambda_0 = \frac{1}{\psi} \inf_{S^*} \{(\Phi_1(\mathbf{D}^*)u + \Phi_2(\mathbf{D}^*)v + \Phi_3(\mathbf{D}^*)\phi \} \tag{25}$$

where now

$$u = -\frac{y_\Omega}{F_x (y_C - y_\Omega)}; v = \frac{x_\Omega}{F_x (y_C - y_\Omega)}; \phi = \frac{1}{F_x (y_C - y_\Omega)} \tag{26}$$

and **D*** is the displacement floor vector (24). To a given position of the rotation center Ω the quantity in brackets

$$\{\Phi_1(\mathbf{D}^*)u + \Phi_2(\mathbf{D}^*)v + \Phi_3(\mathbf{D}^*)\phi\} \tag{27}$$

increases linearly with u,v, ϕ, i.e. with the coordinates x_Ω, y_Ω.In the u,v, ϕ space the representative point P of the function (27) moves along a plane face of a convex multiplane surface. When the rotation center Ω changes position and goes inside another frame between others adjacent walls, the functions $\Phi_1(D^*)$, $\Phi_2(D^*)$ and $\Phi_3(D^*)$ change values and the representative point P moves into another face. A vertex of the polyhedral surface corresponds to a rotation center Ω located at a crossing point between two adjacent walls. The minimum of the kinematical multipliers λ can thus be obtained looking this minimum only in the vertex set V* of the rotational mechanism with rotation center located at the intersection of different walls and of the two translational displacements directed along the x and y directions. Thus we obtain:

$$\lambda_0 = \frac{1}{\Psi} \inf_{V^*} \{(\Phi_1(\mathbf{D}^*)u + \Phi_2(\mathbf{D}^*)v + \Phi_3(\mathbf{D}^*)\phi \} \tag{28}$$

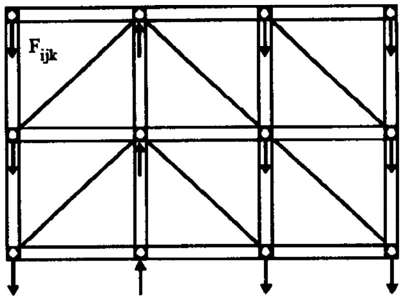

fig. 15

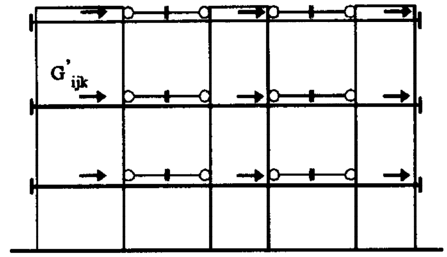

fig.16

The presence of the diagonal steel ties in the floor (fig.13) permits the transfer of horizontal stresses from the weakest walls to the strongest ones. The consequent axial load in these ties can be easily evaluated if the ties are considered as tension rods of an ideal truss set in the plane of the floor; the struts of the truss are the walls orthogonal to the direction of the seismic action (fig.15). The nodal forces F_{ijk} acting on the truss are, if the collapse mechanism is purely translational:

$$F_{ijk} = (\lambda_0 - \lambda_{0k}) \cdot G'_{ijk} \tag{29}$$

where G'_{ijk} is the horizontal seismic load corresponding to $\lambda=1$ and acting on the i-th pier of the k-th wall at the j-th storey (fig.16), λ_0 is the collapse multiplier of the whole building, λ_{0k} is the collapse multiplier of the k-th wall. If the collapse mechanism is rotational there is one wall which does not collapse; the forces F_{ijk} corresponding to this wall must be calculated from the translational equilibrium of the truss in the direction of the seismic action. Anyway, we can observe that the forces F_{ijk} must verify only this translational equilibrium while the rotational one is generally assured also by the reactions of the walls orthogonals to the direction of the seismic action.

CONCLUSIONS

Different systems of reinforcement can be arranged to improve the seismic strength of the historical masonry buildings. The paper has analyzed the one consisting in the insertion of two different systems of steel ties. The first is formed by ties passing through the walls at the height of the architraves; this device first of all prevents the out of plane collapse of the exterior walls, but also increases the seismic strength of the single wall, mostly if masonry panels are inserted between the piers. The second system is formed by diagonal ties inserted within the floors. This device transforms the tridimensional structure in an assemblage of walls connected by rigid horizontal diaphragms and consequently increases the building seismic strength which otherwise should coincide with the weakest wall resistance.

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