DESIGN FOR NONLINEAR SEISMIC RESPONSE COMBINATIONS

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ABSTRACT

A probability based new design method is presented for nonlinear interaction and combination problems in seismic analysis. Due to the nonlinear dependence of the combination on the constituent members, the combined response is a non-Gaussian random process. The paper introduces a concept to estimate the extreme peak distribution of such processes for a specified duration of seismic input. The approach is based on the arbitrary point in time distribution of the nonlinear combination and uses the fundamental result for the maximum distribution of a stationary Gaussian process. The link between the two distributions is established by an equivalent number of square waves representing instantaneous outcomes of a Gaussian random variable. An example involving a two dimensional bending moment-axial force interaction is used to illustrate the application of the method. Monte Carlo simulations are used to demonstrate the surprising accuracy of the developed method.

KEYWORDS

Non-Gaussian; nonlinear response combination; interaction; temporal variability; extreme peak distribution; response spectrum; random process; reliability based design; statistical dependence.

INTRODUCTION

In the past, considerable progress has been made in applying the response spectrum method (Der Kiureghian, 1981; Singh and Maldonado, 1991), a popular procedure for seismic analysis of linear structures, to a broad range of design situations. With recent extensions, the method is applicable to multiple supported, non-uniformly excited structures (Der Kiureghian and Neuenhofer, 1992; Berrah and Kausel, 1992), problems characterized by significant contribution of high frequency modes (Der Kiureghian and Nakamura, 1993) as well as non-classically damped structures (Maldonado and Singh, 1991). The response spectrum method provides an estimate of the maximum of an individual dynamic response quantity that is expressed as a linear combination of modal contributions.

Consider a response quantity

$$R(t) = g[\mathbf{X}(t)] \tag{1}$$

expressed as a nonlinear combination of response components $\mathbf{X}(t) = [X_1(t)\dots X_n(t)]^t$. Examples for a combination according to (1) include the set of principal stresses in a state of plane stress

$$\sigma_{1,2}(t) = \frac{\sigma_x(t)+\sigma_y(t)}{2} \pm \sqrt{\frac{(\sigma_x(t)-\sigma_y(t))^2}{4} + \tau_{xy}^2(t)} \tag{2}$$

or a bending moment-axial force interaction problem

$$R(t) = g[M_y(t),\, M_z(t),\, P(t)] \tag{3}$$

In what follows it is assumed that the structure is linearly elastic, and the seismic excitation is a stationary Gaussian process. Hence modal responses and all response quantities $X_i(t)$ obtained from linear modal superposition are also stationary Gaussian random processes. Interest lies in the random variable $R_\tau = \max_\tau R(t)$ where τ

2 NEUENH.

specifies the duration of seismic input. For design purposes, one is interested in the probability that the combined response $R(t)$ exceeds a specified threshold r at least once, i.e. the first excursion probability

$$p_f = P(R_\tau > r) = 1 - F_{R_\tau}(r) \tag{4}$$

where $F_{R_\tau}(r)$ denotes the cumulative distribution function of R_τ.

The response spectrum methods developed so far are limited to estimating the mean of the maximum value of a single load effect process, such as the stress components in (2) or the individual internal forces in (3). However, they cannot be employed to provide a measure for the maximum of the combined response $R(t)$, because this quantity cannot be expressed as a linear superposition of individual modal responses.

The nonlinear response combination problem, illustrated in (2-3) is identical to the problem of first out-crossing of a safe domain D by a vector process $\mathbf{X}(t)$, or alternatively, the problem of up-crossing of the scalar process $R(t) = g[\mathbf{X}(t)]$ above a specified threshold r. This is because whenever the process $\mathbf{X}(t)$ out-crosses the domain D, the corresponding scalar process up-crosses the threshold r. The expression for the mean up-crossing rate ν_R is given by Rice (1944), and an expression for the mean out-crossing rate ν_D, a generalization of Rice's result, is provided by Belyaev (1968) and Belyaev and Nosko (1969).

Since the mean crossing rate ν provides an upper bound to the probability in (4), it is a key descriptor in time dependent reliability problems. Consequently, extensive research has been performed in the area of load combination to determine this rate. In conjunction with earthquake engineering problems, attention has been focused on stationary Gaussian vector processes (Veneziano et. al., 1977; Pearce and Wen, 1985; Madsen, 1986; Li and Melchers, 1995).

The distribution of the peak response R_τ obtained from a crossing rate approach is only good at its far tail. Thus, the information obtained from crossing statistics does not suffice for practical seismic analysis where prime interest usually lies in the mean value μ_{R_τ} of the maximum R_τ. Consequently, the application of results obtained from up- or out-crossing type analysis is limited, and the need arises to develop procedures yielding sufficiently accurate estimates for design quantities of nonlinear seismic response combinations. Current seismic design codes do not contain provisions which account for the time variability of individual response components that are part of design criteria such as those given in (2-3). It has thus become common to design based on the simultaneous occurrence of the individual maxima.

The objective of this paper is to present a new concept, the ESW (*Equivalent Square Waves*) method, to approximately determine the extreme peak distribution of nonlinear combinations of response processes relevant in seismic analysis. The approach is based on the arbitrary point in time distribution of the combined process R. This distribution is obtained from the multi-normal distribution of the constituent members of the process by employing elementary rules of probability transformation. The link between the point-in-time and maximum distributions is established using an equivalent number of statistically independent square waves. This number is the key quantity in the proposed method, and it is determined using a well known result for the extreme peak distribution of a stationary Gaussian process. In order to determine both the arbitrary point in time distribution and the ESW-number of the combined response R, information is needed, that can be obtained from an ordinary response spectrum analysis of the structure.

EXTREME PEAK OF STATIONARY GAUSSIAN PROCESS

Let $X(t) = \mu_X + \tilde{X}(t)$ denote a stationary Gaussian process in which μ_X represents the mean value due to static or quasi-static actions and $\tilde{X}$ is the fluctuating component due to earthquake loading. A key element of the proposed method is the use of the well known result for the maximum value X_τ of $X(t)$ over a duration τ, i.e. $X_\tau = \max_\tau X(t)$. According to Vanmarcke (1975), a good approximation for the CDF of X_τ is

$$F_{X_\tau}(x) = \left(1 - e^{-\frac{y^2}{2}}\right) \exp\left(-\nu\tau \, \frac{1 - e^{-\sqrt{2\pi}\delta^{1+b} y}}{e^{\frac{y^2}{2}} - 1}\right) \qquad y \geq 0 \tag{5}$$

3 NEUENH.

where $y = (x - \mu_X)/\sigma_X$ is the normalized maximum, $\nu = (\lambda_2/\lambda_0)^{1/2}/(2\pi)$ is the mean zero-up-crossing rate of $\tilde{X}$, $\delta = (1 - \lambda_1^2/\lambda_0\lambda_2)^{1/2}$ is a bandwidth parameter, $b = 2$ is an empirically determined constant, and

$$\lambda_m = \int_0^\infty \omega^m G_{\tilde{X}}(\omega)\, \mathrm{d}\omega \qquad m = 0, 1, 2 \tag{6}$$

are the spectral moments of $\tilde{X}(t)$, where $G_{\tilde{X}}(\omega)$ denotes the power spectral density. If interest lies in the variable $X_\tau^* = \max_\tau \mid X(t) \mid$, (5) has to be modified (Vanmarcke 1975).

THE EQUIVALENT SQUARE WAVE PROCESS

Consider a standard stationary Gaussian process $Y(t)$ with zero mean, unit standard deviation, and extreme value Y_τ defined by $Y_\tau = \max_\tau Y(t)$. The mean value μ_{Y_τ} of Y_τ is

$$\mu_{Y_\tau} = \int_0^\infty y \frac{\mathrm{d}F_{Y_\tau}(y)}{\mathrm{d}y}\, \mathrm{d}y \tag{7}$$

where $F_{Y_T}(y)$ is the Vanmarcke CDF defined in 5. For the standard process, this distribution is completely defined in terms of the parameters ν and δ.

Next consider the random variable Y_N defined as the maximum of N independent outcomes of a standard Gaussian variable Y. These outcomes can be seen as the amplitudes of a sequence of square waves. The CDF of Y_N, the maximum wave amplitude, is given by $F_{Y_N}(y) = [\Phi(y)]^N$, where $\Phi(\cdot)$ is the standard normal CDF. Its mean μ_{Y_N} is obtained from

$$\mu_{Y_N} = \int_{-\infty}^\infty y \frac{\mathrm{d}F_{Y_N}(y)}{\mathrm{d}y}\, \mathrm{d}y = N \int_{-\infty}^\infty y \Phi(y)^{N-1} \varphi(y)\, \mathrm{d}y \tag{8}$$

with $\varphi(\cdot)$ denoting the standard normal PDF. For any standard Gaussian process $Y(t)$ defined by the pair of parameters νT and δ, there exists a number N such that $\mu_{Y_N} = \mu_{Y_\tau}$, i.e.

$$N \int_{-\infty}^\infty y\, \Phi(y)^{N-1} \varphi(y)\, \mathrm{d}y = \int_0^\infty y \frac{\mathrm{d}F_{Y_T}(y)}{\mathrm{d}y}\, \mathrm{d}y \tag{9}$$

This number is referred to as the equivalent number of square waves (ESW-number). The motivation behind (9) is to replace the actual Gaussian process with an equivalent square-wave process by matching the means of the two extremes. The advantage gained from this approach becomes evident when it is applied to non-Gaussian processes for which closed form solutions of the peak distribution are not available.

The subsequent set of figures illustrates the above concept. Fig. 1 shows a sample of two stationary standard Gaussian processes $Y(t)$ of equal duration $\tau = 10\,\mathrm{s}$. The two realizations are responses of single-degree-of-freedom oscillators with mean numbers of crossings $\nu_1 T = 10$ and $\nu_2 T = 20$ to white noise input with $\delta = 0.25$. The two equivalent square wave processes in Fig. 1 are realizations of a sequence of N_1 and N_2 mutually statistically independent standard normal variables Y. The corresponding ESW-numbers N_1 and N_2 are obtained based on the condition in (9) rounded to the next integer value.

Fig. 2 summarizes the relation between the mean number of crossings νT and the ESW-number N. Various values for the bandwidth parameter δ are considered. They correspond to the response of a single-degree-of-freedom oscillator with damping ratios ζ to a white noise input. The values on the abscissa represent the range of νT typically encountered in earthquake engineering practice.

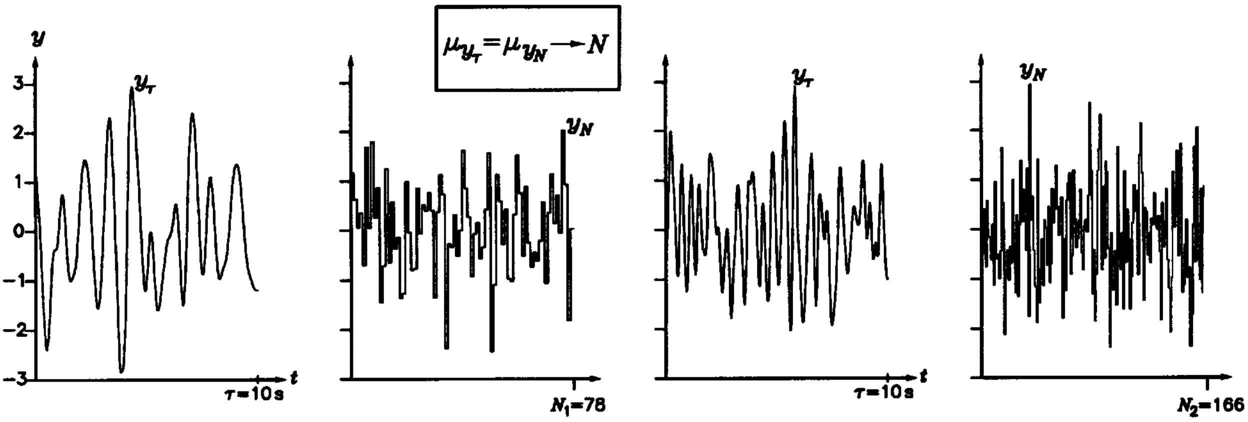

Fig. 1. Illustration of the ESW method

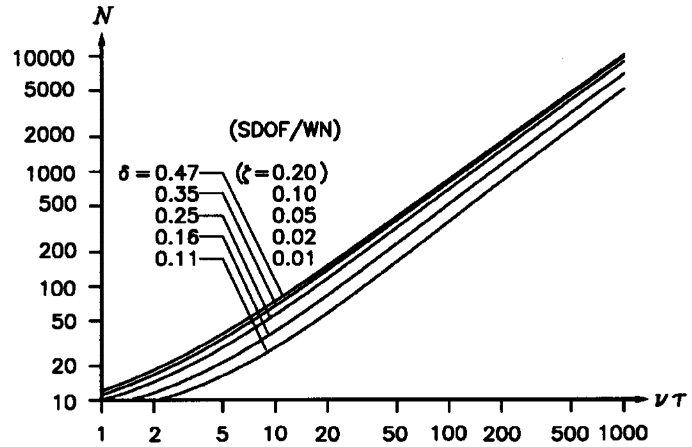

Fig. 2. Equivalent number of square waves versus mean number of crossings

The ESW estimate f_{Y_N} for the Vanmarcke distribution f_{Y_T} is obtained by fitting a single parameter N such that the two mean values are identical. It is interesting to compare the entire shape of the two distributions. Fig. 3 compares the two probability density functions for various values of $\nu\tau$ and δ. The two distributions are found to be remarkably similar in shape. It is apparent that the mean value of the ESW distribution is extremely insensitive to changes in N. This phenomenon is illustrated by the thin dashed lines in Fig. 3. In the case considered in the figure, an increase in N of about 50 % (1469 versus 2173) leads to an increase in μ_{Y_N} of just $\approx$ 2.5 % (3.35 versus 3.43). This behavior turns out to be the most beneficial characteristic of the ESW approach.

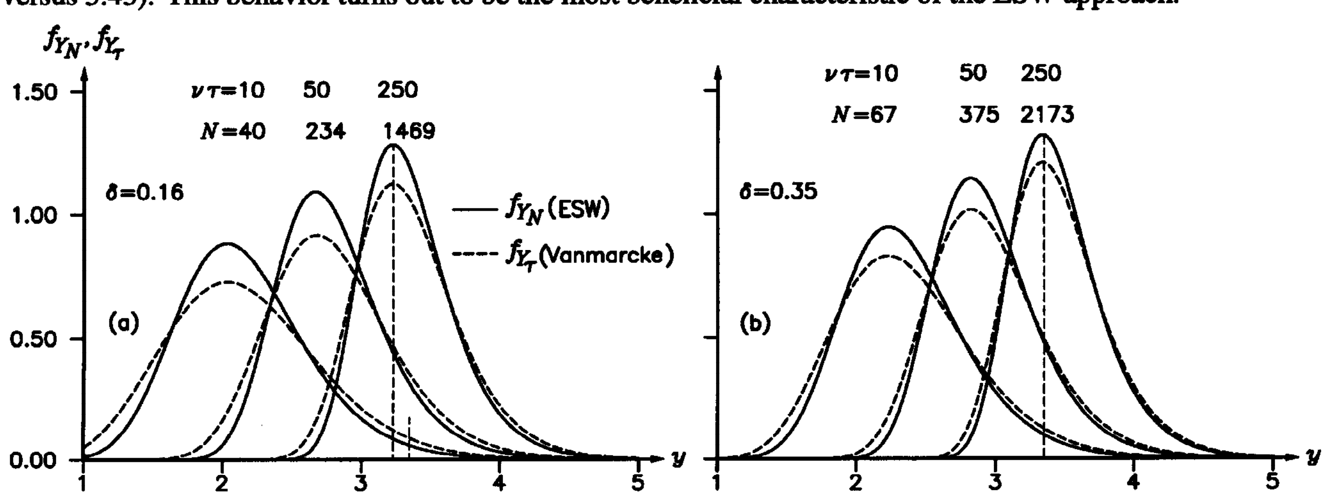

Fig. 3. ESW and Vanmarcke distributions for standard Gaussian process

S NEUENH

ESW METHOD APPLIED TO RESPONSE COMBINATIONS

Generally, different component processes $X_i(t)$ have different ESW-numbers N_i. In most cases, however, major contribution to the components X_i is expected to come from the same modes. Consequently, differences between various ESW-numbers do not tend to be overly large. Yet, it is reasonable to develop a combination rule for the component ESW-numbers N_i in order to estimate the ESW-number N_R of the combined process R. Since N_R is close to N_i, if the corresponding process $X_i(t)$ dominate $R(t)$, a formula for N_R should reflect both the variance σ_i of the individual component and the sensitivity $\partial r/\partial x_i$ of R with respect to X_i. In order to estimate the ESW-number N_R of the combined response, several strategies of different degrees of complexity have been investigated. The weighted average

$$N_R = \frac{\sum_i \left| \frac{\partial r}{\partial x_i}_{|x_{i0}} \right| \sigma_i N_i}{\sum_i \left| \frac{\partial r}{\partial x_i}_{|x_{i0}} \right| \sigma_i} \tag{10}$$

appears to be the most appropriate. Studies have shown that the choice of the expansion point in (10) has only minor influence on the combined ESW-number N_R. It seems natural to select $x_{i0} = \mu_{X^*_{i\tau}}$, i.e. the value obtained from a response spectrum analysis. The combined ESW-number N_R is applied to the instantaneous distribution $f_R(r)$ of the response process $R(t)$ and the mean of the extreme peak of $R(t)$ is

$$\mu_{R_\tau} \approx \mu_{R_N} = N_R \int_{-\infty}^{\infty} r F_R(r)^{N_R-1} f_R(r)\,\mathrm{d}r \tag{11}$$

EXAMPLE APPLICATION AND COMPARISON WITH SIMULATION RESULTS

Consider a rectangular, symmetrically reinforced concrete cross section of dimensions h and b, subjected to a bending moment $M(t)$ and an axial force $P(t)$. For design purposes, well known interaction diagrams are used that plot contours of the necessary reinforcement ratio $\rho = A_s/bh$ as a function of the normalized forces $m = M/(bh^2 f_c)$ and $p = P/(bhf_c)$, where A_s is the total area of reinforcement and f_c is the concrete compressive strength. The present example focuses on comparisons between the ESW method, which accounts for the stochastic nature of $m(t)$ and $p(t)$, and the conventional, deterministic method that assumes simultaneous occurrence of the peak values. Simulation is used to determine the accuracy of the ESW method.

For a given m and p, let the required ρ be given by the following relations

$$\begin{aligned}
\rho &= 0.132\,|\,m\,| - 0.0463p + 0.002 && p < 0.16 \\
&= 0.132\,|\,m\,| + 0.33(p-0.23)^2 - 0.007 && |\,p-0.23\,| \le 0.07 \\
&= 0.132\,|\,m\,| + 0.0463p - 0.0193 && p > 0.30
\end{aligned} \tag{12}$$

where p is considered to be positive if it acts in compression. This relation is obtained by fitting to interaction diagrams available in the literature (e.g., CEB 1982) for a specific steel yield strength and a specific concrete cover. Material laws and failure criteria in terms of ultimate strains for concrete and reinforcing steel on which the interaction diagram is based, can be found in reinforced concrete structural codes (Eurocode 2). We note that the above relation defines ρ as a nonlinear function of the response processes $m(t)$ and $p(t)$.

Three degrees of statistical dependence between the axial force and the bending moment are considered: $\rho_{mp} =$ 0.0, 0.50, and 0.98. The bending moment is assumed to be a zero mean process, i.e. $\mu_m = 0$, whereas for the axial force the mean μ_p arising from static loads is assumed to be α times the mean of the peak of the fluctuating dynamic component, i.e. $\alpha = \mu_P/\mu_{P^*_\tau}$. The two cases $\alpha = 1$ and 3 are considered. It is noted that the former is an extreme case because of the high contribution of the dynamic part of the axial force to the total axial force. Note that for that case the column is just about to uplift when the fluctuating part of the axial force acts in tension. The two response processes $P(t)$ and $M(t)$ are assumed to have equal mean number of crossings $\nu\tau = 100$ and identical bandwidth parameters $\delta = 0.25$.

6 NEUENH.

It is well known in reinforced concrete design practice that there is a range in the interaction plane in which the compressive force acts favorably, i.e. an increasing compressive force leads to a smaller reinforcement ratio. This is because an increasing compressive force unloads the tension side of the cross section. To account for this possibility in conventional design, the two extremes of the axial force $p = \mu_p + \mu_{\bar{p}^*_T}$ and $p = \mu_p - \mu_{\bar{p}^*_T}$ must be considered. Hence,

$$\rho = \max\left[\rho(p = \mu_p + \mu_{\bar{p}^*_T}, m = \mu_{m^*_T}), \rho(p = \mu_p - \mu_{\bar{p}^*_T}, m = \mu_{m^*_T})\right] \tag{13}$$

Fig. 5 shows the interaction diagram (solid lines) obtained from the conventional method (12, 13) for three selected values of ρ (0.01, 0.03, and 0.05) and $\alpha = 1$ and 3. The dotted lines in this figure represent (12) without account of the condition in (13). The results are in terms of the peak values $m = \mu_{m^*_T}$ and $p = \mu_p + \mu_{\bar{p}^*_T}$ that can be obtained from response spectrum analysis. The interaction curves based on the ESW method are generated by computing the required ρ for points on a 26x26 grid and then plotting contours. These are shown as dashed or dashed-dot lines for the correlation coefficients $\rho_{mp} = 0.0, 0.50$, and 0.98.

It is interesting to observe that due to the condition of (13), the parabolic section of the interaction diagram remains irrelevant in the deterministic design for both mean values of p considered in the analysis. For further increasing contribution of the dead load to the total axial force, this would no longer be the case since the two reinforcement ratios appearing in (13) become identical and the deterministic design approaches (12). Furthermore, for the $\alpha = 1$ case the second term in (13) leading to zero axial force is the relevant design quantity up to $\mu_p + \mu_{\bar{p}^*_T} = 0.46$. This is reflected by a straight line in Fig. 5(a). It can be seen that for $p = 0$ or $m = 0$ the contours of the stochastic analysis are all identical regardless of the correlation between the axial force and the bending moment. This is because the correlation becomes immaterial if either m or p approaches zero.

As expected, the results of Fig. 5(b) are similar to those of Fig. 5(a), especially in the range of higher compressive forces. In the lower portion of the diagram (small compressive forces), differences between the contour lines of the ESW analysis in Figs. 5(a) and 5(b), respectively, reflect the differences in the corresponding deterministic analysis. Due to the increased portion of the time invariant load μ_P in the $\alpha = 3$ case, all contour lines in Fig. 5(b) are more closely spaced than in Fig. 5(a).

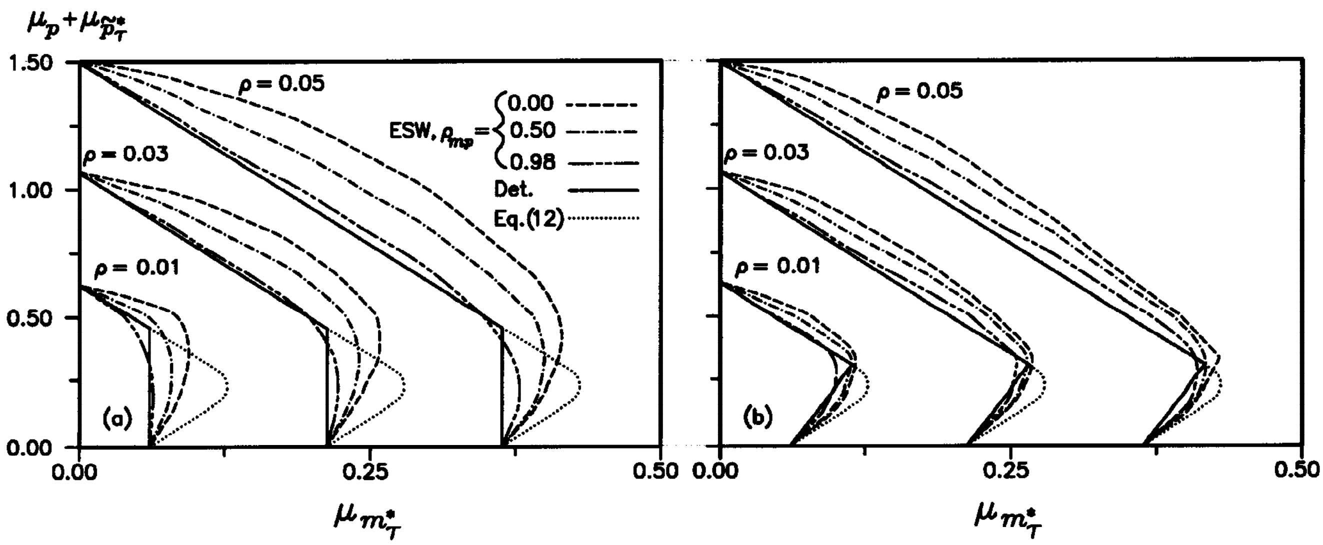

Fig. 5. Contour lines of ρ for deterministic approach and ESW approach with different degrees of correlation; (a): $\alpha = 1.0$; (b): $\alpha = 3.0$

A qualitative comparison between the deterministic and ESW approaches is provided by Fig. 6. For the sake of brevity, illustrations are restricted to the case $\rho_{mp} = 0$. As shown in Fig. 5, this is the case where the differences between the two design strategies are relatively large. The figure shows contours of the difference in reinforcement (in %) between the deterministic and the ESW approach normalized with respect to the latter. As already observed in Fig. 5, a design according to the ESW method leads to less reinforcement than the deterministic design in virtually the entire m-p interaction plane. Differences can be quite large, usually greater

7 NEUENH.

than 20% over a broad range of m-p combinations for the case $\alpha = 1$. In the range of medium eccentricity of the load and medium reinforcement ratios, differences are found to be $\approx 35\%$. For $\alpha = 3$, differences between the two design strategies on the one hand, as well as between different degrees of correlation within the ESW approach on the other hand, are generally smaller. This is because the dynamic m-p interaction problem becomes increasingly static with increasing portion of the dead load. Consequently, the m-p combination problem becomes less critical.

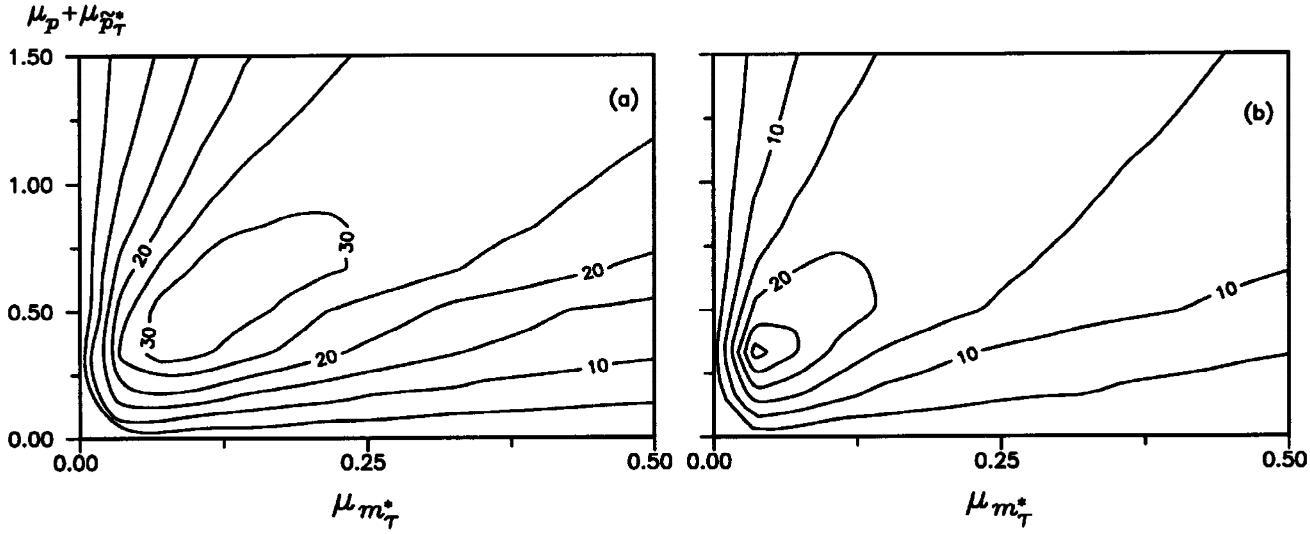

Fig. 6. Normalized difference $(\rho_{DET} - \rho_{ESW})/\rho_{ESW}$ (in %) between deterministic and ESW design for $\rho_{mp} = 0$; (a): $\alpha = 1.0$; (b): $\alpha = 3.0$

The most important conclusion from this example application is the finding that a deterministic design below the balance point, i.e. $p = 0.23$ (this is where most of the seismic design situations occur), is relatively close to an ESW design, especially for medium and large reinforcement ratios and for cases in which the portion of the dead load is large.

The simulation results are considered as exact values. Fig. 7 illustrates the convincing accuracy of the new approach by plotting contour lines of the ratio of the simulation results to the ESW values. It suffices to consider the most critical case $\alpha = 1$ for $\rho_{mp} = 0$. Maximum, minimum, mean and standard deviation of 1.0198, 0.9774, 0.9910 and 0.006, respectively, are the statistics for the ratios of the simulation results to the ESW values corresponding to the 26x26 m-p combinations.

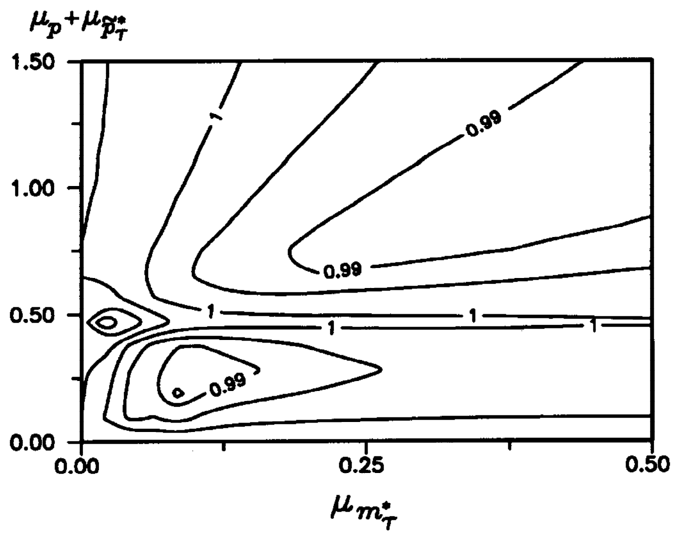

Fig. 7. Accuracy ρ_{ESW}/ρ_{SIM} of ESW Approach in Comparison to Simulation Results ($\rho_{mp} = 0$; $\alpha = 1$)

SUMMARY AND CONCLUSIONS

An design method for nonlinear seismic response combinations and interaction problems is developed. The proposed method is based on a new approach to obtain an estimate for the extreme peak distribution for stationary non-Gaussian scalar response processes. Using the well known and widely used result for the extreme peak distribution of a stationary Gaussian process, an equivalent number of square waves (ESW-number) of Gaussian amplitude is determined for each component. An approximate ESW-number is computed for the combined response using a simple combination rule which properly weighs the influence of the individual components. This combined ESW-number, together with the instantaneous distribution of the combined response, is used to obtain an approximation for the extreme peak distribution of the nonlinear response process. Using the arbitrary point in time distribution of the combined response, the method fully accounts for the non-Gaussian character of the response process as well as any correlation structure between its components. A comprehensive example application is carried out and results obtained are found to be in remarkably close agreement with simulation results.

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