Copyright © 1996 Elsevier Science Ltd
Paper No. 1185. (quote when citing this article)
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

APPLICATION OF A SUBDOMAIN APPROACH TO DYNAMIC SHEET PILE SOIL INTERACTION

G. DEGRANDE * and D. AUBRY

Laboratoire de Mécanique des Sols, Structures et Matériaux, Ecole Centrale de Paris,
Grande Voie des Vignes, F-92295 Chatenay Malabry, France

ABSTRACT

A subdomain approach is employed to analyse the seismic response of a sheet pile wall. The method takes advantage of recent developments in the field of dynamic soil-structure interaction and offers a rigorous alternative to a simplified pseudo-static analysis, without being too demanding on the computational point of view. It is used to propose design criteria for sheet pile walls in seismic regions.

KEYWORDS

Dynamic soil-structure interaction; sheet pile; subdomain approach; seismic wave propagation.

PROBLEM OUTLINE

The twodimensional (in-plane) seismic response of a sheet pile of height L in a horizontally layered soil medium on rigid bedrock is envisaged (figure 1). The origin of the cartesian coordinate system (x, y) is placed on the bedrock below the sheet pile. The top of the sheet pile is located at a height H_1 above the bedrock. The height of the soil at the right and left hand side equals H_2 and H_3 respectively.

The problem domain is subdivided into three subdomains. The domain Ω_1 of the generalized sheet pile consists of the sheet pile and the (layered) soil below, extending to the bedrock. The sheet pile material is isotropic and linear elastic. The sheet pile can be tied back by an anchor. The soil domains are horizontally stratified and denoted by Ω_m where the subscript m is equal to 2 or 3 for the soil domain at the right and left hand side of the sheet pile. Each layer consists of a homogeneous linear elastic material, while hysteretic material damping in the soil is accounted for. The restriction to linear elastic materials allows to rely on the superposition principle as required for a frequency domain analysis.

The free surface Γ_0 is horizontal and defined as $\Gamma_0 = \Gamma_{01} \cup \Gamma_{02} \cup \Gamma_{03}$ with $\Gamma_0 \cap \partial\Omega_1 = \Gamma_{01}$ and $\Gamma_0 \cap \partial\Omega_m = \Gamma_{0m}$. The rigid bedrock Γ_b is also horizontal and defined as $\Gamma_b = \Gamma_{b1} \cup \Gamma_{b2} \cup \Gamma_{b3}$ with $\Gamma_b \cap \partial\Omega_1 = \Gamma_{b1}$ and $\Gamma_b \cap \partial\Omega_m = \Gamma_{bm}$. Both soil domains are bounded by infinite boundaries on

*Presently at the Department of Civil Engineering, K.U.Leuven, W. de Croylaan 2, B-3001 Heverlee

$\Gamma_\infty = \Gamma_{\infty 2} \cup \Gamma_{\infty 3}$ with $\Gamma_\infty \cap \partial\Omega_1 = \emptyset$ and $\Gamma_\infty \cap \partial\Omega_m = \Gamma_{\infty m}$. The restriction to horizontally layered soil domains, bounded by a horizontal free surface and rigid bedrock enables an integral transformation of the horizontal coordinate x to the horizontal wave number domain k_x which facilitates the development of radiating boundary conditions on $\Gamma_{\infty m}$. However, the restriction to a horizontal rigid bedrock limits the seismic loading to a vertical incident P-wave or SV-wave.

A virtual work condition will be used to impose traction boundary conditions at the interface Σ between the sheet pile and the soil domains. This interface is defined as $\Sigma = \Sigma_{12} \cup \Sigma_{13}$ with $\Sigma \cap \partial\Omega_1 = \Sigma$ and $\Sigma \cap \partial\Omega_m = \Sigma_{1m}$. Perfect contact between the sheet pile and the soil is assumed on the interface Σ.

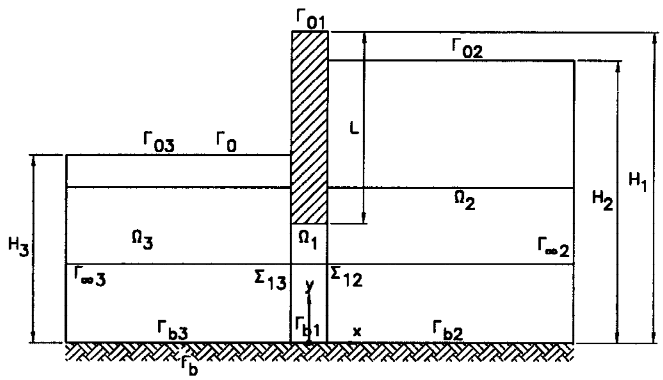


Fig. 1: Dynamic sheet pile soil interaction

DECOMPOSITION OF THE DISPLACEMENT VECTORS

The displacement vector $\mathbf{u}_p$ in the domain Ω_1 of the generalized sheet pile is written as the sum of the rigid base motion $\mathbf{a}$ and a modal decomposition on the basis of N_1 eigenvectors $\mathbf{\Phi}_{1j}$ $(j = 1, N_1)$ of the generalized sheet pile, clamped at its base:

$$\mathbf{u}_p = \mathbf{a} + \mathbf{\Phi}_1 \alpha_1 \tag{1}$$

The columns of the matrix $\mathbf{\Phi}_1$ are equal to the the eigenvectors $\mathbf{\Phi}_{1j}$, i.e. $\mathbf{\Phi}_1 = [\mathbf{\Phi}_{11}\, \mathbf{\Phi}_{12} \cdots \mathbf{\Phi}_{1N_1}]$, while the modal participation factors α_{1j} are collected in a vector $\alpha_1 = \{\alpha_{11}\, \alpha_{12} \cdots \alpha_{1N_1}\}^T$.

Similarly, a decomposition of the problem into simpler subproblems allows to write the soil displacement vector $\mathbf{u}_{sm}$ in the domain Ω_m as the following superposition (Clouteau, 1990; Aubry and Clouteau, 1992):

$$\mathbf{u}_{sm} = \mathbf{u}_{am} + \mathbf{u}_{im} + \mathbf{u}_{d0m} + \mathbf{\Phi}_m \alpha_m \tag{2}$$

$\mathbf{u}_{am}$ is an elastodynamic field in Ω_m, radiated by the rigid body motion of the sheet pile. It satisfies $\mathbf{u}_{am} = \mathbf{a}$ on Σ_{1m} and Sommerfeld's radiation conditions on $\Gamma_{\infty m}$. $\mathbf{u}_{im}$ is the (one-dimensional) incident wave field in Ω_m, due to a vertical incident P-wave or SV-wave. $\mathbf{u}_{d0m}$ is the locally diffracted elastodynamic field in Ω_m, satisfying $\mathbf{u}_{d0m} = -\mathbf{u}_{im}$ on Σ_{1m} and Sommerfeld's radiation conditions on $\Gamma_{\infty m}$. The scattered wave field in Ω_m is expanded on the basis of the N_m surface wave modes $\mathbf{\Phi}_{mj}$ $(j = 1, N_m)$ of the multilayered soil on rigid bedrock. The columns of the matrix $\mathbf{\Phi}_m$ are equal to the eigenvectors $\mathbf{\Phi}_{mj}$, i.e. $\mathbf{\Phi}_m = [\mathbf{\Phi}_{m1}\, \mathbf{\Phi}_{m2} \cdots \mathbf{\Phi}_{mN_m}]$, while the modal participation factors α_{mj} are collected in a vector $\alpha_m = \{\alpha_{m1}\, \alpha_{m2} \cdots \alpha_{mN_m}\}^T$.

WEAK VARIATIONAL FORMULATION

On the interface Σ, perfect contact between the sheet pile and the soil domains Ω_m is assumed:

$$\mathbf{u}_p = \mathbf{u}_{sm} \quad \text{on } \Sigma \tag{3}$$

A weak variational approach is used to impose the traction boundary condition on the interface Σ:

$$< \mathbf{t}_p(\mathbf{u}_p) + \mathbf{t}_{s2}(\mathbf{u}_{s2}) + \mathbf{t}_{s3}(\mathbf{u}_{s3}), \mathbf{v} >_\Sigma \;=\; 0 \qquad \forall \mathbf{v} \tag{4}$$

$\mathbf{v}$ are the weighting functions and $\mathbf{t}(\mathbf{u}) = \sigma(\mathbf{u})\mathbf{n}$ are the tractions on a boundary with unit outward normal $\mathbf{n}$. $< \mathbf{a}, \mathbf{b} >_\Sigma$ is the integral $\int_\Sigma \mathbf{a}^T \mathbf{b} d\Sigma$ defined on the interface Σ for two vectors $\mathbf{a}$ and $\mathbf{b}$. Similarly, $(\mathbf{a}, \mathbf{b})_\Omega$ will be used in the following to denote the integral $\int_\Omega \mathbf{a}^T \mathbf{b} d\Omega$ in the domain Ω.

The decompositions (1) and (2) for the displacements $\mathbf{u}_p$ and $\mathbf{u}_{sm}$ can be introduced in conditions (3) and (4). Accounting for the identities $\mathbf{u}_{am} = \mathbf{a}$ and $\mathbf{u}_{im} + \mathbf{u}_{d0m} = 0$ on Σ, the displacement continuity (3) reduces to:

$$\mathbf{\Phi}_1 \alpha_1 \;=\; \mathbf{\Phi}_m \alpha_m \qquad \text{on } \Sigma \tag{5}$$

Based on similar arguments, the virtual work statement (4) becomes:

$$\begin{aligned} & < \mathbf{v}, \mathbf{t}_p(\mathbf{\Phi}_1\alpha_1) >_\Sigma + < \mathbf{v}, \mathbf{t}_{s2}(\mathbf{\Phi}_1\alpha_1) >_{\Sigma_{12}} + < \mathbf{v}, \mathbf{t}_{s3}(\mathbf{\Phi}_1\alpha_1) >_{\Sigma_{13}} \\ = & \; - < \mathbf{v}, \mathbf{t}_p(\mathbf{a}) >_\Sigma - < \mathbf{v}, \mathbf{t}_{s2}(\mathbf{u}_{a2}) >_{\Sigma_{12}} - < \mathbf{v}, \mathbf{t}_{s2}(\mathbf{u}_{i2}) >_{\Sigma_{12}} - < \mathbf{v}, \mathbf{t}_{s2}(\mathbf{u}_{d02}) >_{\Sigma_{12}} \\ & \; - < \mathbf{v}, \mathbf{t}_{s3}(\mathbf{u}_{a3}) >_{\Sigma_{13}} - < \mathbf{v}, \mathbf{t}_{s3}(\mathbf{u}_{i3}) >_{\Sigma_{13}} - < \mathbf{v}, \mathbf{t}_{s3}(\mathbf{u}_{d03}) >_{\Sigma_{13}} \end{aligned} \tag{6}$$

The terms on the left hand side are the impedances of the sheet pile and both soil domains, while the terms on the right hand side can be considered as the virtual work originating from external forces. In the following sections, general expressions for $< \mathbf{v}, \mathbf{t}_p(\mathbf{u}_p) >_\Sigma$ and $< \mathbf{v}, \mathbf{t}_{sm}(\mathbf{u}_{sm}) >_{\Sigma_{1m}}$ will be derived.

RITZ VARIATIONAL APPROACH FOR THE SHEET PILE

In this section, the boundary term $< \mathbf{v}, \mathbf{t}_p(\mathbf{u}_p) >_\Sigma$, representing the virtual work of external forces on the generalized sheet pile, will be elaborated. A strong integral formulation for a twodimensional beam element, accounting for shear deformation and rotational inertia, immediately follows from the partial differential equations and the boundary conditions. A weak integral formulation can be obtained after integration by parts on the relevant terms and is equivalent to the following virtual work expression:

$$< \mathbf{v}, \mathbf{t}(\mathbf{u}_p) >_\Sigma \;=\; (\epsilon(\mathbf{v}), \sigma(\mathbf{u}_p))_{\Omega_1} + (\mathbf{v}, \mathcal{M}\ddot{\mathbf{u}}_p)_{\Omega_1} \tag{7}$$

where a notation has been used which is similar to the one usually employed in continuum mechanics. The term on the left hand side of equation (7) collects the virtual work of the distributed and concentrated external forces on the sheet pile. It is convenient to define a generalized displacement vector $\mathbf{u}_p = \{u_x \, u_y \, \beta\}^T$ with u_x and u_y the axial and transverse displacements and β the rotation due to bending. The first term on the right hand side of equation (7) is the strain energy. $\epsilon = \{\epsilon_x \, \gamma \, \kappa\}^T$ is a generalized strain vector with ϵ_x the axial strain, γ the shear deformation and κ the curvature of the beam. The strains are related to the displacements by:

$$\epsilon \;=\; \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial x} & 1 \\ 0 & 0 & \frac{\partial}{\partial x} \end{bmatrix} \mathbf{u}_p = \mathbf{L}\mathbf{u}_p \tag{8}$$

The constitutive equation relates the generalized stress vector $\sigma = \{N \, T \, M\}^T$, with N the normal force, T the shear force and M the bending moment to the generalized strain vector ϵ:

$$\sigma \;=\; \mathbf{D}\epsilon \tag{9}$$

Herein $\mathbf{D} = \mathcal{D}\{EA \;\; kGA \;\; EI\}$ is a diagonal matrix with constitutive coefficients. E is the Young modulus, G the shear modulus, A the cross-sectional area, I the moment of inertia and k a correction factor to account for the cross-sectional area effective in shear. The second term on the right hand side of equation (7) denotes the virtual work of the inertia forces, where $\mathcal{M} = \mathcal{D}\{\rho A \;\; \rho A \;\; \rho I\}$. Equation

(7) is the basis for a mixed finite element formulation based on a discretization of all displacement components of $\mathbf{u}_p$ (Bathe, 1982).

In a weak Galerkin formulation, both the sheet pile displacement vector $\mathbf{u}_p$ and the weighting functions $\mathbf{v}$ are decomposed on the basis of sheet pile eigenvectors, according to equation (1) and $\mathbf{v} = \mathbf{\Phi}_1\alpha_v$ respectively. The vector α_v contains the virtual modal participation factors. Introducing definition (8) of the generalized strain and the constitutive relation (9) and assuming a harmonic variation with respect to time at a circular frequency ω, the weak Galerkin counterpart of the integral expression (7) becomes:

$$< \mathbf{v}, \mathbf{t}(\mathbf{u}_p) >_\Sigma \; = \; \alpha_v^T(\mathbf{L\Phi}_1, \mathbf{DL\Phi}_1)_{\Omega_1}\alpha_1 - \omega^2\alpha_v^T(\mathbf{\Phi}_1, \mathcal{M}\mathbf{\Phi}_1)_{\Omega_1}\alpha_1 - \omega^2\alpha_v^T(\mathbf{\Phi}_1, \mathcal{M}\mathbf{a})_{\Omega_1} \tag{10}$$

It should be noted that the rigid base motion $\mathbf{a}$ does not contribute to the strain energy.

A Ritz variational approach with global piecewize linear and cubic shape functions $\mathbf{N}_1$ introduces the following approximation on the eigenmodes of the generalized sheet pile:

$$\mathbf{\Phi}_1 \; \simeq \; \hat{\mathbf{\Phi}}_1 = \mathbf{N}_1\underline{\mathbf{\Phi}}_1 \tag{11}$$

and results in the following Ritz formulation of the boundary term:

$$< \mathbf{v}, \mathbf{t}(\mathbf{u}_p) >_\Sigma \; = \; \alpha_v^T\underline{\mathbf{\Phi}}_1^T\mathbf{K}_1\underline{\mathbf{\Phi}}_1\alpha_1 - \omega^2\alpha_v^T\underline{\mathbf{\Phi}}_1^T\mathbf{M}_1\underline{\mathbf{\Phi}}_1\alpha_1 - \omega^2\alpha_v^T\underline{\mathbf{\Phi}}_1^T\mathbf{M}_1\mathbf{a} \tag{12}$$

$\mathbf{K}_1 = (\mathbf{LN}_1, \mathbf{DLN}_1)_{\Omega_1}$ and $\mathbf{M}_1 = (\mathbf{N}_1, \mathcal{M}\mathbf{N}_1)_{\Omega_1}$ are the sheet pile's stiffness and consistent mass matrix. The eigenpairs (ω_{1j}, Φ_{1j}) are solutions of the generalized eigenvalue problem:

$$\mathbf{K}_1\Phi_{1j} = \omega_{1j}^2\mathbf{M}_1\Phi_{1j} \tag{13}$$

If the eigenvectors are orthonormized with respect to the mass matrix, $\underline{\mathbf{\Phi}}_1^T\mathbf{M}_1\underline{\mathbf{\Phi}}_1 = \mathbf{I}$ and $\underline{\mathbf{\Phi}}_1^T\mathbf{K}_1\underline{\mathbf{\Phi}}_1 = \mathbf{\Lambda}$, with $\mathbf{\Lambda} = \mathcal{D}\{\omega_{11}^2\,\omega_{12}^2\cdots\omega_{1N_1}^2\}$, the virtual work expression finally becomes:

$$< \mathbf{v}, \mathbf{t}(\mathbf{u}_p) >_\Sigma \; = \; \alpha_v^T\left[(\mathbf{\Lambda} - \omega^2\mathbf{I})\alpha_1 - \omega^2\underline{\mathbf{\Phi}}_1^T\mathbf{M}_1\mathbf{a}\right] \tag{14}$$

RITZ VARIATIONAL APPROACH FOR THE SOIL

In this section, the boundary term $< \mathbf{v}, \mathbf{t}_{sm}(\mathbf{u}_{sm}) >_\Sigma$, representing the virtual work of external forces on the soil domain Ω_m along the interface Σ, will be elaborated. The dynamic behaviour of the soil can be described within the frame of elastodynamics. The equilibrium equation of the medium is:

$$\operatorname{div}\sigma + \rho^s\mathbf{b} \; = \; \rho^s\ddot{\mathbf{u}}_{sm} \tag{15}$$

σ is the total stress tensor, $\rho^s\mathbf{b}$ the body force and ρ^s the density of the solid skeleton. The natural boundary conditions on the part $\partial\Omega_{mN}$ of the boundary $\partial\Omega_m$ of the domain Ω_m are:

$$\sigma\mathbf{n} - \bar{\mathbf{t}} \; = \; 0 \quad \text{on } \partial\Omega_{mN} \tag{16}$$

As the imposed tractions $\bar{\mathbf{t}}$ are only nonzero on the part Σ of the boundary $\partial\Omega_{mN}$, $\partial\Omega_{mN}$ will be replaced by Σ. The small strain tensor ϵ^s is equal to the symmetric part of the soil displacement gradient. The constitutive equations can be summarized as follows:

$$\sigma \; = \; 2\mu^s\epsilon^s + \lambda^s\operatorname{tr}\epsilon^s\mathbf{1} \tag{17}$$

where μ^s and λ^s are the Lamé coefficients and $\mathbf{1}$ is the unit tensor. In the frequency domain, the correspondence principle allows to account for hysteretic material damping in the solid skeleton by using complex Lamé coefficients μ^{s*} and λ^{s*}, defined as $\mu^{s*} = \mu^s(1 + 2i\beta_s^s)$ and $(\lambda^{s*} + 2\mu^{s*}) = (\lambda^s + 2\mu^s)(1 + 2i\beta_p^s)$. Herein, $i = \sqrt{-1}$ is the imaginary unit and β_s^s and β_p^s are the hysteretic material damping ratios for rotational and dilatational deformation respectively.

The strong integral formulation, equivalent to the partial differential equations (15) and the boundary conditions (16) is:

$$(\mathbf{v}, \operatorname{div}\sigma - \rho^s \ddot{\mathbf{u}}_{sm})_{\Omega_m} - < \mathbf{v}, \sigma\mathbf{n} - \bar{\mathbf{t}} >_\Sigma \;=\; 0 \tag{18}$$

The influence of body forces has been neglected. In view of a discretization, it is convenient to replace the symmetric stress and strain tensors σ and ϵ by two vectors. The strain-displacement relation becomes $\epsilon = \mathbf{L}_m \mathbf{u}_{sm}$ and the constitutive relation reduces to $\sigma = \mathbf{D}\epsilon$. After reordering of terms, assuming a harmonic variation in time and using vector notation, equation (18) becomes:

$$< \mathbf{v}, \bar{\mathbf{t}} >_\Sigma \;=\; -(\mathbf{v}, \mathbf{L}_m^T \mathbf{D} \mathbf{L}_m \mathbf{u}_{sm})_{\Omega_m} - \omega^2 (\mathbf{v}, \rho^s \mathbf{u}_{sm})_{\Omega_m} + < \mathbf{v}, \mathbf{D}\mathbf{L}_m \mathbf{u}_{sm} \mathbf{n} >_\Sigma \tag{19}$$

A Ritz variational approach with global piecewize linear shape functions is used to approximate the soil displacements. A Galerkin approximation is obtained if the same displacement functions are used as weighting functions. If the horizontal coordinate x is transformed to the horizontal wave number k_x, the following discretized system of equations is finally obtained:

$$\mathbf{f}_m \;=\; \left[k_x^2 \mathbf{A}_m + i k_x \mathbf{B}_m + \mathbf{G}_m - \omega^2 \mathbf{M}_m\right] \underline{\mathbf{u}}_{sm} \tag{20}$$

with $k_x^2 \mathbf{A}_m + i k_x \mathbf{B}_m + \mathbf{G}_m = -(\mathbf{N}_m, \mathbf{L}_m^T \mathbf{D} \mathbf{L}_m \mathbf{N}_m)_{\Omega_m} + < \mathbf{N}_m, \mathbf{D}\mathbf{L}_m \mathbf{N}_m \mathbf{n} >_\Sigma$ and $\mathbf{M}_m = (\mathbf{N}_m, \rho^s \mathbf{N}_m)_{\Omega_m}$. Assuming linear interpolation, an expression for the elements of the matrices $\mathbf{A}_m$, $\mathbf{B}_m$, $\mathbf{G}_m$ and $\mathbf{M}_m$ and the vector $\mathbf{f}_m$ has been presented by several authors (Waas, 1972; Kausel, 1974; Kausel, 1986; Kausel and Peek, 1982; Tassoulas and Kausel, 1983). It is customary to write $\mathbf{C}_m = \mathbf{G}_m - \omega^2 \mathbf{M}_m$. The eigenpairs $(k_{xj}, \mathbf{\Phi}_{mj})$ are solutions of the corresponding eigenvalue problem for the soil:

$$\left[k_{xj}^2 \mathbf{A}_m + i k_{xj} \mathbf{B}_m + \mathbf{C}_m\right] \mathbf{\Phi}_{mj} \;=\; 0 \tag{21}$$

or

$$\begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{A}_m^{-1}\mathbf{C}_m & \mathbf{A}_m^{-1}\mathbf{B}_m \end{bmatrix} \begin{Bmatrix} \mathbf{\Phi}_{mj} \\ \lambda_{mj}\mathbf{\Phi}_{mj} \end{Bmatrix} \;=\; \lambda_{mj} \begin{Bmatrix} \mathbf{\Phi}_{mj} \\ \lambda_{mj}\mathbf{\Phi}_{mj} \end{Bmatrix} \tag{22}$$

with $\lambda_{mj} = i k_{xj}$. In the soil domain Ω_2, $\mathrm{Im}(k_{xj})$ should be negative as waves propagate to the right ($x > 0$), while in the soil domain Ω_3, $\mathrm{Im}(k_{xj})$ should be positive as waves propagate to the left ($x < 0$).

All elements are now present to derive an expression for the term $< \mathbf{v}, \mathbf{t}_{sm}(\mathbf{u}_{sm}) >_\Sigma$. If the displacement field $\mathbf{u}_{sm}$ is decomposed on the basis of the eigenmodes $\mathbf{\Phi}_{mj}$, the virtual work expression for the boundary term becomes:

$$< \mathbf{v}, \mathbf{t}_{sm}(\mathbf{u}_{sm}) >_\Sigma \;=\; \sum_{j=1}^{N_m} < \mathbf{v}, \mathbf{t}_{sm}(\mathbf{\Phi}_{mj})\alpha_{mj} >_\Gamma \tag{23}$$

In a Galerkin formulation, the weighting functions $\mathbf{v}$ are decomposed on the basis of sheet pile eigenmodes, i.e. $\mathbf{v} = \mathbf{\Phi}_1 \alpha_v$ where α_v contains the virtual modal participation factors. If we use a Ritz variational approach:

$$\mathbf{\Phi}_m \;\simeq\; \hat{\mathbf{\Phi}}_m = \mathbf{N}_m \underline{\mathbf{\Phi}}_m \tag{24}$$

the boundary term becomes:

$$< \mathbf{v}, \mathbf{t}_{sm}(\mathbf{u}_{sm}) >_\Sigma \;=\; \mp \sum_{j=1}^{N_m} \alpha_v^T \underline{\mathbf{\Phi}}_1^T < \mathbf{N}_m, \mathbf{D}\mathbf{L}_m \mathbf{N}_m >_\Gamma \underline{\mathbf{\Phi}}_{mj} \alpha_{mj} \tag{25}$$

where the $-$ and $+$ sign are used in the domain Ω_2 and Ω_3 respectively. Elaborating the integral and using matrix notation (Tassoulas and Kausel, 1983):

$$< \mathbf{v}, \mathbf{t}_{sm}(\mathbf{u}_{sm}) >_\Sigma \;=\; \pm \sum_{j=1}^{N_m} \alpha_v^T \underline{\mathbf{\Phi}}_1^T \left[i k_{xj} \mathbf{A}_m + \mathbf{D}_m\right] \underline{\mathbf{\Phi}}_{mj} \alpha_{mj} \tag{26}$$

or

$$< \mathbf{v}, \mathbf{t}_{sm}(\mathbf{u}_{sm}) >_\Sigma \;=\; \pm\alpha_v^T \underline{\mathbf{\Phi}}_1^T \left[\mathbf{A}_m \underline{\mathbf{\Phi}}_m \mathbf{K}_m + \mathbf{D}_m \underline{\mathbf{\Phi}}_m\right] \alpha_m \tag{27}$$

where $\mathbf{K}_m = \mathcal{D}\{ik_{x1}\, ik_{x2} \cdots ik_{xN_m}\}$. As we want an expression in terms of $\mathbf{u}_{sm}$, rather than the unknown participation factors α_m, we can finally write:

$$< \mathbf{v}, \mathbf{t}_{sm}(\mathbf{u}_{sm}) >_\Sigma \;=\; \pm\alpha_v^T \underline{\mathbf{\Phi}}_1^T \left[\mathbf{A}_m \underline{\mathbf{\Phi}}_m \mathbf{K}_m \underline{\mathbf{\Phi}}_m^{-1} + \mathbf{D}_m\right] \mathbf{u}_{sm} \;=\; \pm\alpha_v^T \underline{\mathbf{\Phi}}_1^T \mathbf{R}_m \mathbf{u}_{sm} \tag{28}$$

This expression allows us to write the virtual work of the boundary tractions at the interface Σ for a general displacement field $\mathbf{u}_{sm}$. The formulation is equivalent to the derivation of a consistent boundary condition (Waas, 1972).

MATRIX FORMULATION

Expressions (14) and (28) allow to elaborate the virtual work statement (6). After some algebra and stipulating that the formulation should hold for any set of virtual modal participation factors, the following system of equations is obtained:

$$\begin{aligned}
&[(\mathbf{\Lambda} - \omega^2\mathbf{I}) + \mathbf{\Phi}_1^T\mathbf{R}_2\mathbf{\Phi}_1 - \mathbf{\Phi}_1^T\mathbf{R}_3\mathbf{\Phi}_1]\alpha_1 \\
&\quad = \omega^2\mathbf{\Phi}_1^T\mathbf{M}_1\mathbf{a} + \mathbf{\Phi}_1^T\mathbf{R}_2\mathbf{u}_{a2} + \mathbf{\Phi}_1^T\mathbf{D}_2\mathbf{u}_{i2} + \mathbf{\Phi}_1^T\mathbf{R}_2\mathbf{u}_{d02} - \mathbf{\Phi}_1^T\mathbf{R}_3\mathbf{u}_{a3} - \mathbf{\Phi}_1^T\mathbf{D}_3\mathbf{u}_{i3} - \mathbf{\Phi}_1^T\mathbf{R}_3\mathbf{u}_{d03}
\end{aligned} \tag{29}$$

The solution of this system of equations gives the complex participation factors α_1 on the eigenmodes of the generalized sheet pile. Equation (1) allows to recover the displacements of the sheet pile and, consequently, the member forces. The displacements in the soil domains Ω_m can be calculated as:

$$\mathbf{u}_{sm} \;=\; \mathbf{u}_{im} + \mathbf{\Phi}_m(\alpha_{am} + \alpha_{dom} + \alpha_m)\mathcal{D}\{\exp(-ik_{xj}x)\} \tag{30}$$

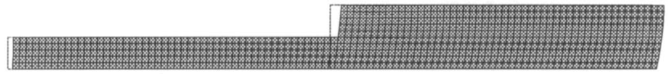
where α_{am}, α_{dom}, α_m are the complex participation factors on the eigenmodes of the soil for the field radiated by the rigid body motion of the sheet pile, the locally diffracted field and the scattered wave field respectively. They follow from the solution of $\mathbf{\Phi}_m\alpha_{am} = \mathbf{u}_{am}$, $\mathbf{\Phi}_m\alpha_{dom} = \mathbf{u}_{dom}$ and $\mathbf{\Phi}_m\alpha_m = \mathbf{\Phi}_1\alpha_1$.

NUMERICAL EXAMPLE

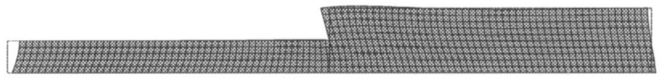
The seismic response of a sheet pile due to a vertical incident plane harmonic P-wave and SV-wave will be considered. Referring to the nomenclature introduced in relation to figure 1, the relevant dimensions are $L = 15\,\mathrm{m}$, $H_1 = 20\,\mathrm{m}$, $H_2 = 20\,\mathrm{m}$ and $H_3 = 10\,\mathrm{m}$. A sheet pile Arbed PU 20 with a cross sectional area $A = 0.0018\,\mathrm{m}^2/\mathrm{m}$, inertia $I = 0.00043\,\mathrm{m}^4/\mathrm{m}$, Young modulus $E = 2.1 \times 10^{11}\,\mathrm{N/m}^2$ and density $\rho = 7850\,\mathrm{kg/m}^3$ is embedded in a sand soil with Young modulus $E^s = 2.983 \times 10^8\,\mathrm{N/m}^2$, Poisson coefficient $\nu^s = 1/3$ and density $\rho^s = 1621.8\,\mathrm{kg/m}^3$. Hysteretic material damping in the soil is represented by $\beta^s = \beta^s_s = \beta^s_p = 0.05$.

The interface Σ between the soil and the sheet pile is discretized into 20 elements. The first 10 eigenmodes of the generalized sheet pile are accounted for, leading to a 10 by 10 system of equations (29) and resulting in the participation factors α_1 on the generalized sheet pile's eigenmodes. Postprocessing allows for the calculation of the displacements and member forces in the sheet pile, as well as the displacement fields in the soil.

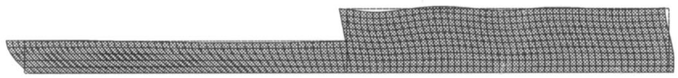
As a matter of illustration, figure 2 shows the displacement fields in the soil for a vertical incident plane harmonic SV-wave at excitation frequencies $f_1 = 2.5\,\mathrm{Hz}$, $f_2 = 5.0\,\mathrm{Hz}$ and $f_3 = 7.5\,\mathrm{Hz}$. It is important to repeat that only a discretization of the interface Σ between the sheet pile and the soil is required; the mesh in figure 2 has been used for postprocessing only. The first resonance frequencies of the soil domains Ω_m on rigid bedrock are equal to $f_m^{res} = C_{sm}/4H_m$ where C_{sm} is the shear wave velocity in the domain Ω_m, giving $f_2^{res} = 3.28\,\mathrm{Hz}$ and $f_3^{res} = 6.56\,\mathrm{Hz}$. It is apparent from figure 2 that wave



a. Frequency 2.5 Hz



b. Frequency 5.0 Hz

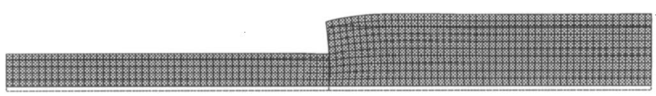


c. Frequency 7.5 Hz

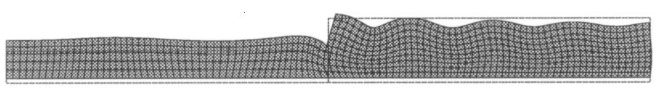
Fig. 2: Displacements for a vertical incident plane harmonic SV-wave.

propagation and, consequently, dissipation of energy, only occurs if the excitation frequency is higher than the first resonance frequency of the layer on rigid bedrock.

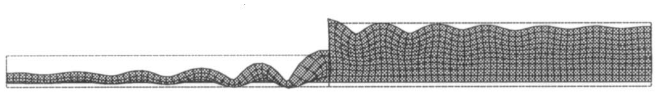
Figure 3 shows similar displacement fields for a vertical incident plane harmonic P-wave at excitation frequencies $f_1 = 5.0\,\mathrm{Hz}$, $f_2 = 10.0\,\mathrm{Hz}$ and $f_3 = 15.0\,\mathrm{Hz}$. The first resonance frequencies of the soil domains Ω_m on rigid bedrock are equal to $f_m^{res} = C_{pm}/4H_m$ where C_{pm} is the longitudinal wave velocity in the domain Ω_m, giving $f_2^{res} = 6.56\,\mathrm{Hz}$ and $f_3^{res} = 13.125\,\mathrm{Hz}$. Similar observations as in the previous case can be made.



a. Frequency 5.0 Hz



b. Frequency 10.0 Hz



c. Frequency 15.0 Hz

Fig. 3: Displacements for a vertical incident plane harmonic P-wave.

A more thorough analysis requires calculations for a range of frequencies so that the variation of the sheet pile's member forces with varying frequency can be studied. The transient response for an actual earthquake can be obtained if an FFT algorithm is employed to calculate the inverse transformation from the frequency to the time domain.

CONCLUSION

Relying on a multidomain approach, the decomposition of the general problem into several simpler subproblems and the use of modal bases to represent seismic fields emitted at the interfaces, the seismic response of sheet pile walls has been considered. The resulting formulation is based on a terminology close to engineering practice and efficient from a computational point of view. In view of the proposition of design rules for sheet pile walls in seismic regions within the frame of part 5 (*"Foundations, retaining structures and geotechnical aspects"*) of Eurocode 8 (*"Structures in seismic regions - design"*), a parameter study is presently undertaken. The influence of several parameters such as the type of incident wave, the frequency content of the seismic loading, local soil conditions, material damping, the retaining height and the presence of a tieback are investigated. Alternative formulations, based on analytical solutions rather than a Rayleigh-Ritz discretization will be envisaged in the near future.

ACKNOWLEDGEMENTS

Part of this research has been carried out by the first author during a postdoctoral stay at Ecole Centrale de Paris. The financial support of the European Community within the frame of the program "Human Capital and Mobility" (Research Contract Number ERBCHBGCT930403) is kindly acknowledged. We also wish to thank Didier Clouteau for his contribution to many discussions and Jacques Ciabrini and Jose Fernandez, who have contributed to the present developments.

REFERENCES

Aubry, D. and Clouteau, D. (1992). A subdomain approach to dynamic soil-structure interaction. In *Recent advances in earthquake engineering and structural dynamics*, pages 251–271. Ouest Editions.

Bathe, K. (1982). *Finite element procedures in engineering analysis.* Prentice-Hall, Englewood Cliffs, New Jersey.

Clouteau, D. (1990). *Propagation d'ondes dans des milieux hétérogènes. Application à la tenue des ouvrages sous séismes.* PhD thesis, Laboratoire de Mécanique des Sols, Structures et Matériaux, Ecole Centrale de Paris.

Kausel, E. (1974). Forced vibrations of circular foundations on layered media. Research report R74-11, Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts.

Kausel, E. (1986). Wave propagation in anisotropic layered media. *International Journal for Numerical Methods in Engineering*, 23:1567–1578.

Kausel, E. and Peek, R. (1982). Dynamic loads in the interior of a layered stratum: an explicit solution. *Bulletin of the Seismological Society of America*, 72(5):1469–1481.

Tassoulas, J. and Kausel, E. (1983). Elements for the numerical analysis of wave motion in layered strata. *International Journal for Numerical Methods in Engineering*, 19(7):1005–1032.

Waas, G. (1972). *Linear two-dimensional analysis of soil dynamic problems in semi-infinite layer media.* PhD thesis, Department of Civil Engineering, University of California, Berkeley.