A STUDY ON ESTIMATION METHODS FOR RESPONSES OF NONLINEAR HYSTERETIC STRUCTURES

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ABSTRACT

Construction of seismically isolated bridges will possibly increase in the future, which consequently suggests problems on how to evaluate inelastic responses conveniently. Three different methods for estimating the response of single-degree-of-freedom system with bilinear hysteretic restoring force are discussed in this report, and estimated values for response of each method are compared with the results of time history analysis.

KEYWORDS

Bilinear model; response spectrum; equivalent linearization; seismic isolation bearings; inelastic response.

INTRODUCTION

In recent years, the design and constructions of seismically isolated bridges, otherwise known as menshin bridges have increased. These bridges utilize seismic isolation bearings for dispersing and reducing seismic load. Further, after the tragic Hyoukoken-nanbu Earthquake, it is thought that application of seismic isolation bearings to existing bridges as antiseismic reinforcement will increase. Since seismic isolation bearings have nonlinear hysteretic characteristic, it is very important for practical design how it should be modelled and how its elasto-plastic response could be estimated accurately and conveniently. In this paper, three methods of estimation for responses are discussed and evaluated for single-degree-of-freedom model with bilinear hysteretic restoring force. These are the following.


3 Method C. A method which estimates maximum responses by balancing the total input energy of the earthquake with the total absorbed energy of the seismic isolation bearing.

Each estimation method is evaluated by comparing its results with that of time history analysis. The level of seismic coefficient method (Specifications for Highway Bridges, 1990) and the level of design ultimate horizontal strength method during an earthquake (Manual for Menshin Design of Highway Bridges, 1992) are considered as seismic load levels for evaluation. Herein, these levels are referred to as Level I and Level II, respectively. Also, the same investigation is conducted according to three types of ground, i.e. Type I, Type II and Type III (Specifications for Highway Bridges, 1990). Type I corresponds to the base rock ground, Type III to the weak ground among alluvial soil layers, and Type II to those of diluvial soil layers or alluvial soil.
layers which do not belong to types I nor III.

METHOD OF ANALYSIS

The responses estimated by using methods A, B, and C are compared with the results of time history analysis. In the time history analysis, the structure is subjected to standard seismic waves according to the types of ground. The analytic model and the three estimation methods are discussed below.

Analytic Model

The bilinear model is widely used for elasto-plastic behaviour of materials and members because of its simplicity. It is thought that this would be a good model for materials with elasto-plastic restoring force, like isolation bearings which are composed of steel damper and laminated rubber (e.g. lead rubber bearing or high damping rubber bearing).

Figure 1 shows the bilinear model for single-degree-of-freedom system and its restoring force characteristic.

![Bilinear Model Diagram]

\[
W = Mg = 1000 \text{ (tf)}
\]

\[
k_1 = \frac{4\pi^2}{T_1} \text{ (tf/cm)}
\]

\[
k_2 = \gamma k_1 \text{ (tf/cm)}
\]

\[
h = 0 \%
\]

\[
T_1 : \text{first natural period}
\]

\[
k_1 : \text{elastic stiffness}
\]

\[
k_2 : \text{post-yield stiffness}
\]

\[
\gamma : \text{stiffness ratio}
\]

\[
Q_y : \text{yield load}
\]

\[
M : \text{mass of the system}
\]

\[
g : \text{gravitational acceleration}
\]

\[
h : \text{damping constant}
\]

Fig. 1. Single-degree-of-freedom model with bilinear hysteretic restoring force.

The values of the parameters of bilinear model are indicated in Table 1. The combination of 9 cases of hysteresis and 6 cases of yield load gives a total of 54 cases for analysis. The same cases are considered for each type of ground.

<table>
<thead>
<tr>
<th>Hysteresis No.</th>
<th>(T_1) (sec)</th>
<th>(T_2) (sec)</th>
<th>(k_1) (tf/cm)</th>
<th>(k_2) (tf/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-a</td>
<td>0.60</td>
<td>1.20</td>
<td>111.90</td>
<td>27.98</td>
</tr>
<tr>
<td>1-b</td>
<td>0.60</td>
<td>1.47</td>
<td>111.90</td>
<td>18.65</td>
</tr>
<tr>
<td>1-c</td>
<td>0.60</td>
<td>1.70</td>
<td>111.90</td>
<td>13.99</td>
</tr>
<tr>
<td>2-a</td>
<td>0.80</td>
<td>1.60</td>
<td>62.94</td>
<td>15.74</td>
</tr>
<tr>
<td>2-b</td>
<td>0.80</td>
<td>1.96</td>
<td>62.94</td>
<td>10.49</td>
</tr>
<tr>
<td>2-c</td>
<td>0.80</td>
<td>2.26</td>
<td>62.94</td>
<td>7.87</td>
</tr>
<tr>
<td>3-a</td>
<td>1.00</td>
<td>2.00</td>
<td>40.28</td>
<td>10.07</td>
</tr>
<tr>
<td>3-b</td>
<td>1.00</td>
<td>2.45</td>
<td>40.28</td>
<td>6.71</td>
</tr>
<tr>
<td>3-c</td>
<td>1.00</td>
<td>2.83</td>
<td>40.28</td>
<td>5.04</td>
</tr>
</tbody>
</table>

Six values of yield load are considered for each hysteresis, i.e.

\[
Q_y = 25, 50, 75, 100, 125, 150 \text{ tf}
\]
Estimation Methods for Inelastic Responses

Static Response Estimation Method based on Seismic Coefficient (Method A). This is a static analysis based on seismic coefficient described in Manual for Monohin Design of Highway Bridges (1992), and a simple method for estimating inelastic responses — the use of seismic coefficient deletes the application of dynamic analysis. For bridges with seismic isolation bearings, where the stiffness of the bearing is generally small compared to that of substructure, the primary vibration mode is predominant. For such reason, simple estimation method is applicable. The left side of Fig. 2 shows the flow chart of Method A.

There are upper limits set for horizontal seismic coefficients for design. In Level I, the limits for ground types I, II, and III are 0.2G, 0.25G, and 0.3G, respectively. In Level II, upper limits are 0.7G, 0.85G, and 1.0G, respectively.

![Flow chart of Method A](image)

**Fig. 2. Flow chart of Method A (left) and Method B (right).**

Estimation Method using Response Spectra (Method B). This estimates maximum response values by using method of equivalent linearization and the acceleration response spectra described in Specifications for Highway Bridges (1990). The acceleration response spectrum is calculated by modifying the basic acceleration response spectrum with correction factors according to zone, importance classification, and damping constant. There are upper limits set for acceleration response spectra. In Level I, the limits for ground types I, II, and III are 0.4G, 0.5G, and 0.6G, respectively. In Level II, upper limits are 1.4G, 1.7G, and 2.0G, respectively. Figure 2 (right side) and Fig. 3 shows the flow chart of Method B and the basic acceleration response spectra for Level II, respectively.
Fig. 3. Basic acceleration response spectra for Level II. (Damping constant is 5%.)

Estimation Method by Equilibration of Energies (Method C). This method uses the equilibration of energies for estimating maximum responses. That is, the total input energy from seismic waves is balanced to the total absorbed energy of the seismic isolated bearing.

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**Flow Chart of Method C**

1. Determine the hysteretic characteristic of restoring force of the structure
   \[ Q = f(Q_p, k_1, k_2) \]

2. Assuming maximum displacement, \( \delta_{\text{max}} \) (initial value = yield displacement)

3. Calculate the equivalent natural period and equivalent damping constant
   \[ T_e, h_e, W = W_e + \alpha W_{ip} \]

4. Calculate the total input energy due to earthquake
   \[ E(t_e) = RT_e, h_e \]

   - If \( E(t_e) \) is not zero, then
     - \( W_{\text{total}} = E(t_e) \)
   - If \( E(t_e) \) is zero, then
     - \( W_{\text{total}} = 0 \)

5. Determine the estimated response values
   \[ \delta_{\text{max}} \]

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Fig. 4. The total absorbed energy of the structure and flow chart of Method C.

The total absorbed energy is described in Fig. 4 with reference to *Recommendation for the Design of Base Isolated Buildings* (1993). The total absorbed energy \( W_{\text{total}} \) of the structure is defined as the sum of elastic strain energy \( W_e \) (i.e. shaded area in Fig. 4) due to post-yield stiffness \( k_2 \), and hysteretic absorbed energy.
during one cycle $W_{1p}$ (i.e. area of the loop in Fig. 4) multiplied by a certain factor, hysteretic absorbed energy factor $\alpha$.

In Recommendation for the Design of Base Isolated Buildings (1993), $\alpha$ is equal to 2. In this analysis, the value of $\alpha$ is computed from

$$E(t_0) = W_e + \alpha W_{1p},$$

where $E(t_0)$ is the total input energy of seismic waves. The average values of $\alpha$, for the 54 cases considered, are calculated according to Level I and Level II, as well as to the types of ground; provided that those cases which do not satisfy antiseismic conditions indicated in Recommendation for the Design of Base Isolated Buildings (1993) are neglected. In Level I, the values of $\alpha$ for ground types I, II, and III are 1.62, 3.29, and 6.85, respectively. In Level II, these are 2.40, 3.26, and 10.31, respectively. The computed average values of hysteretic absorbed energy factor $\alpha$ are then used in Method C. The flow chart of Method C is shown in Fig. 4 (left).

**Time History Analysis for Nonlinear Models**

In order to evaluate the three estimation methods mentioned above, time history analysis for bilinear model is applied and method of linear acceleration is used in the analysis. The seismic waves described in Manual for Menshin Design of Highway Bridges (1992), i.e. for Level I, and in Specifications for Highway Bridges (1990), i.e. for Level II, are inputted according to types of ground. The maximum acceleration response and maximum relative displacement are computed with respect to the input seismic waves.

**RESULTS**

The maximum response acceleration and relative displacement estimated by using the three methods discussed are compared and checked with the results of time history analysis, assuming that the estimated values correlates linearly with the results of time history analysis. Figure 5 and Fig. 6 shows the results for Level I and Level II, respectively, where $R$ is coefficient of correlation. Figure 7 indicates the values of hysteretic absorbed energy factor $\alpha$ of ground Type II. From these results, the following have been ascertained.

1. Method A. In Level I, the estimated values show good correlation with the results of time history analysis. These also exceed the results of time history analysis, thus, indicate safety estimations (Fig. 5). However, in Level II, the estimated values become lesser than the results of time history analysis as response values increase. Also, coefficient of correlations decrease as ground becomes weak (Fig. 6).

2. Method B. In both levels, estimated values correlate closely with results of time history analysis. In ground Type III of Level II, estimations are less than the analytic results.

3. Method C. In both levels, estimated values are approximately equal to the results of time history analysis, irrespective to types of ground. Moreover, safety estimated values can be obtained for small values of $\alpha$, and good results can be obatained for $\alpha \approx 2$, in both levels.

**CONCLUSIONS**

To summarize, the results of the three estimation methods for inelastic responses of structure, assumed as single-degree-of-freedom model with bilinear hysteretic restoring force, confirm the following:

1. Estimation method for static response based on seismic coefficient is a simple estimation method. But since there are upper limit values, its degree of correlation with respect to the results of time history analysis decreases and its results show small values, in cases where response values are large.

2. Estimation method using response spectra and equivalent linear method is effective and reveals good results with time history analysis. However, since estimations depend on acceleration response spectra, establishment of standard acceleration response spectra is very important.

3. Estimation method by equilibration of energies is effective, irrespective to seismic load level and types of ground, although requires adequate value for hysteretic absorbed energy factor $\alpha$. 
Fig. 5. Comparison of maximum estimated response values of methods A, B, and C for Level I. (In here, R represents the coefficient of correlation.)
Fig. 6. Comparison of maximum estimated response values of methods A, B, and C for Level II. (In here, R represents the coefficient of correlation.)
Fig. 7. Hysteretic absorbed energy factor of ground Type II, for Level I and Level II.

REFERENCES