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RELIABILITY BASES FOR THE DETERMINATION OF SEISMIC DESIGN RESPONSE SPECTRA

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ABSTRACT

A method is presented for the determination of site response spectra for seismic design of nonlinear structural systems. It is consistent with a reliability-based approach that accounts for uncertainties associated to both, earthquake ground motion characteristics and structural properties. For systems with multiple failure modes the acceptance criterion is expressed in terms of either a computed failure probability or a safety index, both related to the exceedance of the deformation capacity of a critical section or sub-assemblage of the structural system. Several illustrative examples are presented of the computation of failure probabilities and safety indexes for multistory buildings frames. The possibility of using simple equivalent nonlinear systems to replace the corresponding detailed systems is explored, and a comparison is made of the safety levels produced with both the detailed models and their simplified counterparts.

KEYWORDS

Deformation-based criteria; dynamic response; nonlinear systems; site response spectra; system reliability.

INTRODUCTION

The studies reported in this paper were stimulated by the need to develop a systematic approach to establish site response spectra at a specific location in the Valley of Mexico, so that the resulting system reliability levels should either coincide with a specified value or be consistent with those implicit in design spectra for sites taken as reference points for calibration.

The Technical Norms for seismic design of the Federal District Building Code of Mexico (Departamento del Distrito Federal, 1995) specify design response spectra for the zones designated in that document as I to III. In addition, the latter is divided into two sub-zones, in order to account, at least approximately, for the spatial variability of the intensity of ground motion observed during the 1985 Michoacán earthquake. However, in spite of this division equal design spectra are assumed to apply at sites characterized by quite different local conditions, described in simple form by the thickness and the stiffness of the soft soil sediments that overlie the firm ground. Because these spectra do not explicity account for the local dominant ground periods, they show wide plateaus of maximum ordinates which are considered to be conservative envelopes to the spectra that may be expected at different sites, in accordance with their specific local conditions. Approximate criteria are included in the Appendix to the Norms, to account for the value of the local dominant ground period, but these criteria still fail to pay attention to more detailed information about frequency content of ground motion records. Thus, site design spectra determined in accordance with these criteria are probably too conservative for wide intervals of vibration periods, as a consequence of the intention of the code-writing body of covering uncertainties associated with a large number of concepts, such as the characteristics of future ground motions; the constitutive properties of structural materials and members subjected to random, alternating loads; the approximations implicit in the methods employed to estimate seismic responses; the possibility of occurrence of different failure modes, and others. The influence of soil-structure interaction is approximately covered by increasing the fundamental period of the system, but the changes in the configuration of natural modes that may arise from local soil deformation (sway and rocking) are ignored. Accordingly, the reduction of spectral ordinates associated with radiation damping is disregarded, in order to compensate for the reductions in effective available ductility that result from the fact that, while the contributions of structural deformations to the lateral displacements during high intensity earthquakes may include significant non linear components, those arising from foundation response probably remain within the linear range of behavior.

The last decade has witnessed considerable progress in our knowledge about the mechanical properties of the soft soil layers in the Valley of Mexico, as well as about their seismic response. The progress is based on both the records obtained by a large number of accelerographs during several moderate intensity earthquakes, and the mathematical models that have been developed. This has fostered the interest of structural designers for the use of design spectra that take into account in an explicit manner detailed site specific information. This includes, for instance, ground motion records and theoretical or empirical transfer functions relating Fourier spectra on firm ground and on the site of interest. Before accepting this approach for systematic application in structural design practice, it is necessary to study the general requirements to be imposed in order to guarantee that the new criteria lead to safety levels not lower than those implicit in the current Norms for several reference structures standing on the area most severely damaged during the 1985 eartquake. These structures are used for the purpose of calibration, because their behavior during the mentioned earthquake served as a reference to the group of experts who established the current design spectra. For the same reason, the criteria to be adopted to take into account the influence of soil-structure interaction must recognize that the design spectra proposed by the code committee for the reference structures implicitly consider the response reductions associated with that influence.

Establishing consistent-reliability response spectra and design requirements implies examining a wide variety of concepts, ranging from seismic hazard models to life-cycle optimization of structural systems. This paper concentrates on the analysis of system reliability for given intensities and on the development and calibration of simple "equivalent" systems that may be used to estimate the reliabilities of the complex systems usually found in engineering practice.

SEISMIC RELIABILITY OF BUILDING FRAMES

The approach adopted here was proposed by Esteva and Ruiz (1989). Its assets and limitations were discussed by Esteva (1995). According to this approach, the determination of system reliability for a given intensity is based on the assumption that under the action of an earthquake the structure may fail in n different modes; for instance, each failure mode may correspond to exceedance of the capacity for ductile deformation at a given story. If S_i represents the maximum amplitude of the response variable governing the occurrence of the *i*-th failure mode (here, story deformation) and R_i is the value of S_i causing failure, the ratio $Q_i = S_i/R_i$ is the reciprocal of a random safety factor such that $Q_i \geq 1$ means failure in the *i*-th mode. It is also assumed that failure occurs precisely in the *i*-th mode and not in any other if $Q_i \geq Q_j$ for all j = 1, ..., n. This is a simplifying assumption that ignores the possibility that during the response process the condition $Q_j \geq 1$ may be reached before the condition $Q_i \geq 1$. From these assumptions, the probability of failure for a given intensity (y) equals the probability that the maximum of all the values Q_i exceeds unity. Thus, if that maximum is called Q, then $P_F(y) = P(Q \geq 1 \mid y)$. In the article by Esteva and

Ruiz, this probability is estimated after fitting a lognormal distribution to the values of Q generated by Monte Carlo simulation, taking into account uncertainties about the detailed characteristics of the ground motion for a given intensity, as well as about the mechanical properties of the system (masses, stiffnesses, strengths, cyclic behavior, deformation capacity).

SIMPLIFIED MODELS FOR RELIABILITY ASSESSMENT OF COMPLEX SYSTEMS

In order to establish reliability-based seismic design spectra it is necessary to study the relations between mechanical properties of complex systems and safety levels (or their complement, failure probabilities) for given intensities. The study according to the approach presented in the previous section may entail an extremely large computational effort. A practical way to limit this effort consists in representing the complex systems of interest by simplified models that retain some of their essential traits, such as the relations among average values of the most significant mechanical properties. A process of calibration is then required; *i.e.*, the derivation of rules to transform the output of the simplified model into the required output of its complex counterpart (system reliability, in this case). This problem was previously studied by Díaz-López and Esteva (1991), who proposed rules to transform a multistory, multibay building frame with uncertain properties into two types of simplified, reference models, also with uncertain properties: a single-bay, multistory model and a single-degree-of-freedom model. By Monte Carlo simulation they obtained failure probability functions (in terms of intensities) for the detailed model as well as for its two simplified representations. For the cases studied, failure probabilities computed on the basis of the simplified models may differ from those derived from the detailed model by factors of the order of 2 for high intensities, and much larger for low intensities.

The simplified reference model adopted in this study is a single-story, single-bay frame, whose properties and corresponding uncertainties are derived from those of the detailed system as described in the following. (It differs from the simplified model of the preceding paragraph.) The detailed model will be designated by DM, and the simplified reference model by SM.

Mass and vertical load

If M is the mass matrix of the DM, the mass m of the SM is given by

$$m = Z^T MZ \tag{1}$$

where Z is the shape of the fundamental mode of the DM; as an approximation, it can be taken as the deformed configuration of DM under a system of lateral forces with a distribution approximately equal to the inertial forces associated to that mode.

The vertical load w is obtained replacing in the foregoing equation the mass matrix by the vertical load matrix of the DM.

Lateral stiffness

The lateral stiffness of SM is designated by k, and calculated as follows:

$$k = Z^T KZ \tag{2}$$

Here, K is the stiffness matrix of the DM. In ordinary structural dynamics, it is assumed that K in Eq. 2 is expressed in terms of lateral displacements, and that joint rotations are eliminated through a process of condensation. Consequently, in general Z is a vector constituted by lateral displacements. However, in order to derive the uncertainty on k from that on the properties of the members of DM it is convenient to work with the uncondensed system. Thus, both K and Z will contain elements associated with lateral displacements and joint rotations. This permits us to replace Eq. 2 by one expressed in terms of the member stiffnesses of DM.

Let u_i be the vector of deformations (two rotations and one transverse displacement) of the *i*-th structural member, and let A be a displacement transformation matrix such that

$$u = \begin{Bmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ u_n \end{Bmatrix} = AZ \tag{3}$$

Then, we can define a diagonal matrix K′ formed by the stiffness matrices of the elements of DM

$$K' = \begin{bmatrix} K'_1 & & & & \\ & K'_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & K'_n \end{bmatrix} \tag{4}$$

From the foregoing equations it can be easily shown that

$$K = A^T K' A \tag{5}$$

and, therefore, that

$$k = u^T K' u \tag{6}$$

or in expanded form

$$k = \sum_i \sum_j K'_{ij} u_i u_j \tag{7}$$

Frame height

If SM is symmetric with respect to a vertical axis, its two joints will have equal rotations. Therefore, it will be represented by a two-degree-of-freedom system, whose deformed configuration will be determined by the lateral displacement Δ, relative to the ground surface, and the rotation θ of each of its joints.

We want to preserve the ratios of the stiffnesses of beams to those of columns. For this purpose, we will assume the following:

$$\kappa_c = \frac{1}{2} \sum \kappa_{ci} \quad , \kappa_b = \sum \kappa_{bj} \tag{8a, b}$$

where κ is used in general to denote the ratio EI/L, for a structural member, and the subscripts mean the following: ci identifies the *i*-th column of DM, bj its *j*-th beam, c stands for each column of SM, and b for its beam. The summations in Eqs. 8a and 8b are extended to all the columns and all the beams in the DM, respectively.

Under the assumptions in the preceding paragraphs, the length of the columns of the SM is h, given by the following equation:

$$h^2 = \frac{24\ \kappa_c}{k}\left[1 - \frac{3}{4+6\frac{\kappa_b}{\kappa_c}}\right] \tag{9}$$

Span of reference frame

The span, l, of the beam of the SM is determined on the basis of the following condition:

$$\frac{Vh}{M_{obl}+M_{obr}} = \frac{\sum_{j=1}^{n} V_j h_j}{\sum_{k=1}^{n_b} (M_{oblk} + M_{obrk})} \tag{10}$$

where V is the base shear, equal for both, DM and SM, M_{obl} and M_{obr} are the fixed-end moments due to load w at the left and right ends of the beam of SM, respectively, V_j and h_j are respectively the shear and the height of the j-th story, M_{oblk} and M_{obrk} the fixed-end moments due to vertical load at the left and right ends of the k-th beam of DM, n the number of stories, and n_b the number of beams.

Member strengths

The criteria proposed here correspond to the particular case which complies with the following:

a) The resisting bending moments at both ends and in both directions in each column are equal.

b) For the SM system, the resisting positive bending moments at both ends of the beam are equal, as well as the negative bending moments, but the former may differ from the latter.

From these conditions, the resisting moments m_{cR} at the ends of each column of SM are derived from the resisting moments M_{cRk} at the column-ends of the first story of DM, as follows:

$$m_{cR} = \frac{1}{2}\ \frac{h}{h_1}\ \frac{\overline{w}}{\overline{W}} \sum_{k=1}^{n_{c1}} M_{cRk} \tag{11}$$

Here, h_1, is the height of the first story, $\overline{W}$ and $\overline{w}$ are the expected values of the weights of the masses of DM and SM, respectively, and n_{c1} is the number of columns at the first story of DM.

The resisting positive and negative moments at the ends of the beam of SM are obtained in accordance with the following:

$$\frac{m^{+}_{bR}}{m_{cR}} = \frac{\sum_{i=1}^{n_b} (M^{+}_{bRli} + M^{+}_{bRri})}{\sum_{j=1}^{n_c} M_{cRj}} \tag{12a}$$

$$\frac{m^{-}_{bR}}{m_{cR}} = \frac{\sum_{i=1}^{n_b} (M^{-}_{bRli} + M^{-}_{bRri})}{\sum_{j=1}^{n_c} M_{cRj}} \tag{12b}$$

In these equations, the + and - define the sign of the resisting moment being considered, m_{bR} and m_{cR} denote resistances of beam and columns in SM, M_{bRli} and M_{bRri} denote resistance of the left and right ends of the *i*-th beam, M_{cRj} the resistance of the *j*-th column and n_b and n_c designate the number of beams and of columns in DM.

Uncertainties about properties of reference system

In this study h and *l* are deterministic. Their values are calculated by means of Eqs. 9 and 10, replacing all uncertain variables by their mean values.

Means and variances of m, w, k, m_{cR}, m^+_{bR} and m^-_{bR} as given by Eqs. 1, 7, 11, 12a and 12b are obtained by straightforward application of well known formulas of the theory of Statistics. The resulting expressions will not be presented here.

CONSISTENT-RELIABILITY SPECTRA FOR SYSTEMS WITH UNCERTAIN PROPERTIES

Mendoza, Díaz and Esteva (1995) used Monte Carlo simulation to obtain response spectra corresponding to specified values of the safety index β (Cornell, 1969) for elastoplastic and stiffness degrading (Takeda-type) non linear shear systems with uncertain mass, stiffness, lateral strength and deformation capacity. Uncertainties on the detailed time histories for given intensities were incorporated by using a family of simulations of the EW component of the SCT record of the Michoacán, Mexico, earthquake of September 19, 1985 (soft soil in Mexico City); all simulated records were scaled up or down to the same Arias intensity as the recorded accelerogram. Typical results are presented in Figs. 1 and 2, taken from the reference mentioned above, which show values of the required expected strengths of some elasto-plastic and stiffness-degrading systems for a safety index β=2. The sets of these values as functions of the vibration period constitute the **expected-strength design spectra** (ESDS in the figures). They are compared with the average response spectra of the sample of simulated accelerograms and with the spectra of required expected strength that result from the design spectra of the current Mexico City seismic design requirements. The spectra specified by the code are much wider than those arrived at by our simulations. The additional width proposed by the code is intended to cover uncertainties about the dominant periods of ground motion, as well as some related to the structural properties and the structural response.

APPLICATIONS

Two buildings, seven- and fourteen-story high, were studied for three values of the seismic design coefficient, c: 0.05, 0.075 and 0.1. The buildings were designed as ductile frames, following the requirements of the Mexico City building code of 1987 (Departamento del Distrito Federal, 1987). Step-by-step dynamic response studies were carried out on two-dimensional systems, each representing the middle frame of one of the buildings, including a tributary mass corresponding to a half-bay width at each side of the frame. Details about the properties of the buildings studied are given by Esteva et al, 1996.

For the DM, the following variables were taken as uncertain: the concrete compressive strength, f_c' ; the steel yield stress, f_y ; the dimensions and cover of the structural members; the available story ductility and the vertical loads. The correlation coefficient between values of the same variable for different elements was taken as 0.8. The expected value of the available story ductility, μ, was related to its nominal value, μ^*, and to its coefficient of variation, V_μ , though the equation $\mu=\mu^* \exp(1.65\, V_\mu)$; μ^* was taken as 4, and V_μ as 0.25. Uncertainties about dead and live loads were taken in accordance with Díaz, Esteva and Flores, 1990.

Hysteretic bilineal behavior was assumed for both, the DM and the SM of the frames studied. These models were assumed to be subjected to a family of simulated accelerograms, with statistical properties similar to those of SCT 850919EW (EW component of the accelerogram recorded at the SCT site in Mexico City during the earthquake of September 19, 1985). The simulations were obtained in accordance with the algorithm of Grigoriu et al (1988). The records were normalized so that they led to the same maximum ordinate of the acceleration spectrum for 0.05 damping as the original SCT record.

RESULTS

The reliability of all the systems for the family of excitations considered was expressed in terms of Cornell's index β for each of the seismic design coefficients considered. For this purpose, external actions and structural capacities were represented by story ductility demands and available ductilities, respectively. The statistical properties of these variables were obtained by the Monte Carlo method. Thirteen simulations were generated for the DM; a larger number was used in the study of the SM. The values of β obtained are shown in Table 1.

The results of the DM show larger values of β for the seven-story systems than for the fourteen-story ones, for all the seismic design coefficients considered. Such results are in agreement with what should be expected, given the shape of the average response spectrum of the records used, with a peak near a period of 2s.

Comparing the results of the DM with those of the SM, it is found that the reliability indices are systematically larger for the latter, in particular for low seismic design coefficients. This variation is even more pronounced for the fourteen-story systems, for which the SM show too large values of β, even for low seismic design coefficients.

The foregoing results show a limitation of the use of simplified systems to estimate the reliabilities of their detailed counterparts. Probably, the differences found may be mainly due to the incapacity of simple systems to account for the possibility of occurrence of several failure modes.

Table 2 shows a comparison of the values of β for seven-story systems with approximately equal fundamental periods of vibration, but different values of the ratio of beam and column stiffness (κ_b/κ_c). For the case with a larger value of this ratio, the differences between the results of the DM and the SM are much smaller than for the other case. Cases with larger values of the stiffness ratio were not studied, as they were not consistent with the strong-column weak-beam design criterion implicit in modern seismic design practice.

CONCLUSIONS

An exploratory study was presented about the capacity of a type of simplified reference system to estimate the reliability of mdof building frames subjected to ground motion. The results show that the reliability indices predicted by the reference system are significantly larger than those corresponding to the real systems they are supposed to represent.

A comparison of the reliability indices obtained for the detailed models of the seven- and fourteen-story frames show that the design code used does not lead to consistent reliability levels.

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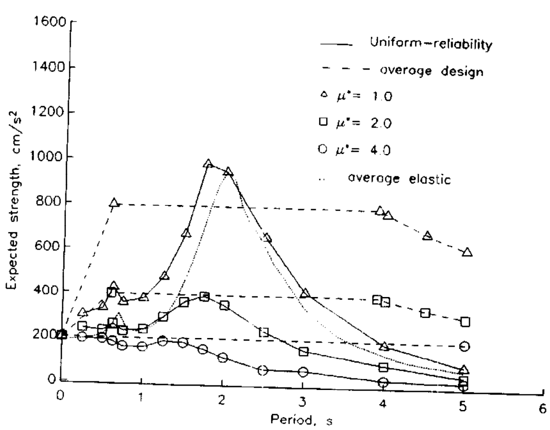

Fig 1. Uniform-reliability ESDS for elastoplastic systems, compared with Mexico City Building Code requirements. V_R=0.15, β=2.

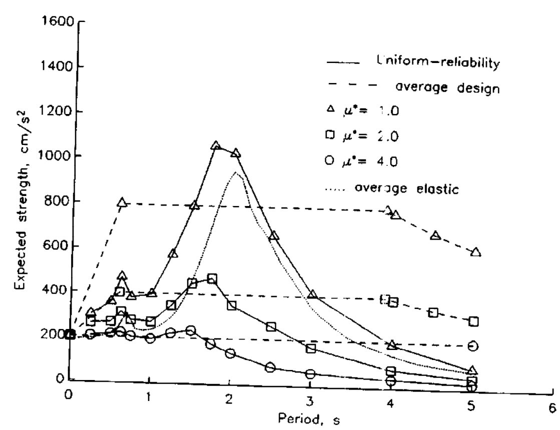

Fig 2. Uniform-reliability ESDS for stiffness-degrading systems, compared with Mexico City Building Code requirementes. V_R=0.15, β=2.

Building	c	DM	SM
7-story	0.1	4.5	7.5
	0.075	2.4	5.1
	0.05	0.2	3.7
14-story	0.1	2.9	7.0
	0.075	1.3	6.2
	0.05	0.2	5.9

Table 1. Values of β.

κ_b/κ_c	c	DM	SM
0.465	0.1	4.5	7.5
	0.075	2.4	5.1
	0.05	0.2	3.7
1.388	0.1	7.0	7.5
	0.075	2.0	4.0
	0.5	0.6	2.7

Table 2. Comparison of β values for seven-story systems with two different values of beam/column stiffness ratio