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PROPOSED DISPLACEMENT METHOD TO PREDICT THE LATERAL AND TORSIONAL RESPONSE OF A CLASS OF MULTISTORY STRUCTURES

Karen M. Houghton, Ph.D., P.E. and Steven L. McCabe, Ph.D., P.E.

Ellerbe Becket, 605 West 47th Street, Suite 200, Kansas City, MO 64112
University of Kansas, 2008 Learned Hall, Lawrence, KS 66045

ABSTRACT

An analytical procedure is developed that enables design engineers to predict the lateral and torsional response of a multistory structure utilizing the plane frame eigenvalues (translational periods) of the multistory structure and the three-dimensional response spectrum results of a similar single story structure. The key advantage of the analytical procedure is that it provides a simple, yet accurate means to predict seismic lateral displacements inclusive of torsion in multistory structures without doing a full scale three-dimensional dynamic analysis. In addition the analytical procedure is a displacement based method which has significant advantages when compared to a conventional force based method.

KEYWORDS

Torsion; displacement method.

INTRODUCTION

This paper presents an analytical procedure capable of determining the lateral and torsional response of low-rise multistory structures subjected to seismic motion . An upper envelope of 20 stories is imposed in keeping with the typical United States (U.S.) codes definition for low-rise structures. The U.S. codes typically classify a structure as low-rise if it is no more than 20 stories or is 240 feet or less at the maximum height. The development of this procedure was two-fold: (1) identify the lateral and torsional responses at each level in an N-story structure and determine how to predict these responses with respect to the first floor lateral and torsional responses and (2) identify a relationship between the first floor lateral and torsional response of a N-story to that of a similar single story structure. It is demonstrated in this paper that by identifying the two key relationships above, the lateral and torsional responses at any level in an N-story structure can be estimated accurately by using the lateral and torsional responses of a similar single story structure in conjunction with the plane frame eigenvalues of the N-story structure. The word "similar" in the remaining text implies that the two structures, single and N-story systems, are being considered for the same e/r ratio and that they both maintain approximately the same uncoupled fundamental torsional to lateral frequency ratio, $\omega_\theta / \omega_\Delta$. In order to identify the above relationship, the lateral and torsional responses of N-story and similar single story structures were obtained and examined for various e/r ratios.

ANALYSIS

In order to evaluate the lateral and torsional response of single or multistory structures the equations of motion must be determined and solved in a response spectrum analysis. It should be noted that there are two types of response spectrums and hence analytical approaches. One conventional approach is to use a ***acceleration*** response spectrum such as that provided by most U.S. codes. The second type of response spectrum is a ***displacement*** response spectrum which is typically identified by a tripartite logarithmic graph. The following analysis utilizes a displacement response spectrum.

In order to proceed with the analysis of the multistory structures the following N-story models and assumptions were defined with the intent to obtain results which were generally generic and easy to evaluate. Once a basic understanding of the lateral and torsional response in these multistory structures with respect to similar single story systems was understood, the analytical method could be developed. The N-story models and analytical assumptions are discussed below.

The N-story structures evaluated were 2, 3, 4, 5, 10, 15, and 20 stories with 10 foot floor heights. This number of structures was selected to provide a reasonable range of structures between 1 and 20 stories. In addition, the following assumptions were implemented for each structures.

- *Each floor is reduced to 3-DOF based upon the assumption that the floor systems behave as rigid diaphragms and that the columns are axially inextensible. In general this is the case for concrete slabs and composite decks which are considered rigid diaphragms.*
- *The type of eccentricity is limited to a single component eccentricity, thereby, reducing the potential 3-DOF per floor to 2-DOF per floor. The 2-DOF per floor are θ, the angular rotation of the diaphragm and Δ, the lateral displacement of the diaphragm.*
- *In addition, it will be assumed that the centers of mass and stiffness per floor coincide so that the eccentricity per floor is the same in magnitude and location. The stiffness between any two consecutive floors will be constant along the height of the structure in keeping with the UBC code definition of a regular structure*

Equations of Motion

Using the analytical assumptions above the equations of motion can be obtained for single and multistory systems. The equations of motion can be derived using rigid body dynamics which enables the mass distribution to be represented through the radius of gyration of the rigid diaphragm (Kan et al., 1977, Houghton et al., 1993). This approach simplifies the analysis procedure as compared to discrete nodal masses (Houghton et al., 1993). The equations of motion for a single story structure can be written in matrix format shown in Eq. 1

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{z}_{cg} \\ r\ddot{\theta}_{cg} \\ \ddot{x}_{cg} \end{Bmatrix} + \begin{bmatrix} K_z & \frac{e_x}{r}K_z & 0 \\ \frac{e_x}{r}K_z & \frac{K_\theta}{r^2} & -\frac{e_z}{r}K_x \\ 0 & -\frac{e_z}{r}K_x & K_x \end{bmatrix} \begin{Bmatrix} z_{cg} \\ r\theta_{cg} \\ x_{cg} \end{Bmatrix} = \begin{Bmatrix} -m\ddot{u}_{gz} \\ 0 \\ -m\ddot{u}_{gx} \end{Bmatrix} \quad (1)$$

The equations of motion can be obtained for N-story structures in a similar fashion as they are for single story structures. If each level of the multistory structure is isolated and considered as a single story system then the effective lateral and translational stiffness per floor can be obtained using the same equations as defined for the single story. However when determining the effective stiffness in the equations of motion, the coupling that occurs between adjacent floors must be accounted for. This coupling occurs between both the translational and rotational responses of adjacent levels. In addition, the system eccentricity per level is defined exactly as that for a single story system. Hence, a N-story system can be conveniently envisioned as "N" single story systems stacked upon each other. The primary difference is that the number of equations to solve has increased and more importantly the coupling effects are two-fold. One coupling effect occurs at each floor level between θ and Δ as a result of the floor eccentricity. A second coupling effect will occur between θ and Δ of adjacent levels which is independent of the eccentricity.

The equations of motion for a multistory system can be expressed in matrix form similar to the single story equations of motion by utilizing submatrices. In Eq. (1), the matrix notation and format of the equations of motion for a single story structure are maintained for the N-story structures by defining subvectors for the displacement, mass and stiffness of the multistory structure.

The displacement submatrices can be defined as the following:

$$u_z = \begin{Bmatrix} u_{1z} \\ u_{2z} \\ \cdot \\ \cdot \\ \cdot \\ u_{Nz} \end{Bmatrix} \qquad r u_\theta = \begin{Bmatrix} u_{1\theta} \\ u_{2\theta} \\ \cdot \\ \cdot \\ \cdot \\ u_{N\theta} \end{Bmatrix} \qquad u_x = \begin{Bmatrix} u_{1x} \\ u_{2x} \\ \cdot \\ \cdot \\ \cdot \\ u_{Nx} \end{Bmatrix} \tag{2}$$

The mass submatrix can be defined as:

$$m = \begin{bmatrix} m_1 & & & & \\ & m_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \\ & & & & & m_N \end{bmatrix} \tag{3}$$

The stiffness submatrix in the y direction can be defined as:

$$K_z = \begin{bmatrix} (K_{z1} + K_{z2}) & -K_{z2} & & & & \\ -K_{z2} & (K_{z2} + K_{z3}) & -K_{z3} & & & \\ & -K_{z3} & (K_{z3} + K_{z4}) & -K_{z4} & & \\ & & \cdot & \cdot & \cdot & \\ & & & \cdot & \cdot & -K_{zN} \\ & & & & -K_{zN} & K_{zN} \end{bmatrix} \tag{4}$$

The stiffness submatrices for K_x and K_θ have the same format as Eq. (4) except the definitions for K_x and K_θ are substituted for K_x above. The submatrices are then put into the generalized equations of motion in Eq. (1) to complete the equations of motion for the N-story structures.

Damping is not included in the equations of motion since it will be included in the response spectrum. The ground accelerations $\ddot{u}_{gz}(t)$ and $\ddot{u}_{gx}(t)$ along the z and x axes, respectively, are assumed to be uniform across the base of the structure. The torsional ground acceleration, θ_{cg}, will be assumed to be zero so that the lower bound of the torsional response is obtained. It should be noted that in reality this acceleration component may be non-zero and may contribute to the torsional response.

Response Spectrum Analysis

The normalized acceleration response spectrum presented in most U.S. codes was converted to an equivalent normalized displacement response spectrum and then used for the ground motion in the three-dimensional dynamic response spectrum analysis. The resulting spectral quantities were S_D=13.75 in., S_v=25.57 in/s, and S_A=1.0g. This response spectrum was selected since it is a widely accepted design response spectrum suggested by both the U.S. UBC and SEAOC building codes. The equations of motion above were solved for each N-story structure for twelve different magnitudes of normalized eccentricity, e/r, where $0 \leq e/r \leq 0.35$. This range on e/r is equivalent to a maximum system eccentricity of 30 per cent which would typically be considered an upper bound for a system eccentricity in an actual structure.

RESULTS

The objectives for evaluating the lateral and torsional responses were two-fold. First, the lateral and torsional response at each level within the N-story structures were examined in order to establish a means of estimating each response per upper level with respect to level one. Secondly, a method which relates the lateral and torsional response of a 3-DOF single story structure to a N-story structure was developed. In order to meet the above objectives, the lateral and torsional response of each floor and corresponding frequency was examined in each N-story structure and cross examined with that of a single story structure. The results pertaining to the first objective above are presented first while those pertinent to the second objective are presented last.

Lateral and Torsional Response in N-Story Structures

In order to establish a basis for the lateral and torsional displacements per level of an N-story system, the lateral and torsional displacements were calculated at each level for the twelve various e/r ratios. Based upon the results, a best-fit-curve utilizing the Least Squares Method was obtained for each response at each story level as a function of e/r. This effectively provided two best-fit-curves, one for torsion and one for translation, per story level of each N-story system. The curves made it possible to examine the lateral and torsional response of each floor as a function of the normalized eccentricity ratio, e/r, story level, and total number of floors, N.

Typical plots of the actual lateral and torsional responses versus the corresponding best-fit-curves at each level within the various N-story structures are not shown for brevity. However, it was found that for any one e/r ratio the torsional response increases while the translational response decreases with story height regardless of the number of stories involved. Furthermore, the best-fit-curves for each response at each level coincide with the actual responses obtained from the three-dimensional dynamic analysis. This suggests that the best-fit-quadratic-curve is an accurate model of the lateral and torsional responses at each level in the N-story structures. However, a more scientific means to verify that the quadratic curves are indeed representative of the true responses is to examine the correlation coefficients obtained from the Least Squares Method. An examination of the correlation coefficients verified that the best-fit-curves captured 98 per cent or more of the data. Hence, the quadratic curve is an accurate mathematical model of the translational and torsional responses.

Frequency Relationships

In order to establish a basis in understanding the lateral and torsional responses of N-story structures, the translational and torsional frequencies associated with each floor were examined. Since it was desired to find a relationship between the upper levels with respect to level one of each N-story structure, a ratio of the upper level frequencies to the first level frequency was computed for each of the twelve eccentricity cases. This effectively produced two sets of data per story level, one for translation and one for torsion with twelve ratios per set.

The lateral and torsional frequency ratios were computed and plotted with e/r as the abscissa for each of the N-story systems. It was found that the frequency ratios of each floor with respect to level one remains constant regardless of the eccentricity, e/r, in each of the N-story structures. In addition, the frequency ratios increase with floor level, i.

It is interesting to note that a comparison of the translational frequency ratios to the corresponding torsional frequency ratios revealed that the ratio magnitudes are identical in each N-story structure and remain constant regardless of the system eccentricity magnitude. More simply, the frequency ratio for level i to level one in a N-story structure is the same regardless if the torsional or translational frequencies are used in computing the frequency ratio. It will be demonstrated subsequently, that this will be a convenient relationship in simplifying the proposed analytical method.

Lateral and Torsional Response Behavior Between Upper Levels with Respect to Level One

A ratio for each eccentricity case utilizing the individual floor responses to the first level responses was computed. The two response ratios, one for torsion and one lateral, were plotted with respect to the system eccentricity. It was found that the lateral and torsional response of level one to any level above it remains constant regardless of the system eccentricity and multistory system. In addition, the torsional response (absolute) ratio increases with the level designation while the lateral response decreases. The behavior of the torsional response ratio indicates that the torsional response of any one level with respect to the first level in the structure is a function of the level number or designation which implies the uncoupled floor frequency.

PROPOSED ANALYTICAL METHOD

Evaluating the Lateral and Torsional Response Between Single and Multistory Structures

The final task was to establish a lateral and torsional response relationship between level one of each N-story structure to the corresponding responses of a similar single story structure. In order to establish this relationship the lateral and torsional responses in addition to the corresponding frequencies at level one in each N-story structure were examined with respect to those of a similar single story structure.

It was found that the lateral and torsional responses at the first level of each N-story structure to the single story structure increases with the total number of floors, N (Houghton et al., 1993). This behavior in the first level lateral and torsional responses with respect to the total number of stories can be explained by examining the frequencies and the number of contributing modes of vibration involved in the responses. With each floor addition, the contributing modes of vibration explicitly increases when computing the responses for any one level. In addition, the fundamental lateral frequencies decrease as N increases since the structure becomes softer with additional floor heights. The taller the structure becomes, the lateral periods become larger, and hence the frequencies decrease; therefore, the lateral spectral displacements used in the modal combination method are smaller. However, the effects of the higher modes of vibration due to additional stories out weigh the smaller spectral displacements and hence the first level responses increase as a function of N for any normalized eccentricity, e/r,.

Linking the Torsional and Lateral Response of Level One with the Upper Levels in a N-Story Structure

In general when the torsional response of the upper levels was evaluated with respect to the first level, it was found that the torsional response could be expressed as a function of two parameters. The first parameter was the ratio of the upper level frequencies to that of the first level. It is convenient to note that the frequency ratio can be computed from the translational frequencies or torsional frequencies of the two floors, since the ratio will be the same regardless of the approach. Secondly, the torsional response of the upper level with respect to that experienced by the first level is a function of the story number, i, and the maximum story level, N. To predict the torsional response of the upper levels in a N-story structure knowing the torsional response of level one the following relationships were found with Eq. (5) applicable for N < 4 and Eq. (6) for N > 5.

$$\theta_i = \frac{\sqrt{\omega_i / \omega_1}}{[N+i]^x} * \theta_1 \qquad (i=2...N) \tag{5}$$

$$\theta_i = \frac{\omega_i / \omega_1}{[N+i]^x} * \theta_1 * \beta \qquad (i=2...N) \tag{6}$$

The first term of Eq. (5) and (6) merely estimates the coupling effects of level one with each level above and is not restricted to the torsional response of the system. Therefore, the first term in Eq. (5) and (6) can be used to modify the lateral displacements as well. This enables one to estimated not only the torsional response but the lateral response at each level of the structure. Hence, the translational displacements per level can be defined as:

$$\Delta_i = \frac{\sqrt{\omega_i / \omega_1}}{[N+i]^x} * \Delta_1 \qquad (i=2...N) \tag{7}$$

$$\Delta_i = \frac{\omega_i / \omega_1}{[N+i]^x} * \Delta_1 * \beta \qquad (i=2...N) \tag{8}$$

The exponent on the denominator in Eqs. (5) through (8) was found to vary with respect to the maximum number of stories the structure had. The values for x are tabulated in Table 1.

In order to verify Eqs. (5) through (8), the equations were computed for the applicable values of N for each of the levels in the N-story structures. The resulting estimated torsional and lateral displacements of each floor level were plotted on one graph with the actual system response obtained from a three-dimensional response spectrum analysis. This verification was carried out for each of the N-story structures. Figures 1 and 2 are example plots of this verification for the five story structure. It can be seen in Figs. 1 and 2 that the curves resulting from Eqs. (5) through (8) are similar in shape and magnitude to the actual torsional response curves obtained from the three-dimensional analysis. The resulting error obtained in the comparison did not exceed 1% for any N-story system.

Presently, Eqs. (5) through (8) are limited since any application of the equations requires the torsional and lateral response of the first level within a N-story structure to be known. In order to obtain the responses of the first level, a three-dimensional dynamic analysis is required of the N-story system and thus the analytical computations become less economical and more time consuming. Since a three-dimensional analysis on a single story structure is much simpler than that of a multistory system, it would be beneficial if the lateral and torsional response of the multistory system could be related to that of a similar single story system. Therefore, the next task was to develop this relationship between the single and multistory system.

Lateral and Torsional Response Relationships Between N-Story and Single Story Structures

The most complicated task was to develop a relationship between the responses of a single and similar multistory structure subjected to the same ground motion. The benefit of establishing such a relationship would require two analytical steps to determine the response of N-story structures; (1) complete a three-dimensional analysis of a similar single story structure to obtain the lateral and torsional response, (2) using the responses of the similar single story structure, estimate the first level torsional and lateral response of a multistory system. Once the first level responses were known for the multistory system, Eqs. (5) through (8) could be utilized to estimated the responses of the upper levels within the system.

In order to identify a means of adjusting the first level torsional response of a single story system to match that of a multistory system, it was convenient to look at a ratio of the first level torsional response of the single story to multistory structure. It was concluded that the ratio of the first level torsional response of the single story to multistory structure remains constant regardless of the eccentricity considered. In addition, the ratio increases with an increase in N or total number of floors in a structure. It was found that the amplification of the first level torsional response of the multistory structures with respect to the single story system was the following:

$$\theta_{1,N} = \frac{\omega_{1,N} / \omega_{1,1}}{[N]^{y}} * \theta_{1,1} \tag{9}$$

Similarly the translational displacements can be obtained from:

$$\Delta_{1,N} = \frac{\omega_{1,N} / \omega_{1,1}}{[N]^{y}} * \Delta_{1,1} \tag{10}$$

Where N is the total number of floors in the multistory structure. In addition, the first subscript designates the floor level, while the second subscript designates the maximum story number. The ratio of the circular frequencies can be based on the translational or torsional frequencies corresponding to level one of the single and multistory systems. It has previously been noted that either of the of frequency ratios will provide the same ratio. In addition, this ratio is constant regardless of the eccentricity and therefore it is convenient to examine the systems when the system eccentricity is zero.

The exponent y serves to adjust the response predictions with respect to the N value. The exponent value, y, and the corresponding maximum error obtained by using Eq. (9) and (10) was well below 0.50 per cent. The error was based on the true three-dimensional response spectrum results and the corresponding results of Eq. (9) and (10). Figure 3 compares the torsional response at the first level of the N-story systems obtained with Eq. (9) to the actual torsional response obtained from a three-dimensional response spectrum analysis. Although it is not presented similar results were obtained when Eq. (10) was utilized to predict the first level translational displacements.

CONCLUSION

Several key relationships concerning the lateral and torsional response of multistory structures to similar single story systems have been identified and discussed herein. As a result of these relationships a simplified analytical method for N-story was developed. The two main phases in developing the analytical method were as follows: (1) identifying a relationship between the lateral and torsional responses of the first level with respect to the upper levels in a N-story system and (2) identifying a relationship between the first level lateral and torsional responses of a single story structure to the corresponding first level responses in a similar N-story system. An advantage of the procedure is that the three-dimensional analysis of the multistory structures has been eliminated without compromising the accuracy of estimating the lateral or torsional response within the N-story system..

Hence, the lateral and torsional response at any level in a N-story system can be predicted based upon a three-dimensional response spectrum analysis of a similar single story structure and a plane frame eigen solution of the N-story system. The accuracy of the proposed procedure for obtaining the torsional response at each level of a N-story system has been verified (Houghton et al., 1993) and shown to be less than 1 per cent.

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Table 1. Exponent Values for Multistory Structures

N	2	3	4	5	10	15	20
x	0.353	0.339	0.322	0.33	0.25	0.225	0.2
y	0.45	0.53	0.58	0.61	0.69	0.73	0.75

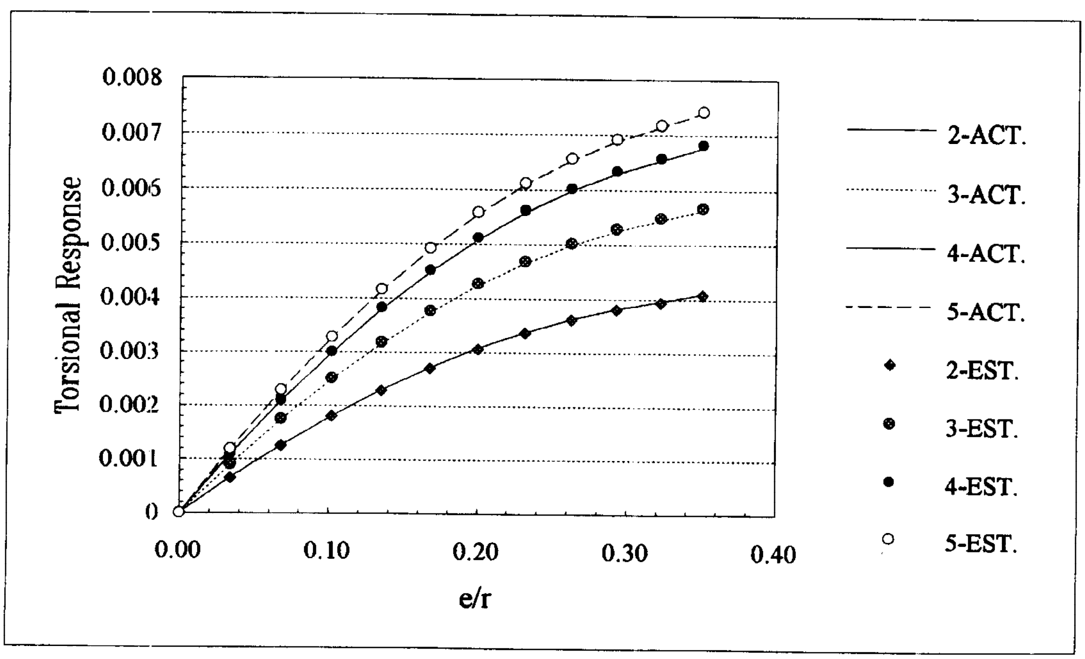

Figure 4.33 Actual Verus Estimated Torsional Response
Per Floor of 5-Story

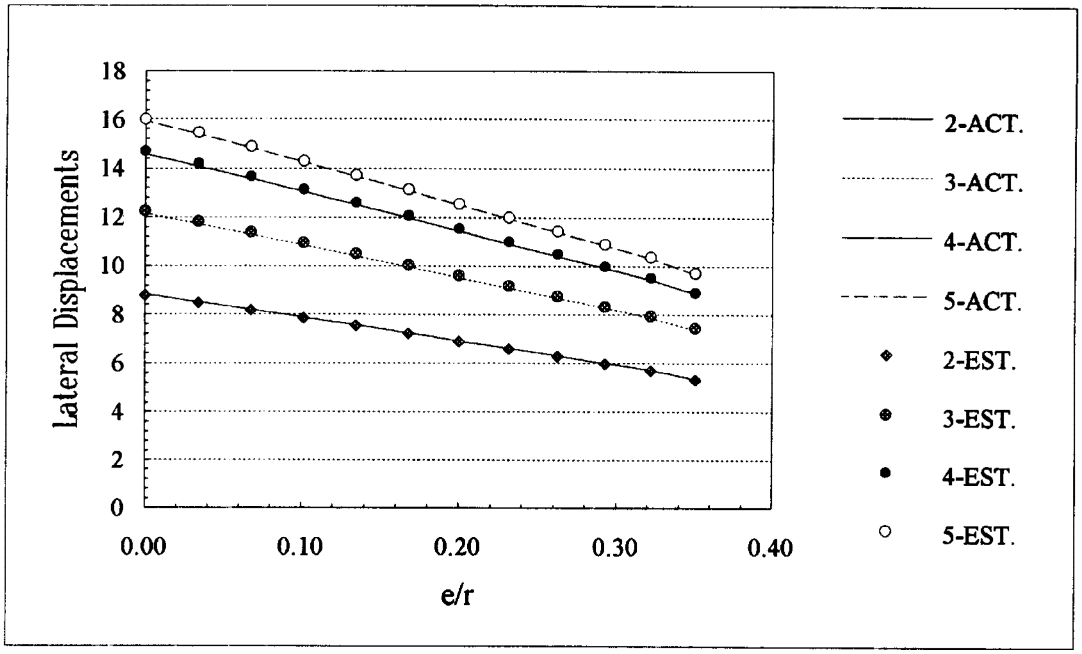

Figure 4.40 Actual Verus Estimated Lateral Displacements Per Floor of 5-Story

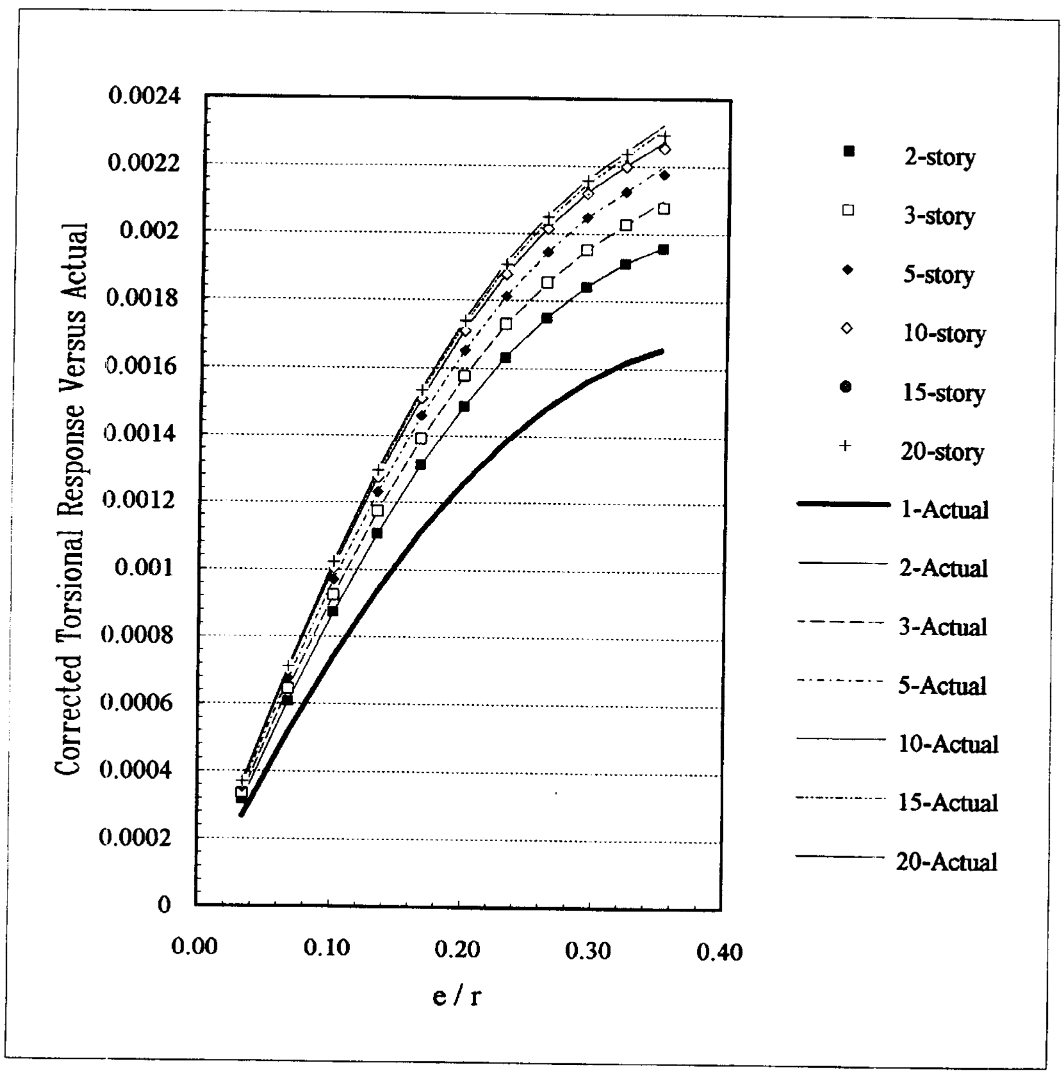

Figure 4.45 Corrected First Floor Torsional Response Versus Actual Torsional Response of N-Story Structures