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Paper No. 1137. (quote when citing this article)
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

SEISMIC INSTABILITY AND THE FORCE REDUCTION FACTOR OF YIELDING STRUCTURES

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ABSTRACT

Under seismic action, the destabilizing effects of gravity loads on the laterally displaced structure (P-Δ effects) lead to earlier collapse due to inordinate increase in displacements. In torsionally unbalanced systems floor rotations induce destabilizing torques, thereby increasing P-Δ effects. This paper investigates the seismic behaviour of yielding asymmetric single storey models by studying the effects of the governing system parameters and ground motion characteristics on the bounds of the force reduction factor *R* at the onset of instability.

KEYWORDS

Earthquake engineering; seismic instability; yielding asymmetric structures.

INTRODUCTION

The action of gravity loads on the laterally displaced configuration of the structure (P-Δ effects) can lead to an earlier collapse due to inordinate increase in the lateral deflection and, hence, in the ductility demand when the post-yield stiffness, as modified by the P-Δ effect, is negative. Therefore, when substantial P-Δ effects are present, the strength of the structure may not be sufficient to limit the deflections to an acceptable level. It follows that the onset of seismic instability imposes an upper bound on the force reduction (or response modification) factor *R*, which may be lower than its values prescribed in seismic codes for different structural systems. Moreover, in torsionally unbalanced systems, destabilizing torques due to floor rotations lower this bound even further. However, because of its threshold nature, this limiting value of *R*, R_l, cannot be substituted for the code value, but it can be taken as a basis for a further reduced design value.

A number of studies were carried out to predict the strength level at the onset of dynamic instability for single-degree-of-freedom (1-dof) systems. Bernal (1987) proposed a statistically derived modified stability coefficient for elastic-perfectly plastic (EPP) models. However, as is known (Fig.1), most structural systems have positive secondary slope (= $r_0 K_0$, where K_0 = elastic stiffness) in their force-displacement relation, and therefore can become unstable only when the secondary slope, as modified by the P-Δ effect, becomes negative. The effect of the gravity load *P* on the force-displacement relation for a bilinear system is shown in Fig. 1, from which it can be observed that the P-Δ modified secondary slope K_Δ'' is given by:

$$K_\Delta'' = K_0(r_0 - \theta) = K_\Delta \frac{r_0 - \theta}{1 - \theta} \tag{1}$$

where $\theta = P/(K_0 h)$ is the stability coefficient and $K_\Delta = K_0 (1 - \theta)$ is the second order stiffness. The effect of P-Δ on degrading models has been studied by several investigators (Mahin and Boroschek, 1991; Rahnama and Krawinkler, 1993), who concluded that P-Δ effects are lesser for stiffness or strength degrading systems than for bilinear ones. It is thus seen that when more realistic response curves are used, P-Δ effects appear to be less pronounced than for EPP systems. A survey of the available P-Δ methods is given in a paper by MacRae (1994). However, in most studies the problem of seismic instability has been addressed mainly with respect to 1-dof models or planar inelastic systems, an approach that is applicable to structures responding mainly in translation, i.e. with negligible rotation. For a common structure - which is likely to be torsionally unbalanced, i.e. eccentric - floor rotations induce destabilizing torques, thereby amplifying the P-Δ effect.

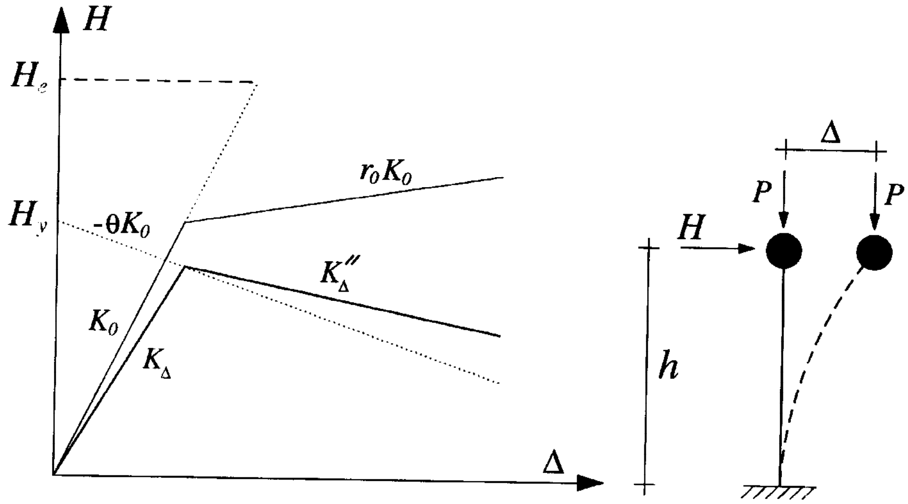

Fig. 1. Force-displacement relation accounting for P-Δ effect

The seismic stability of asymmetric structures has not been studied extensively. The more recent work by Bernal and Sordo (1992) is the only relevant study which has come to the attention of the authors. Using an equivalent 1-dof model they concluded that seismic torsional instability is not likely to be significantly affected by torsional eccentricity provided that the overstrengths of the various elements do not significantly deviate from the system average. When this condition is not satisfied, failure occurs by pivoting about the strong element.

The aim of the present study is mainly to evaluate the effect of the asymmetry on the limiting value of R at the onset of instability (R_l) by presenting the results of a parametric study on the seismic response of code-designed eccentric mono-symmetric structural models, i.e. the same models used extensively in torsional response studies (e.g. De Stefano *et al.*, 1993; Rutenberg *et al.*, 1992). By studying such 'standard' models it becomes easier to assess the significance of instability, measured by an effective stability coefficient $\theta_{eff} = \theta - r_0$ (> 0), as the main system parameters, namely the uncoupled lateral period of the system, stiffness eccentricity and torsional-to-lateral frequency ratio, are varied. Several earthquake time histories, all typical of stiff soil conditions, are considered in order to improve the statistical significance of the results.

THE MODEL EQUATIONS OF MOTIONS AND PARAMETRIC STUDY

The floor plan of the monosymmetric one-storey structure chosen for this study is shown in Fig. 2. Several simplifying assumptions were made: (a) the mass is uniformly distributed with the centroid at C_M; (b) the floor slab is rigid in its own plane; (c) the lateral load resisting system consists of three elements. These elements are arranged so that C_M lies in the line of action of Element 2. This stiffness eccentric arrangement is sometimes denoted as the CM model. The distance from the stiffness center C_R to the center of mass C_M is the stiffness eccentricity e.

The relative yield strength levels of the three elements were computed using the relevant UBC (ICBO, 1994) seismic provisions. For the purpose of design, these provisions replace e by a design eccentricity e_d as follows:

$$e_d = e + 0.05A_x b \tag{2}$$

$$e_d = e - 0.05A_x b \leq 0 \tag{3}$$

in which b is the building dimension perpendicular to the direction of excitation, and A_x is given by:

$$1.0 \le A_x = \left(\frac{\delta_{max}}{1.2\delta_{avg}} \right)^2 \le 3.0 \tag{4}$$

where δ_{max} = maximum lateral displacement of the floor at the considered level, δ_{avg} = average of the horizontal displacements at the extreme positions of the floor at the same level, i.e. $\delta_{avg} = (\delta_{max} + \delta_{min})/2$, and δ_{min} = minimum lateral floor displacement. The more severe loading on each element resulting from using either (2) or (3) shall be considered in design. Note that δ_{max} and δ_{min} are to be computed based on e_d obtained from (2) and (3) when $A_x = 1.0$. The rationale for incorporating an amplification factor A_x in the design eccentricity expressions is given by the NEHRP Commentary (Building Seismic Safety Council, 1992), as based on indications that the 0.05 b accidental eccentricity may not be adequate for protecting the structure against torsional instability.

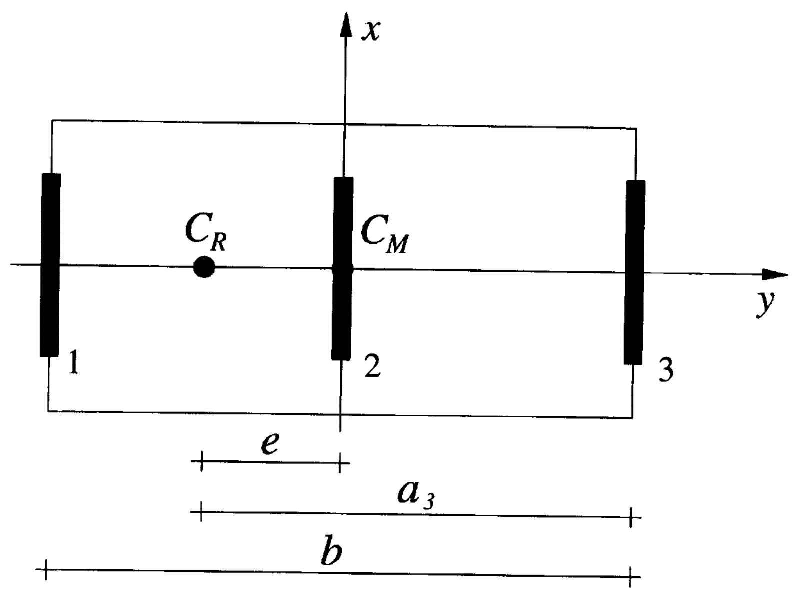

Fig. 2. Idealized one-storey system

The strength F_i of the i-th Element (Fig. 2) is obtained from the well known static linear formula:

$$F_i = F_0 \left[\frac{k_i}{\sum K} \pm \frac{e_d a_i k_i}{\sum K_\Phi} \right] \qquad i = 1,2,3 \tag{5}$$

where F_0 = design base shear or yield strength of the reference symmetric system (i.e. with $e_d = 0$), k_i = lateral stiffness of the i-th Element, $\sum K$ = total lateral stiffness, $\sum K_\Phi$ = total rotational stiffness with respect to C_R and a_i = perpendicular distance of the i-th Element from C_R. It is useful to normalize the length dimensions with respect to the mass radius of gyration about C_M, ρ, so that the results are not dependent on the dimensions of the floor slab. Letting $a_i^* = a_i/\rho$, $e_d^* = e_d/\rho$ and $\Omega^2 = \sum K_\Phi/(\rho^2 \sum K)$, (5) takes the following form:

$$F_i = F_0 \frac{k_i}{\sum K} \left[1 \pm \frac{e_d^* a_i^*}{\Omega^2} \right] \tag{6}$$

Since e_d takes different values for the elements located on either side of the floor, $\sum F_i > F_0$, i.e an overstrength $OS = \sum F_i / F_0$ exists relative to the pure symmetric case. This overstrength may be quite large for torsionally flexible systems, i.e. systems having Ω, which is the uncoupled torsional-to-lateral frequency ratio, smaller than unity. Furthermore, the code overstrength increases with the normalized eccentricity $e^* = e/\rho$, whereas it remains constant as the system uncoupled lateral period $T = 2\pi\sqrt{M/\sum K}$ changes. Models with normalized strength, i.e. whose element yield forces F_i are obtained from (6) by dividing through OS, were also studied in order to isolate the effects of the parameters Ω and e^*.

It is known (e.g. Rahnama and Krawinkler, 1993; Rutenberg and De Stefano, 1995) that the onset of instability is governed by the parameter $\theta_{eff} = \theta - r_0$ (Fig. 1 and Eq. 1). The value of θ depends on P, which in turn is related to the natural period T. Following Bernal (1987), the stability coefficient θ is taken as the ratio of the admissible interstory drift ratio to the seismic design coefficient. Using the UBC (ICBO, 1994) expressions for these two parameters lead to:

$$\theta = min(0.005;0.04/R_w)\frac{T^{2/3}R_w}{1.25ZIS} \qquad T \leq 0.7\,\text{sec} \qquad (7a)$$

$$\theta = min(0.004;0.03/R_w)\frac{T^{2/3}R_w}{1.25ZIS} \qquad T > 0.7\,\text{sec} \qquad (7b)$$

in which R_w = force reduction factor at working load level, Z = seismic zone factor, I = importance factor and S = site coefficient. For $I = S = 1.0$ and a given T, θ depends on Z and on R_w when its value is larger than 8.0 (7a) or 7.5 (7b). The maximum values of the two parameters were chosen, namely $Z = 0.4$ and $R_w = 12$ in order to demonstrate that, even for very low θ, the resulting R_t can already be rather low so that designs with high R_w values may not be safe. Since for 0.7 *sec* < T < 1.08 *sec* the use of (7b) results in θ values lower than those obtained from (7a), it was assumed that in the this range $\theta = \theta$ ($T = 0.7$ *sec*, from Eq. 7a). Figure 3 shows θ_{eff} vs T for $r_0 = 0.03$, which is believed to be a realistic ratio for the secondary slope for structures with bilinear force-displacement response (e.g. steel).

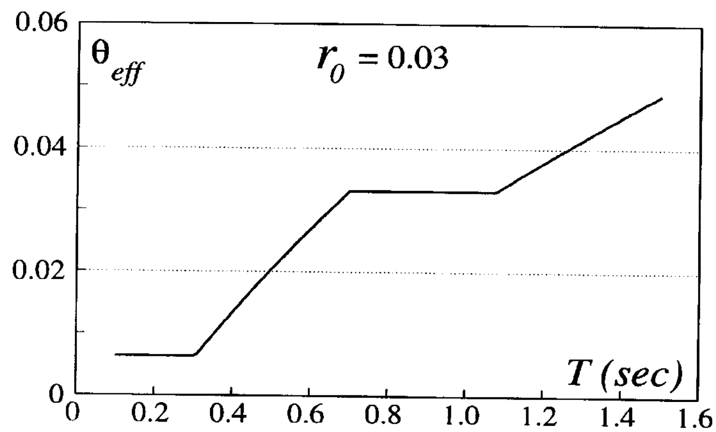

Fig. 3. Assumed variation of θ_{eff} with the system uncoupled period T

The effects of torsion were examined by varying the parameters Ω and e^* within their expected ranges: Ω = 0.8, 1.0, 1.25; e^* = 0.25, 0.50, 0.75. Note that for rectangular buildings loaded normal to their larger plan dimension the width b ranges between 2.5 ρ and 3.5 ρ, i.e. $e^* = 0.75$ corresponds to $0.2\,b \leq e \leq 0.3\,b$.

Table 1. Ω, e^* and OS for systems studied

ID	Ω	e^*	OS	ID	Ω	e^*	OS	ID	Ω	e^*	OS
O1E1	0.8	0.25	1.3192	O2E1	1.0	0.25	1.1991	O3E1	1.25	0.25	1.1574
O1E2	0.8	0.50	1.5244	O2E2	1.0	0.50	1.3619	O3E2	1.25	0.50	1.2695
O1E3	0.8	0.75	1.7137	O2E3	1.0	0.75	1.4882	O3E3	1.25	0.75	1.3528

Table 1 lists the nine models thus created, together with their overstrength (OS) values. It can be seen that OS increases with falling Ω and increasing e^*. As will be demonstrated subsequently, this property is responsible for the very weak dependence of R_t on Ω and e^*. In all these cases R_t was evaluated for a wide range of the fundamental period T: $0.1 \leq T \leq 1.5$ *sec*. Systems with code overstrength as well as strength-normalized ones (i.e. with no OS) were analyzed: the former were used to examine the response of actual systems, the latter ones were studied to better understand the influence of system parameters. For comparison, reference symmetric models, i.e. the 1-dof systems having a fundamental period equal to the uncoupled period of the considered asymmetric systems, were also analyzed.

For input, several earthquake time histories were chosen, all apparently having similar frequency content characteristics as measured by their practically equal a/v (a = peak ground acceleration, v = peak ground velocity) ratios, which was close to unity, and representing stiff soil conditions. The records are listed in Table 2, and their 5% response spectra, scaled to 0.4 g PGA, are shown in Fig. 4.

Table 2. Characteristics of the selected earthquake records

Earthquake	Record	Component	Duration (*sec*)	a (g)	v (*m/sec*)
Imp. Valley 1940	El Centro	S00E	53.8	0.348	0.334
Montenegro 1979	Petrovac	NS	19.6	0.438	0.413
Taft 1952	Lincoln Tunnel	S69E	54.4	0.179	0.177
Friuli 1976	Tolmezzo	EW	36.4	0.313	0.300
Chile 1985	Valparaiso	N50E	72.0	0.284	0.264

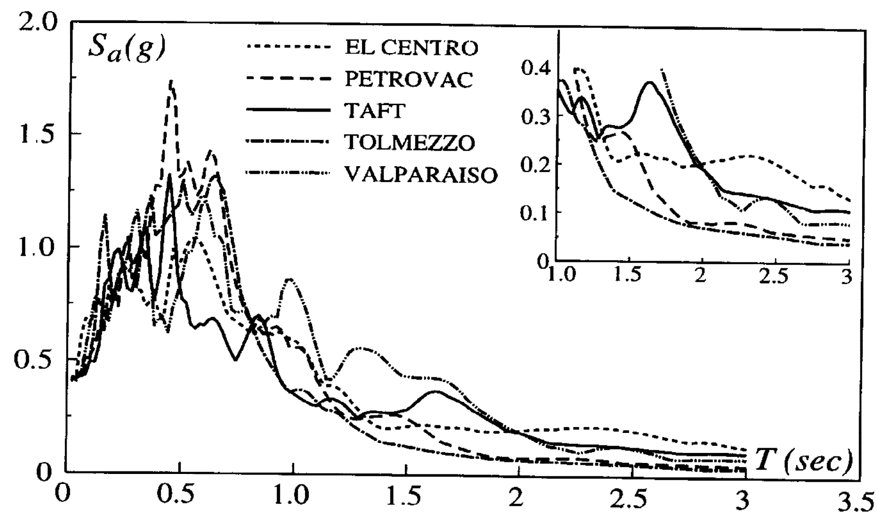

Fig. 4. Scaled acceleration response spectra of the selected earthquake records

The equations of motion in the linear range for the system in Fig. 2 can be written as:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \rho\ddot{\phi} \end{Bmatrix} + [C]\begin{Bmatrix} \dot{x} \\ \rho\dot{\phi} \end{Bmatrix} + \omega^2\begin{bmatrix} 1 & e^* \\ e^* & \Omega^2 + e^{*2} \end{bmatrix}\begin{Bmatrix} x \\ \rho\phi \end{Bmatrix} - \omega^2\theta\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{Bmatrix} x \\ \rho\phi \end{Bmatrix} = -\begin{Bmatrix} \ddot{x}_g \\ 0 \end{Bmatrix} \tag{8}$$

where: x = translation of C_M along x- direction relative to ground, ϕ = clockwise rotation about the vertical axis through C_M, $\ddot{x}_g$ = ground acceleration, $\omega = 2\pi/T$ = uncoupled lateral frequency. The matrix [C] is taken as proportional to the mass and stiffness matrices, and is calibrated to produce 5% damping in each of the two coupled modes of the system. In the nonlinear range, the equations of motion, when written in the incremental form, are similar, but the stiffness matrix is replaced by its tangent counterpart, and the functions x, ϕ and $\ddot{x}_g$ are replaced by their respective increments Δx, $\Delta\phi$ and $\Delta\ddot{x}_g$.

The system response was analyzed by a version of the computer code DRAIN-2D, and the effect of the geometric stiffness matrix was considered by means of fictitious elements with negative stiffness properties (Rutenberg, 1982).

RESULTS

Results for El Centro and Taft records are shown in Figs. 5 and 6 respectively for models with (5a and 6a) and without (5b and 6b) overstrength. Each of these figures displays R_l against the lateral period T, i.e R_l spectra, for the nine models described in Table 1. These figures also give R_l spectra for the reference 1-dof systems. The main feature of the results with *OS* appears to be their narrow spread throughout the wide range of periods. This is a reflection of the adequacy of (2) and (3) to control the response through the provision of overstrength and the judicious distribution of strength among the resisting elements.

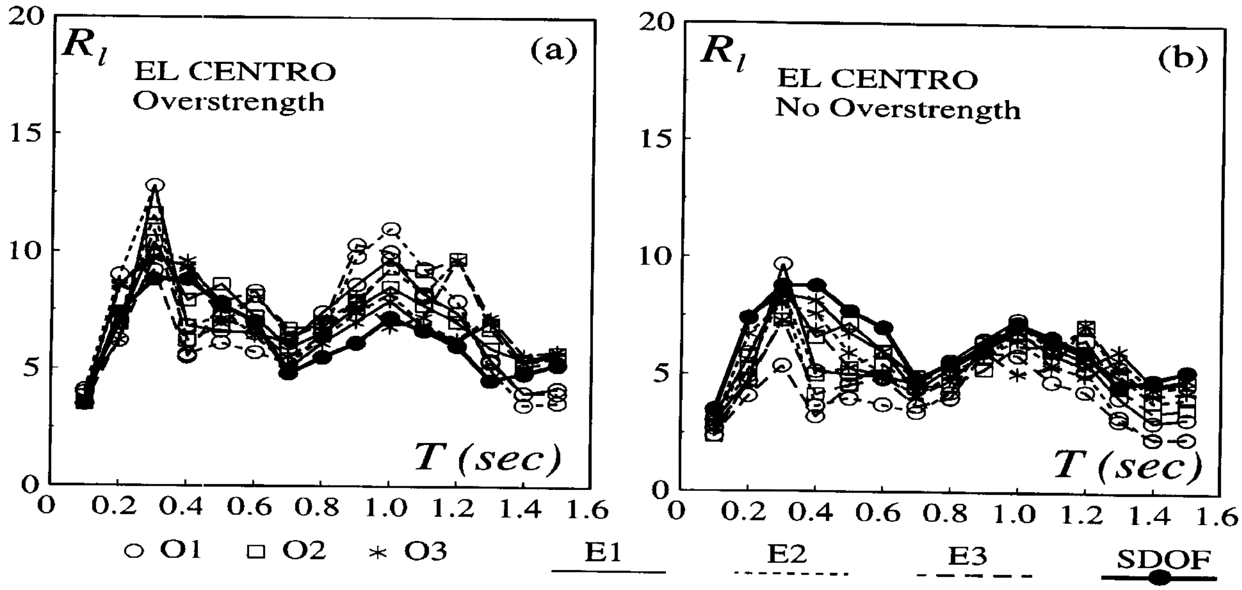

Fig. 5. Limiting reduction factors R_l for systems subjected to El Centro record

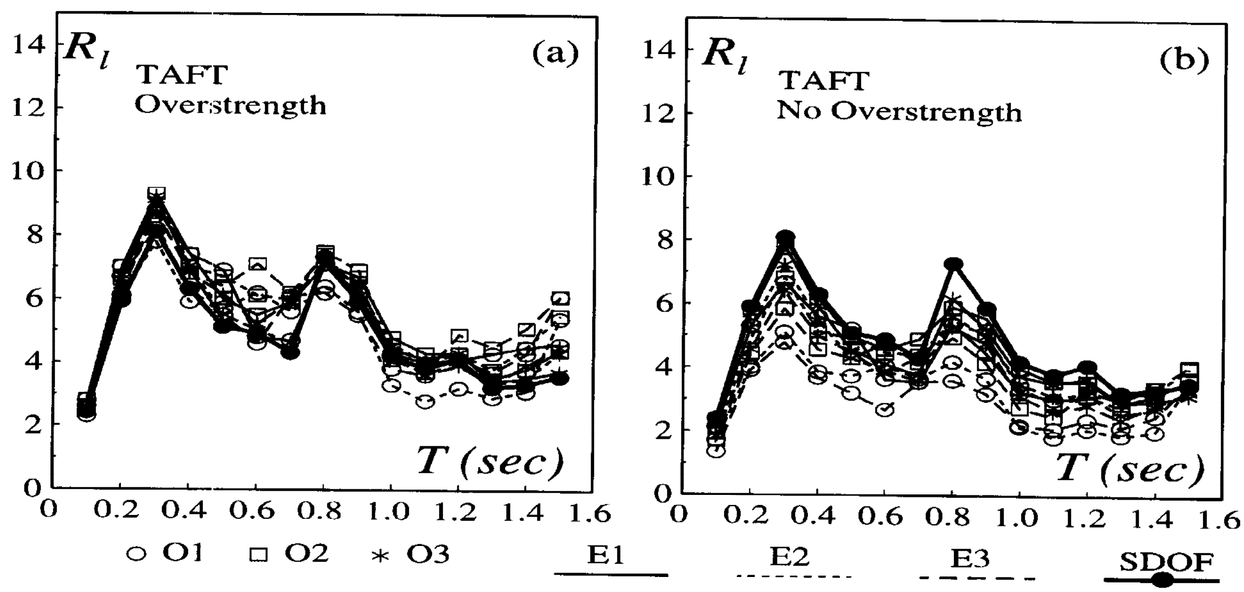

Fig. 6. Limiting reduction factors R_l for systems subjected to Taft record

The effects of *OS* on the different models can even better be appreciated from Figs. 7a and 7b, in which the mean ratio R_l (asymmetric) / R_l (symmetric) - or R_l ratio - of the five records listed in Table 2 is plotted against T for models with and without overstrength respectively. The beneficial effects of *OS* are manifested in Fig. 7a. Indeed, it might be claimed that the UBC overstrength is excessive, in particular at the larger T range of the R_l spectrum. Figure 7b shows quite clearly that for systems without *OS* having large eccentricities ($e^* \geq 0.5$) combined with low Ω (= 0.8), the reduction in R_l relative to the symmetric case is most pronounced (usually more than 30%). Note also the mild increase in the mean ratios as the period T elongates.

Some insight into the expected level of R_l can be gained from Figs. 5 and 6. Noting that $R_w = 12$ is commensurate with $R = 8$ (at limit state level), and that for the purpose of design R_l should be appreciably larger than R, it is evident that the R_l level computed for Taft (Fig. 6) is very low, and to a lesser extent this is also the case for El Centro.

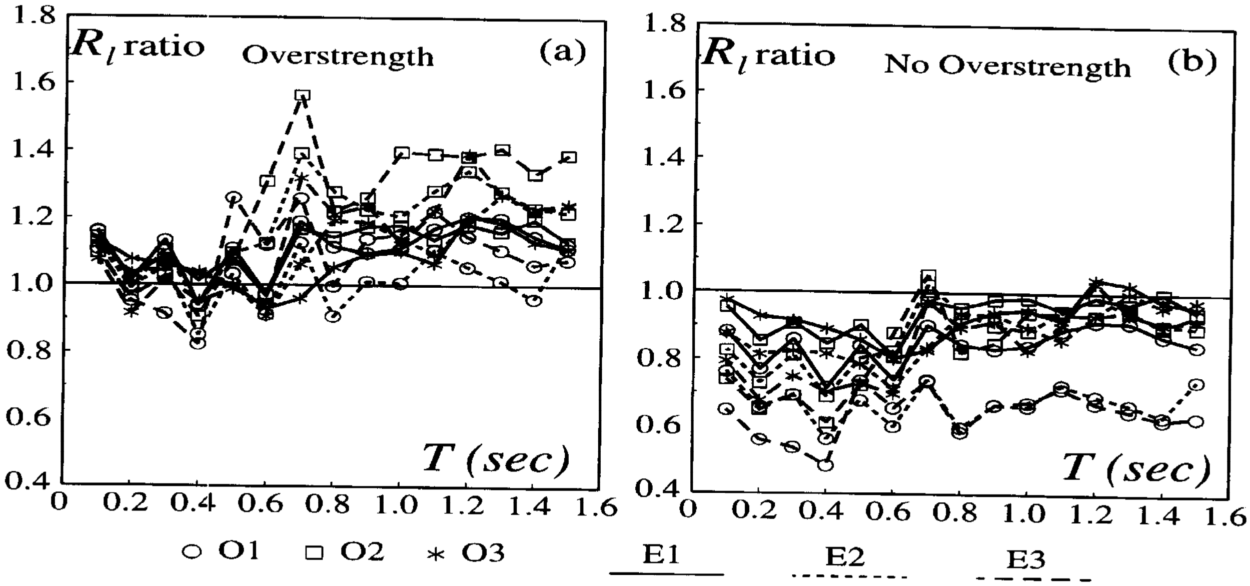

Fig. 7 Mean ratios R_l (asymmetric) / R_l (symmetric) of the five selected records

On the other hand, the R_l spectra for Petrovac and Tolmezzo records (Figs. 8 and 9 respectively) display relatively high values, when the low period range - for which the ductility demand is known to be high even for systems without noticeable P-Δ effects - is excluded. A comparison of the high period response spectra (Fig. 4 inset) of Taft and El Centro on the one hand and of Petrovac and Tolmezzo on the other, shows that the S_a levels of the latter two spectra are much lower. This, together with the shorter duration of the two records, may partially explain their high R_l values.

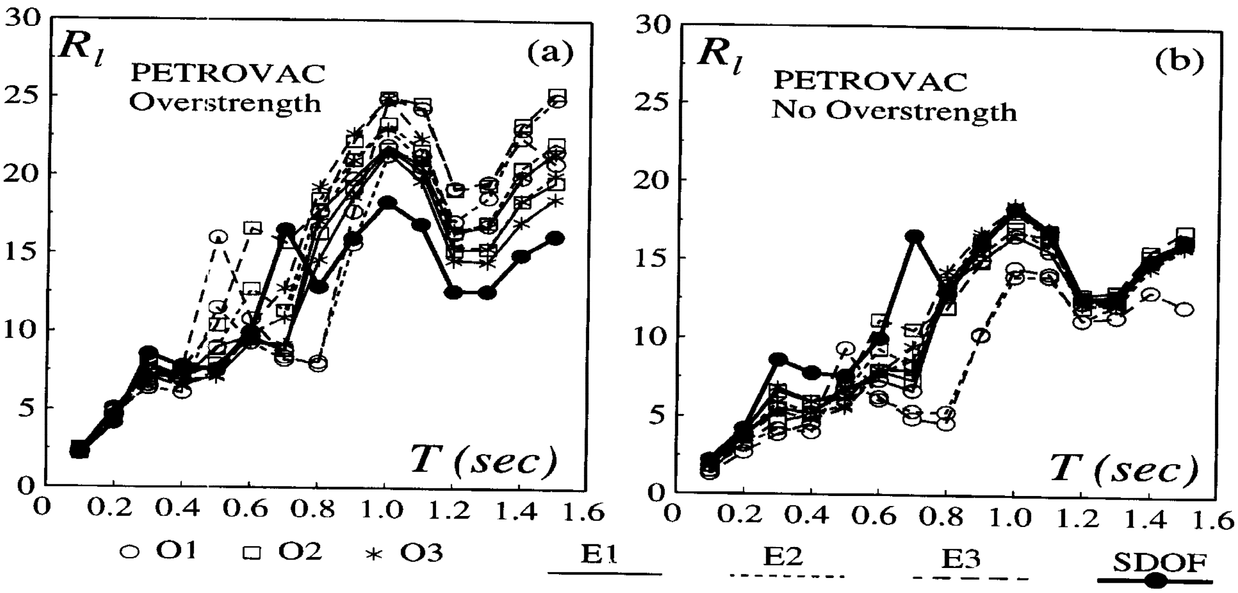

Fig. 8. Limiting reduction factors R_l for systems subjected to Petrovac record

SUMMARY AND CONCLUSIONS

Results of a study on the effects of asymmetry on the instability limits of the force reduction factor R_l have been presented. It has been shown that, for single storey models designed by the seismic provisions of the 1994 edition of the UBC, the torsional effects, which characterize the behaviour of asymmetric structures, are practically neutralized by the overstrength and by the judicious distribution of strength among the resisting elements. As a result, the R_l values obtained are similar and even larger than those computed for similar but symmetric (i.e 1-dof) systems when eccentricities e^* and torsional-to-lateral frequency ratios Ω are within the practical range of these parameters. This study has shown that for asymmetric as well as for symmetric structures P-Δ effects impose an upper bound on the force reduction factor R to be used in seismic design. For systems with moderately positive secondary slope in their response curve this upper bound may be much lower than the higher R values specified by seismic codes. Therefore, the choice of R does not depend only on the structural type and detailing, but also on the secondary response curve as modified by P-Δ effects. Finally, it was also observed that, although all the earthquake records used in this study have similar frequency content characteristics as manifested by almost identical a/v ratios, this does not ensure similar R_l spectra. In fact, substantial differences were present both in shape and in absolute levels.

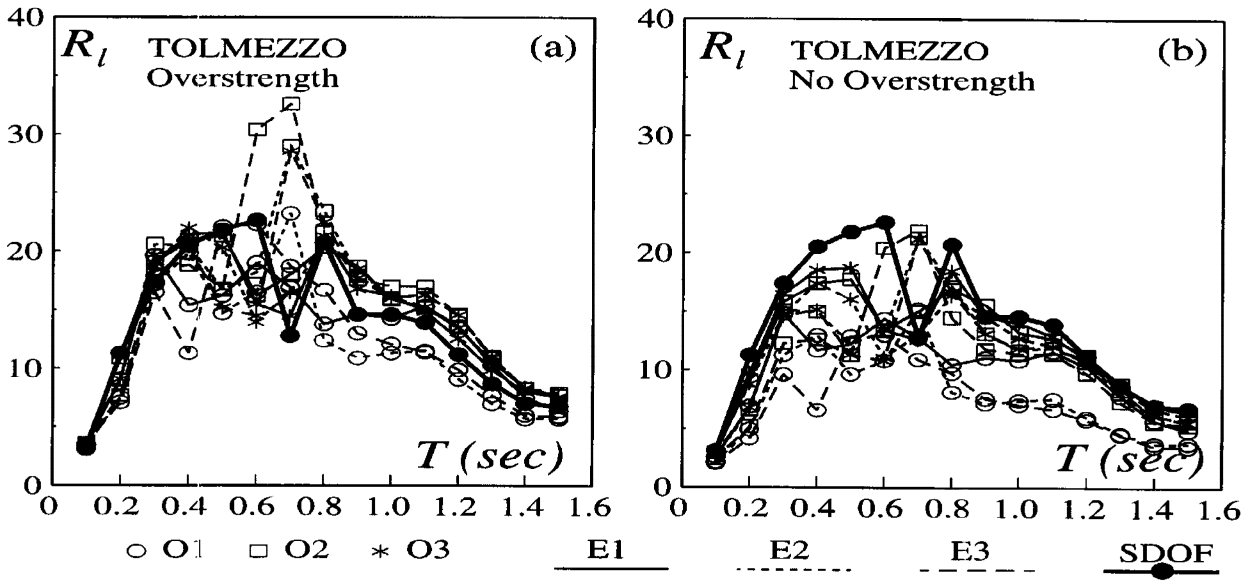

Fig. 9. Limiting reduction factors R_l for systems subjected to Tolmezzo record

ACKNOWLEDGEMENTS

The financial support of the Università di Napoli Federico II through its program for short duration exchange of researchers is gratefully acknowledged by the first author. The support of the Israeli Ministry of Construction and Housing and the Italian CNR for the continuous co-operative research of the authors is also acknowledged.

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