THE INFLUENCE OF BUILDING PHASES IN THE POST-ELASTIC BEHAVIOR
OF MULTISTORIED REINFORCED CONCRETE FRAMES

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ABSTRACT

For a correct prediction of the post-elastic behavior of r.c. frames, an accurate analysis of the stress state in the various structural elements, due to static loading only, is fundamental. This stress state is very sensitive to the building phases and depends on the effectiveness of the successive structural schemes adopted.

In general practice the final configuration of a multistoried frame is considered as the only configuration for structural analysis; it is obvious that the results obtained in this way are very different to those that would have been obtained by considering the building phases and taking into account the actual structural schemes. Assuming that each building phase consists in the construction of a single storey, it is necessary to make use of an analysis procedure that is able to correctly define, for each phase, the stress state in the structural elements. In fact, these elements are stressed by transient static bending moments greater than the final static ones, while the corresponding axial loads are much smaller than the final values.

In the present paper a post-elastic analysis of multistoried frames is carried out. Each building phase is analyzed and the corresponding response is obtained for each structural element; seismic loading is then simulated via a quasi-static cyclic load history.

Some interesting results in terms of post-elastic response are obtained. In particular it is possible to evaluate the effectiveness of an earthquake-resistant design in terms of the ultimate loads and displacements, the sectional and global ductility and the amount of hysteretic energy. Frames analyzed taking into account the building phases are very sensitive to the soft storey mechanism, with plastic hinges in the columns rather than in the beams; the global ductility determined is smaller than the theoretical ductility obtained using a single analysis; the plastic hinge formation path due to a cyclic load history is very different according to the building phases. These results appear very useful, and it is suggested that this approach could form a correct layout for an earthquake-resistant design.

KEYWORDS

Building phases; multistoried r.c. frames; post-elastic behavior; quasi-static cyclic load history; global ductility; plastic hinge; plastic hinge formation path.

BUILDING PHASES MODELLING

In multistoried r.c. frame analysis the geometrical model usually taken into account is that of the final configuration considering the skeleton-frame and no consideration is given to the building phases in terms of either structural schemes or stress state.
An approximate evaluation of the stress state in the external column of a four-storey frame is illustrated in Fig. 1. Considering the final configuration, a bending moment of about $M/2$ and an axial load of $4N$ are obtained (case a.), on the other hand, if loads only up to the second building phase are taken into account, a bending moment of $\alpha M$ and an axial load of $\beta N$ are derived (case b.), in which the factors $\alpha$ and $\beta$ are generally not far from unity.

![Diagram](image)

Fig. 1. Four storey r.c. frame: final configuration (a.), $2^{nd}$ building phase (b.)

Not taking rheological effects into consideration, the building phases lead to a different superimposition of effects in the case that the successive stress states remain elastic, or even to the possibility of yielding in some element due to static loading only. In fact, assuming that each building phase consists in the construction of a single storey, the dead load of the storey under construction (which is as yet non load bearing) is discharged onto the precedent storey through the temporary supports; when a storey is completed and the supports are removed, the storey below is subjected to unloading. This sequence of static loading is valid for all but the top storey.

Considering superimposition of effects, this load sequence leads to a final stress state in the various structural elements which can be very different from that computed considering dead load, permanent load and live load all acting upon the final geometrical configuration; a different post-elastic behavior would be therefore predicted.

Regarding a $N_S$-storey frame it is possible to consider $N_S$ building phases, $N_S + 1$ loading phases and $N_S - 1$ transient phases during which the last storey bears no load, the storey below is subjected to its own dead load, the dead load of the storey above and the live load due to supports, scaffolding and workers, and the remaining storeys are subjected to their own dead loads.

In Fig. 2, a two-span and four-storied frame is considered regarding $N_S + 1$ loading phases, $q_a$ is the dead load, $q_b$ is the live load due to building procedures and $q_c$ is the global load due to permanent load and live load. Loads at each building phase are therefore:

$$q_1 = 2q_a + q_b; \quad q_2 = q_a + q_b; \quad q_3 = q_a + q_c$$  \hspace{1cm} (1)

If the stress fields in all structural elements remain in the elastic regime during the $i^{th}$ building phase ($1 \leq i \leq N_S - 1$), the principle of superimposition remains valid and the following considerations can be made:

- the $i^{th}$ storey is subjected to $q_1$, since the $(i+1)^{th}$ storey is under construction;
- the $(i-1)^{th}$ storey is subjected to the negative distributed load $q_2$ due to support removal;
- the remaining storeys bear no load.

![Diagram](image)

Fig. 2. Loading phases for a multistoried r.c. frame according to building phases.

In the $N_S^{th}$ building phase the frame is in the final configuration:

- the $N_S^{th}$ storey is subjected to its own dead load $q_a$;
- the $(N_S-1)^{th}$ storey is subjected to the negative distributed load $q_2$. 

A \((N_S + 1)^{th}\) loading phase should next be considered in which each storey is subjected to the permanent and live loads; the \(N_S^{th}\) and \((N_S + 1)^{th}\) loading phases have the same geometrical configuration and can be unified, as shown in Fig. 3, considering for the last-but-one storey a negative distributed load:

\[
q_4 = q_3 - q_1
\]  

(2)

Fig. 3. Building phases for a multistoried r.c. frame.

According to Fig. 3 it is therefore possible to express the influence of building phases in terms of the number of structural schemes \(N_S\) and to control the possibility of yielding arising in the structural elements.

ANALYTICAL MODEL

The analytical model adopted in this paper (Albanesi et al., 1996) is a macroscopic finite element model (Breyssse et al., 1987, Riva et al., 1990); both girder and column elements have zero length rotational springs representing the fixed-end rotations at the joint interfaces and rigid offset zones to account for the joint zone (Keshavarzian et al., 1985, Mulas et al.), the macroelement nonlinear hysteretic moment-curvature relation is determined by means of a layered model for each end section (Albanesi et al., 1992), while the nonlinear moment-rotation law for rotational springs is determined using a finite element model (Albanesi et al., 1991). In the aim to reduce computational time the layered model and the bond FEM model are used in an a priori analysis and data storage, it is possible to generalize these results for any load history for each sub-element via a simplified approach (Albanesi et al., 1994a).

In terms of geometrical and mechanical characteristics it is possible to evaluate the length and the global behavior of inelastic zones on the basis of the bending moment diagram; the simplified approach used in the nonlinear hysteretic moment-curvature relation definition is sensitive to cracking, yielding and material failure.

The actual moment-curvature relationship is defined in terms of the peak curvatures \(\chi\) and \(\chi''\) obtained through the a priori analysis. Increments from a given state \([M_0, X_0]\) are determined by interpolation, considering the two possible paths; in each section the current state is updated in terms of equilibrium.

A structural element including beam and column is considered, as schematically shown in Fig. 4; it consists of five parts: two rigid offset zones, two nonlinear zero length rotational springs and a non linear element subdivided into macroelements.

Fig. 4. Adopted girder and column element

Assuming that each storey behaves as a rigid plane and consequently postulating axial indeformability of girder element, it is possible to define, for beam and column respectively, both the actual state:

\[
M_0 = M_0(z); X_0 = X_0(z)
\]  

(3.a)

\[
M_0 = M_0(z,N); X_0 = X_0(z,N)
\]  

(3.b)
and the force and deformation vectors:
\[ f_{b_0} = \left[ -T_0(z_i), T_0(I), -M_0(z_i), M_0(I), -N_0(z_i), N_0(I) \right]^T \]
\[ f_{c_0} = \left[ -T_0(z_i), T_0(I), -M_0(z_i), M_0(I), -N_0(z_i), N_0(I) \right]^T \]
\[ \eta_{b_0} = \left[ \eta_0(z_i), \eta_0(I), \phi_0(z_i), \phi_0(I) \right]^T \]
\[ \eta_{c_0} = \left[ \eta_0(z_i), \eta_0(I), \phi_0(z_i), \phi_0(I), u_0(z_i), u_0(I) \right]^T \] (4.a)

If the same assumption is made for the adopted global element of Fig. 4, the deformation vectors are six-element and eight-element for beam and column respectively taking into account the length of the rigid offset zones.
Assuming that dead and live loads and column axial loads remain constant in the post-elastic analysis, the stress-state determination consists in an iteration process beginning at the current state as defined in (3) \( \div (4) \), and considering incremental deformation vectors; in the case of a column therefore:
\[ \Delta \eta_{c_0} = \eta_c - \eta_{c_0} \rightarrow \Delta f_{c_0} = k_{c_0} \Delta \eta_{c_0} \rightarrow f_{c_1} = f_{c_0} + \Delta f_{c_0} \] (5)

and consequently:
\[ f_{c_1} \rightarrow M_1(z, N) \rightarrow \chi_1(z, N) \rightarrow \delta_1(z) = \int_0^\ell \left[ \int_0^{\chi_1(z)} d\zeta \right] \right] dz \rightarrow \delta_1(l) = \int_0^\ell \left[ \int_0^{\chi_1(z)} d\zeta \right] dz \] (6)

\[ v_1(z) = \frac{l-z}{l} v_1(0) + \frac{z}{l} v_1(l) + \frac{z}{l} \delta_1(l) - \delta_1(z) \]
\[ \eta_{c_1} = \left[ v_1(0), v_1(l), \phi_1(0), \phi_1(l), u_1(0), u_1(l) \right]^T \] (7)

The error in the solution of equilibrium equations in terms of incremental deformation vector is checked against a specified tolerance value and the stiffness matrix is consequently updated considering the tangent stiffness and the curvature increment.

The structural model and the degree of freedom scheme are shown in Fig. 5; in these models the geometrical axis of each element coincides with the deformed axis even though the layered model takes into account cross section decay (as concrete cover failure). Considering \( N_C - 1 \) spans the number of elements and of degrees of freedom is respectively \( N_E = 3N_S(2N_C - 1) \) and \( N_D = N_S(6N_C - 1) \).

The sign conventions used for these degrees of freedom are: nodal and slipping rotations, positive if counterclockwise; vertical beam-column displacements, positive if downward; horizontal beam-column displacements and lateral storey displacements, positive if towards the right. Particular choices in the numbering of these unknowns were made so as to obtain banded stiffness matrices, thus reducing as much as possible the storage required for solution.

Static analysis is carried out for each of the building phases and for the global configuration, and superimposition of effects is verified; a cyclic load is then applied to the global structure up to conventional failure. During post-elastic analysis yielding in structural elements is detected and local failure effects are monitored. The lateral storey loads are defined by means of a constant load multiplier and global state determination is detected by means of an iteration strategy in terms of lateral load increments, \( \Delta P_m \):
\[ \Delta P_m = h_m(\Delta H_m) \quad ;\quad P_m = P_{m-1} + \Delta P_m \quad ;\quad H_m = H_{m-1} + \Delta H_m \] (8)

where \( h_m \) is a non linear operator connecting the nodal displacement vector to nodal load vector. The step-by-step algorithm is based on the modified Newton Raphson method and considers the elastic global stiffness matrix at each iteration; a step-by-step updating of the global stiffness matrix is thus avoided and data storage is not excessive (see
Papadrakakis 1991). Considering the $m^{th}$ load step, the global displacement vector, $H_m$, is obtained in terms of the $(m-1)^{th}$ global displacement vector, $H_{m-1}$, and balanced force vector, $F_{m-1}$:

$$H_m = H_{m-1} + K_0^{-1}(P_m - F_{m-1})$$

(9)

**NUMERICAL ANALYSIS**

In the present paper five multistoried frames are considered (A.: four-storey, one-span; B.: four-storey, two-span; C.: four-storey, three-span; D.: two-storey, two-span; E.: three-storey, two-span). The geometrical and mechanical characteristics of beams and columns in all cases are:

- **beam:** $b = 300\text{mm} \quad h = 500\text{mm} \quad \delta = 22\text{mm} \quad A_s = A_s' = 1257\text{mm}^2$
- **column:** $b = 450\text{mm} \quad h = 450\text{mm} \quad \delta = 22\text{mm} \quad A_s = A_s' = 1257\text{mm}^2$
- **bars:** $d_s = 20\text{mm}$ (longitudinal) $d_s = 8\text{mm}$ (transversal)
- **concrete:** $f_c = 25\text{MPa} \quad f_{ct} = 2.5\text{MPa} \quad E_c = 1000f_c$
- **steel:** $f_y = 413\text{MPa} \quad f_t = 517\text{MPa} \quad E_s = 200000\text{MPa} \quad E_{sh} = 2270\text{MPa} \quad \varepsilon_{su} = 0.14$

The storey height is $H = 3.30m$ (except for the first storey where $H = 3.50m$), the span is $L = 6.10m$ while the static loads are $q_a = 30kN\cdot m^{-1}$ and $q_c = 20kN\cdot m^{-1}$; the frames are designed, considering the global scheme, in accordance to the Italian Seismic Code (see Earthquake Resistant Regulations. A world list, 1992) and considering Italian Limit Design procedures regarding section control. In the cyclic load history the peak load increment is half the storey lateral load prescribed by the Code; conventional failure is defined as local failure in terms of:

a. ultimate curvature in a macroelement for:
   a.1. ultimate tensile strain in longitudinal reinforcement;
   a.2. ultimate compressive strain in concrete core;
   a.3. longitudinal buckling in reinforcement under compression.

b. ultimate fixed-end rotation at a beam-column interface.

All of these conditions are of local character, and generally would not imply global failure of the structure; this in fact depends upon the generation of a kinematical condition. However they all cause non-reparable damage to the structure and can therefore be conventionally assumed as a stop condition in the analysis. The proximity of this conventional failure to actual structure failure due to a soft storey mechanism can be tested by controlling the plastic hinge locations. Moreover, the conditions a.2. and a.3. can be reasonably considered as a single one, as is shown in Albanesi et al. (1994 a) and Albanesi et al. (1995).

In Fig. 6 some beam and column curvature diagrams at various steps are shown; it is evident that plastic hinge development is more extended in the column.

![Fig. 6](image)

**Fig. 6**  Beam and column curvature diagrams

The transient loads during the building phases are determined according to (1), while the cyclic load history assigned is shown in Fig. 7 in terms of adimensionalized lateral storey load. In the same Fig. 7 the plastic hinge formation paths for frames $A$, $B$, $C$ are shown in terms of the load step and the type of analysis: the white hinges are those actually yielding while the black ones have yielded previously.
It can be seen how in the building phases analysis the first yielding occurs for a lower lateral load and, in frames A. and B., this first yielding occurs in a column instead of in a beam; this different yielding path causes premature failure. For frame C. it is possible to obtain the first yielding in a beam and the influence of building phases is less important in terms of premature failure, due to the greater number of columns.

![Lateral cyclic load history](image)

**Frame A.: global analysis**

![Building phases analysis](image)

**Frame A.: building phases analysis**

**Frame B.: global analysis**

**Frame B.: building phases analysis**

**Frame C.: global analysis**

**Frame C.: building phases analysis**

![Fig. 7. Frames A.+C.: Typical cyclic load history and plastic hinge formation paths for global and building phases analysis](image)

In Fig. 8 the storey displacements at failure are shown; it can be seen that in the global analysis the displacement of the top storey is about double that occurring in the building phases analysis.

The frames analyzed considering the building phases are very sensitive to the soft storey mechanism with plastic hinges forming in the columns rather than in the beams and the global ductility is less than the theoretical value obtained through a global analysis; considering frame B., if global ductility $\mu_A$ is the ratio of failure displacement to
first yielding displacement computed at 2/3 of frame height, the values $\mu_A = 3.63$ and $\mu_A = 1.97$ are obtained by global analysis and building phases analysis respectively. It is therefore seen that this frame, designed according to the Italian Seismic Code with an acceptable global ductility, actually behaves as a fragile structure if analyzed using the building phases approach. Furthermore, the early plastic hinge formation makes these frames very sensitive to second order effects ($P - \Delta$); these have not been taken into account in the present paper.

**Fig. 8.** Frames A - C: storey displacements at failure for global and building phases analysis

In Fig. 9 plastic hinge paths for frames D and E are shown. In this case it is seen that frame D has the lowest strength among all frames considering a building phases analysis in terms of post-elastic steps, even though it also has lower design lateral loads.

**Frame D: global analysis**

![Frame D: global analysis](image)

**Frame D: building phases analysis**

![Frame D: building phases analysis](image)

**Frame E: global analysis**

![Frame E: global analysis](image)

**Frame E: building phases analysis**

![Frame E: building phases analysis](image)

**Fig. 9.** Frames D - E: plastic hinges formation paths
Frame $E.$ exhibits an excellent post-elastic behavior with initial plastic hinge formation in the beams rather than in the columns, if global analysis is employed; this result is reversed when the frame analysis is carried out taking into account the building phases: in this case only one beam plastic hinge at failure is obtained.

CONCLUSIONS

A considerable influence of static stress state which develops in each structural element in common multistoried frames is obtained when the superimposition of effects due to building phases is taken into account. Regular structures with constant geometrical and mechanical characteristics at all storeys were used in the analyses; results are obtained in terms of the ultimate loads and displacements, the sectional and global ductility and the amount of hysteretic energy. Although no substantial differences ensue in the elastic response to both vertical and horizontal loads when a phases analysis is carried out, the post-elastic behavior can be considerably different; an essential weakness in the external columns of the lower storeys is significantly revealed.

These results can be enhanced and enriched in detail by considering other geometrical configurations, with reinforcement differentiation in the various structural elements or by taking into account some aspects regarding the rheological properties of the materials, thus looking into the influence of the time elapsed between the construction of one storey and the next.

It is therefore concluded that the actual sequence of construction of a structure invalidates global frame modeling; a similar result has been argued regarding the influence of curtain walls. However, while in this last case a global skeleton-frame analysis, as generally performed, can be shown to be on the safe side regarding the prediction of the structure's actual strength, the opposite is true regarding the influence of the sequence of construction. In fact, a building phases analysis leads to the prediction of a poorer behavior and to the formation of soft storey mechanisms with a subsequent reduction of the structure's global ductility. These results suggest that this approach could form a correct layout for an earthquake-resistant design.

REFERENCES