MODELLING THE DYNAMIC BEHAVIOUR OF MEXICAN CLAYS

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ABSTRACT

This paper presents a cursory look into some of the results of recent research carried out in the Instituto de Ingeniería, UNAM, into the dynamic behaviour of clayey soils. It shows the development of hyperbolic stress-strain relationships with which the most relevant features of the behaviour of two very different soils can be modelled, clays from the Campeche Sound in the Gulf of Mexico and Mexico City clays. The model depends on parameters that can be expressed in terms of experimental functions of plasticity index and relative consistency. Even though the models are formally the same, the functions that define them are different for the two soils studied. This reflects the influence of soil origin and type, mineralogy and geological formation processes on these functions.

KEYWORDS
Laboratory tests, dynamic properties, marine clays, lacustrine clays, behavior modelling

INTRODUCTION

Among other factors, the dynamic behaviour of clays depends on the magnitude of the strains induced by the application of stresses. For shear strains of the order of $10^{-4}$ %, they behave like viscous elastic solids; for strains of up to about $10^{-1}$ %, stiffness and strength depend on stress history and state, and are influenced strongly by soil plasticity. Experimental results accumulated over the last few years at the Instituto de Ingeniería, UNAM, indicate that relative consistency (or liquidity index) also bears an important influence on the shape of stiffness-strain and damping ratio-strain curves; together with soil plasticity, it is one of the key parameters for studying the behaviour of clayey soils subjected to cyclic dynamic loads. This paper describes the way in which these two index properties can be incorporated into hyperbolic stress-strain relationships to model the dynamic behaviour of two types of clays with highly contrasting plasticity index values.
MATERIALS USED

The first material is a marine clay from the Campeche Sound, in the Gulf of Mexico, off the coast of the state of Campeche. Samples were retrieved from the sea bottom at depths ranging from 16 to more than 120 m. A detailed description of its properties as well as of its static and dynamic characteristics can be found elsewhere (Romo and Ovando, 1993). The clays referred to in this paper had natural water contents that varied between 25 and 72 % (average: 45 %); their plasticity indices ranged from 31 to 59 % (average: 39 %).

Mexico City clay is notorious for its high plasticity, low strength and it is also very compressible. The test results presented here were taken from studies performed using materials sampled from three different sites within the old lake zone; the behaviour observed in these results is a representative example of the one described more thoroughly in other papers (Romo, 1995; Romo and Ovando, 1995). Natural water contents of the samples tested vary between 155 and 366 % (average: 242 %) and their plasticity indices between 149 and 190 % (average: 189 %).

Each of the specimens was consolidated isotropically and subjected to undrained cyclic two-way loading in triaxial cells; the samples underwent 30 cycles of increasing amplitude until failure was attained. Some of the samples were also tested in a resonant column in order to look at their small strain behaviour.

SHEAR MODULUS AT SMALL STRAINS

Values of the shear modulus at small strains provide estimates of its initial or maximum value, $G_{max}$, which can be obtained from geophysical tests in the field or, typically but not exclusively, from the resonant column test in the laboratory. Field tests measure shear wave velocity propagation, $V_s$, and shear modulus is obtained indirectly with an elastic relationship ($G = \rho V_s^2$; $\rho = $ mass density). $G_{max}$ values estimated from a field test are usually higher than those obtained from resonant column tests as the strain levels induced in the former type of test can be at least an order of magnitude smaller than the strains experienced by a soil sample tested in a resonant column. Disturbances due to sampling or handling of soil specimens also affect the results of laboratory tests; so do ageing effects which can not be reproduced in the laboratory. These problems have received the attention of numerous workers in the past, like Anderson and Richart (1976), Anderson and Stokoe (1978), Hight et al (1985), inter alia.

Recent comparative studies have shown that in the highly plastic Mexico city clays with low relative consistencies, differences in field and laboratory estimates of $G_{max}$ are less important than in less plastic materials having higher relative consistencies (Ovando et al, 1993). Calling $G_{max}^I$, the field value of the initial shear modulus and $G_{max}^L$, its value obtained from a laboratory test, the quotient between both of these was found to be (pending further verification):

$$\frac{G_{max}^I}{G_{max}^L} \approx 1.4c'^C$$

(1)

where $c'$ is the relative consistency. Brittle soils having large relative consistencies are more prone to be affected by sampling or handling disturbances; hence the quotient tends to a value of about 4 whilst it tends to 1.4 for soils with water contents equal to their liquid limit.

Initial shear moduli obtained in the laboratory also depend strongly on relative consistency. In order to view this, values of $G_{max}$ obtained from cyclic triaxial cells were plotted against the consolidation stress. The graph presented in fig 1 shows data obtained from tests on soils from the Campeche Sound and the one in fig 2, from the Mexico City clays. A cursory look at the data would indicate that $G_{max}$ is not related to consolidation pressure. However, when data are organized according to the values of relative consistency,
definite trends can be established, as suggested by fig 1 for the case of the soils from the Campeche Sound. The data there were fitted to the following equation:

\[ G_{\text{max}} = G_0 + \frac{95(C_r - 0.023)}{1 - (C_r - 0.023)} \sigma'_c \]  

where \( G_0 \) is the shear modulus obtained when the confining pressure, \( \sigma'_c \), is nil.

The data from the Mexico City clays is better organized when plasticity index, \( PI \), is introduced as an additional parameter and can be fitted to

\[ G_{\text{max}} = 122P_a \left( \frac{1}{PI - C_r} \right)^{\{PI - C_r\}} \left( \frac{\sigma'_c}{P_a} \right)^{0.82} \]  

where \( P_a \) is an arbitrary reference pressure to obtain dimensional homogeneity. The curves of \( G_{\text{max}} \) against \( \sigma'_c \) of figs 1 and 2 can not be fitted in one graph leading, consequently, to a single analytical expression to relate them. This is not surprising as it reflects that soil origin and type, geological formation processes and mineralogy, amongst other factors, affect initial stiffness. Further research is required to clarify the influence and importance of each of them.

![Graph 1](image1.png)  
Fig 1 Initial shear modulus of the Campeche Sound clay as a function of consolidation stress and relative consistency

![Graph 2](image2.png)  
Fig 2 Initial shear of clays from Mexico City as a function of consolidation stress and the difference (PI-Cr)

**SHEAR MODULI AT LARGER STRAINS**

Cyclic triaxial test results were used to obtain values of shear moduli as a function of shea strain to produce the graphs in figs 3 and 4, for the clays from the Campeche Sound and Mexico City, respectively. All the samples were isotropically consolidated under different effective confining stresses. Note the rather small values of \( G_{\text{max}} \) exhibited by the Mexico City clay, which span from about 5 to 20 MPa whereas in the Campeche Sound materials these were mostly around 50 MPa. Referring to fig 3, the flat initial portion of the curves obtained for the stiffer, more brittle materials from the Campeche Sound, covers strain levels of about 0.01 % whilst the more plastic materials retrieved from shallow depths near the sea bed show little
degradation of stiffness for strains well above 0.1 %. In fact, the stiffness-strain behaviour of the softer Campeche Sound clays closely resembles the one shown in fig 4 for the Mexico City clays. At least in this respect, it can be argued that the latter are not unique. The curves in both fig 3 and 4 can be modelled with

\[ G = G_{\text{max}} (1 - H(\gamma)) \]  

(4)

where

\[ H(\gamma) = \left[ \frac{(\gamma/\gamma_r)^{2B}}{1+(\gamma/\gamma_r)^{2B}} \right]^d \]  

(5)

\( G \) is the shear modulus for any shear strain, \( \gamma \) and \( \gamma_r \) is a reference strain. The stiffness-strain curves for the marine clays from the Campeche Sound are adequateley modelled adopting constant values of \( A = 1.0 \) and \( B = 0.5 \). In the case of the Mexico City clays, \( A \) depends on an experimentally determined function of plasticity index, \( A' \), and on relative consistency, i.e. \( A = A' + C_r \); \( B \) is another experimental function that only depends on the former. Plots of \( A' \) and \( B \) against plasticity index are given in figs 5 and 6. As it might have been expected, \( \gamma_r \), also depends on plasticity index or relative consistency (figs 7 and 8 for the Campeche Sound and the Mexico City clays, respectively). The influence of plasticity index on the dynamic stiffness-strain behaviour of clays was noted previously by other researchers (e.g., Dobry and Vucetic, 1987).

**DAMPING RATIO**

Hardin and Drnevich (1972) showed that the damping ratio, \( \lambda \), of viscoelastic materials that obey Masing's rules during cyclic loading is related to shear modulus

\[ \lambda = \lambda_{\text{max}} (1 - G/G_{\text{max}}) \]  

(6)

where \( \lambda_{\text{max}} \) is the maximum value of the damping ratio before soil failure. From equations (4) and (5) and making the necessary substitutions, (6) becomes

\[ \lambda = \lambda_{\text{max}} (1 - \left[ \frac{(\gamma/\gamma_r)^{2B}}{1+(\gamma/\gamma_r)^{2B}} \right]^d) \]
\[ \lambda = (\lambda_{\text{max}} - \lambda_{\text{min}})(H(\gamma)) + \lambda_{\text{min}} \]  

(7)

Fig 5 Effect of plasticity index on the parameter \( A' \)

Fig 6 Effect of plasticity index on the parameter \( B \)

\[ \gamma_r = \frac{1}{\gamma} \]  

Fig 7 Reference strain, \( \gamma_r \), as a function of relative consistency, \( C_r \)

Fig 8 Effect of plasticity index on \( \gamma_r \)

\( \lambda_{\text{min}} \) is the initial or small strain value of the damping ratio. This expression shows that \( \lambda \) also depends on plasticity index and relative consistency by virtue of equation (5). The extreme values of \( \lambda \), \( \lambda_{\text{min}} \) and \( \lambda_{\text{max}} \), can be assigned from the results of experiments like those conducted here. Referring to fig 9, \( \lambda_{\text{min}} \) and \( \lambda_{\text{max}} \) were found to equal 3 and 28 % for the marine clays. In contrast, the extreme values of the damping coefficient of the young lacustrine clays from Mexico City are only 0.5 and 13 %. While the former values fall within the range of those found for many other plastic clays, the latter ones are much smaller than most of the values reported previously in the literature.

OTHER ASPECTS OF DYNAMIC SOIL BEHAVIOUR

Initial stiffness, \( G_{\text{max}} \), the relationships between stiffness and strain, damping and strain, as well as the extreme values of \( \lambda \), can be used to describe many of the aspects of the undrained behaviour of plastic
clays during earthquakes but there are other factors that can modify them. Two of them are briefly discussed in what follows.

Fig 9 Damping-strain curves, Campeche Sound clays

Fig 10 Damping-strain curves, Mexico City clay

**Fatigue effects**

Repetitive loads degrade stiffness due to fatigue. The amount and rate of degradation depend on soil type and state, stress level, cyclic stress amplitude and number of applied cycles of stress. Cyclic angular distortions at the microstructural level bring about fatigue. Normally consolidated or lightly overconsolidated saturated clays generally accumulate positive pore pressures during cyclic loading which accelerates stiffness degradation but are less affected by this effect than loose non-plastic (granular) materials in which it can be catastrophic and may even lead to liquefaction. Fatigue is more important in brittle than in plastic soils. For both the Campeche Sound and the Mexico City clays, the authors have found that the relation put forth by Idriss et al (1978) to evaluate fatigue effects, works well:

\[ G_N = G_0 N^{-\alpha} \]  

(8)

Fatigue and the accumulation of positive pore pressures seem to be closely related and in the Campeche Sound clays it only became important when the cyclic stress amplitude exceeded 80% of its undrained strength which roughly coincided with the cyclic stress level at which stiffness degradation also became significant (Romo and Ovando, 1993). The results of the tests described here and of many others performed in Mexico City clay indicate that pore pressure build up due to the application of repetitive loads only becomes apparent when the cyclic strain amplitude reaches 2 to 3 %. Even then, the magnitude of the excess pore pressure seldom exceeds 0.3σ' (Romo, 1995).

**Permanent displacements**

When a soil is subjected to cyclic loading, it undergoes transient cyclic deformations; after a number of load applications permanent, non-reversible deformations appear. Both cyclic transient and permanent deformations depend on cyclic stress amplitude but the latter also depend on the duration of loading. Experiments in a large variety of soils have shown that these deformations are correlated and that there exists a distinctive strain value beyond which permanent deformations accumulate faster. In the case of the
Mexico City clay, the threshold shear strain is about 2.2 % (Romo, 1995), i.e. one order of magnitude larger than the strain obtained from the $G$ versus $\gamma$ curves of fig 4 to fix the boundary between linear and non-linear clay behaviour. The difference between both thresholds suggests that even if the material behaves non-linearly, it will not necessarily yield plastically. The much higher threshold for the appearance of plastic strains also indicates that irreversible deformations will develop significantly when the material is close to reaching its dynamic failure stress. According to the results of many tests, dynamic failure stresses exceed those found under static conditions by about 30 to 40 % (e.g. Romo and Ovando, 1995), a phenomenon that is most probably due to a loading rate or strain rate effect. Other less rate dependent clayey materials also exhibit different threshold strains to mark the limits of linearity and the appearance of plastic straining. In the case of the Campeche Sound clays the information gathered during the research does not allow for setting the threshold values with certainty but they are definitely smaller that in the Mexico City clays. The same can be expected to be true for other less plastic materials.

CONCLUSIONS

Plasticity index and relative consistency have been shown to be key parameters in defining some of the most relevant features of the dynamic behaviour of two clays having different origins and characteristics: the highly plastic Mexico City clays and the medium plasticity clays from the Campeche Sound. These features can be expressed with a hyperbolic model which depends on properties such as the initial shear modulus, the extreme values of damping and a reference strain. All of these depend on the two key parameters. The model also requires that two experimental parameters be defined, A and B.

Even though the dynamic behaviour of the Campeche Sound and the Mexico City clays can be modelled with expressions that are formally equal, the experimental functions that define the dependency of the properties and experimental parameters of the model on plasticity index and relative consistency are different for these two materials. This reflects that soil origin and type, geological formation processes, microstructure, mineralogy and, indirectly, stress state and history, influence the form of these experimental functions. It also suggests that it may not be possible to obtain general soil models of the type expressed by equations (4) and (7) but it does point out that the approach shown here can be followed to obtain site, or even possibly, region-specific models.

Plasticity index and relative consistency can be obtained from simple tests. Hence, the appeal of the model described here to make preliminary estimations of dynamic properties and dynamic soil response, prior to the actual dynamic testing of soils.

REFERENCES


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