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Development of differential response spectra from spatial variability models

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ABSTRACT: This study analyzes the variability that results in the response of lifelines when two different models are used to describe the spatial incoherence of the seismic ground motions at their supports. The models were developed by Harichandran and Vanmarcke (1986) and Luco and Wong (1986) and are commonly used in lifeline earthquake engineering. Differential displacements and differential response spectra are evaluated from these models and compared. The comparison shows that the spatial variability models induce different lifeline response; the differences correlate with the degree of exponential decay of the models with separation distance and frequency.

1 INTRODUCTION

The seismic response of lifelines, such as bridges and pipelines, may be significantly affected by the spatial variability of the seismic motions at their supports. The spatial variability is usually described as a function that decays exponentially with separation distance and frequency. However, various expressions and degrees of exponential decay appear to fit data recorded at different sites or at the same site but for different earthquakes. As a result there is a multitude of expressions that describe the spatial variability of the seismic motions.

Current developments in lifeline earthquake engineering incorporate the spatial variability of the seismic ground motions in design spectra for multiple support excitations (Berrah and Kausel, 1989; Der Kiureghian and Neuenhofer, 1991). However, it has not been established yet which spatial variability expression is the more appropriate for lifeline design purposes. Furthermore, analyses of the seismic response of extended structures are commonly based on only one of the several existing descriptions for the spatial variation. The results of such analyses, however, may depend on the particular expression that was used. The question that arises then is how similar or diverse is the response of lifelines when different models are used to describe the spatial variability of the seismic ground motions at their supports. A comparison of the effects of different spatial variability models on the response of lifelines becomes then necessary.

This study compares two spatial variability models through the response that they induce in lifelines. The models were developed by Harichandran and Vanmarcke (1986) and by Luco and Wong (1986), and are commonly used in lifeline earthquake engineering. Differential displacements and differential response spectra are evaluated from these models and compared. The analysis provides insight into the variability that results in the response of lifelines when different models are used to describe the spatial variation of the seismic ground motions at their supports.

2 SEISMIC GROUND MOTION MODELS

In stationary random vibration analyses of lifeline systems, the stochastic ground motion is commonly described by its cross spectral density between the motions of two stations at a distance ξ apart from each other as:

$$S(\xi,\omega) = S(\omega)\,\rho(\xi,\omega) \tag{1}$$

$S(\omega)$, with ω indicating frequency in rad/sec, is the power spectral density (PSD) of the motions, which is assumed to be the same at all locations on the ground surface. In this study, the PSD of the seismic motions is described by the Clough-Penzien (1975) spectrum:
-acceleration ($\ddot{u}$) PSD (Fig. 1):

$$S_{\ddot{u}}(\omega)=S_o\,\frac{\omega_g^4+4\zeta_g^2\omega_g^2\omega^2}{(\omega_g^2-\omega^2)^2+4\zeta_g^2\omega_g^2\omega^2}\cdot \frac{\omega^4}{(\omega_f^2-\omega^2)^2+4\zeta_f^2\omega_f^2\omega^2} \tag{2a}$$

-displacement (u) PSD:

$$S_u(\omega) = \frac{S_{\ddot{u}}(\omega)}{\omega^4} \tag{2b}$$

with filter parameters (Hindy and Novak, 1980): ω_g= 15.46rad/sec, ω_f=1.636rad/sec, ζ_g=0.623, ζ_f=0.619;

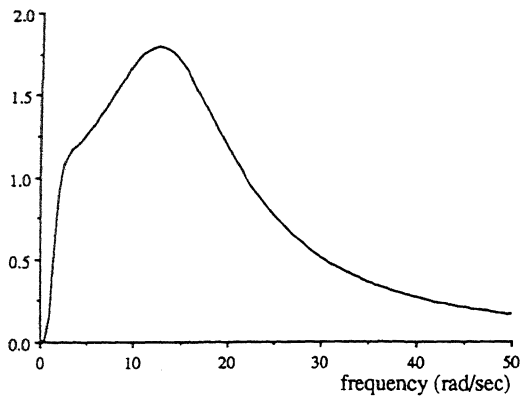

Figure 1. Power spectral density of ground acceleration (in cm^2/sec^3)

these values correspond to firm soil conditions. It is assumed that S_o is equal to $1cm^2/sec^3$.

$\rho(\xi,\omega)$ in Eq. (1) is the frequency dependent spatial correlation function. From the available descriptions for the spatial variability the following two are compared herein, mainly, because they are commonly used in random vibration analyses of lifeline systems. These are: The model developed by Harichandran and Vanmarcke (1986) from the analysis of the recorded data during Event 20 at the SMART-1 array; this model is termed model 1 in this study:

$$\rho(\xi,\omega)=c\,\exp\{\frac{-2|\xi|(1-c+ac)}{a\theta(\omega)}\}+(1-c)\,\exp\{\frac{-2|\xi|(1-c+ac)}{\theta(\omega)}\}$$

$$\theta(\omega)=k\,[1+(\frac{|\omega|}{2\pi f_o})^b]^{-1/2} \qquad (3)$$

in which, c=0.736; a=0.147; k=5210m; f_o=1.09Hz and b=2.78. The second model (model 2) is the one developed by Luco and Wong (1986) from the analysis of wave propagation through random media:

$$\rho(\xi,\omega)=\exp[-\alpha^2\omega^2\xi^2] \qquad (4)$$

in which, $\alpha=2\cdot10^{-4}$sec/m, so that the exponential decay of the analytical expression (Eq. (4)) fits the spatial variability of recorded data (Luco and Wong, 1986). The variation of the two models with frequency at separation distances ξ=100, 300 and 500m is presented in Fig. 2. Zerva (1992a, b) has shown recently through simulations based on the spectral representation method that spatial variability models such as models 1 and 2 represent the spatial incoherence of the seismic motions (change in the shape of the motions); the apparent propagation of the motions on the ground surface is not reproduced by these models. It is noted that models 1 and 2 result in real-valued cross spectral densities.

When the spatial variability effects are neglected, then:

$$\rho(\xi,\omega)=1 \qquad (5)$$

i.e., the spatial correlation is independent of frequency and separation distance. In the deterministic domain, this case corresponds to equal motions at the supports, an assumption commonly used in lifeline earthquake engineering.

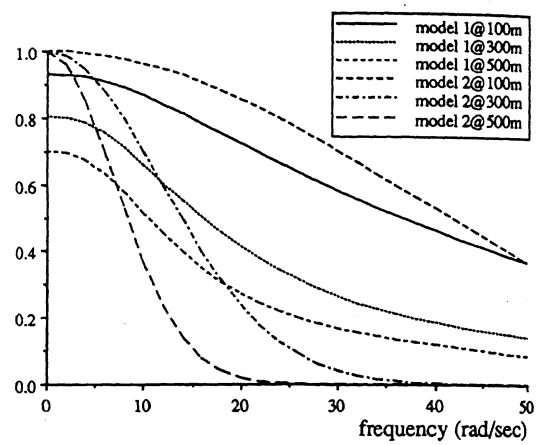

Figure 2. Variation of spatial incoherence models with separation distance and frequency

3 STRUCTURAL RESPONSE FOR MULTIPLE-SUPPORT EXCITATION

The PSD of the seismic response at location y along the axis of an extended structure (e.g., a beam) on multiple supports subjected to spatially variable seismic ground motions is, generally, given by:

$$S(y,\omega)=\sum_{J=1}^{N}\sum_{L=1}^{N} r_J^{(n)}(y)\, r_L^{(n)}(y)\, S_{u_J u_L}(\omega)$$
$$+\sum_{J=1}^{N}\sum_{L=1}^{N}\sum_{k=1}^{K}[\Phi_k^{(n)}(y)\{r_J^{(n)}(y)R_{kL}H_k(\omega)S_{u_J\ddot{u}_L}(\omega)$$
$$+r_L^{(n)}(y)R_{kJ}H_k^*(\omega)S_{\ddot{u}_J u_L}(\omega)\}]$$
$$+\sum_{J=1}^{N}\sum_{L=1}^{N}\sum_{k=1}^{K}\sum_{m=1}^{K}\Phi_k^{(n)}(y)\Phi_m^{(n)}(y)R_{kJ}R_{mL}\cdot$$
$$\cdot H_k^*(\omega)H_m(\omega)S_{\ddot{u}_J\ddot{u}_L}(\omega) \qquad (6)$$

in which, $r_J^{(n)}(y)$ indicates the nth derivative with respect to y of the shape function for unit displacement at support J; $\Phi_k^{(n)}(y)$ is the nth derivative of the mode-shape for mode k; R_{kJ} is the participation factor for mode k and the excitation at support J; N is the number of supports and K the number of modes required for the evaluation for the response quantity under consideration. The frequency transfer function for mode k, $H_k(\omega)$, is given by:

$$H_k(\omega)=[(\omega^2-\omega_k^2)+2i\zeta_k\omega_k\omega]^{-1};\quad i=\sqrt{-1} \qquad (7)$$

ω_k and ζ_k indicate the frequency and the damping coef-

ficient for mode k; and * in Eq. (6) indicates the complex conjugate.

$$S_{u_J u_L}(\omega) = S_u(\omega)\,\rho(\xi_{JL},\omega);\; S_{\ddot{u}_J \ddot{u}_L}(\omega) = S_{\ddot{u}}(\omega)\,\rho(\xi_{JL},\omega);$$

$$S_{\ddot{u}_J u_L}(\omega) = S_{\ddot{u}_J u_L}(\omega) = -S_{\ddot{u}}(\omega)\,\rho(\xi_{JL},\omega)\,/\,\omega^2$$

indicate the cross spectral density between ground displacements at supports J and L, the cross spectral density between ground accelerations, and the cross spectral densities between displacements (u) and accelerations ($\ddot{u}$) at the two supports, respectively. The distance between the two supports is ξ_{JL}. When n=0, Eq. (6) describes the PSD of total displacements. For higher values of n (n>0), the equation provides the PSDs of the derivatives of total displacements with respect to y, and is representative of the PSDs of internal forces in the structure, since internal forces are proportional to the derivatives of total displacements.

In Eq. (6) the double summation represents the quasi-static contribution, the quadruple summation the dynamic contribution and the triple summation the contribution of the cross correlation between the quasi-static and dynamic terms to the total response along the structure. Der Kiureghian and Neuenhofer (1991) indicated that the cross terms between the quasi-static and the dynamic response in Eq. (6) can be neglected if the natural frequencies of the structure are higher than approximately 0.5Hz. Furthermore, they suggested that, for incoherent seismic motions, such as the ones described by models 1 and 2, and for firm soil conditions, as used herein for the evaluation of the filter parameters in the ground motion model (Eq. (2)), the cross terms between modes in the quadruple summation (Eq. (6)) are small for well separated frequencies. Accordingly, the dominant terms in Eq. (6), that control the lifeline response, are the quasi-static ones (double summation in Eq. (6)):

$$S_{QS}(y,\omega)=\sum_{J=1}^{N}\sum_{L=1}^{N} r_J^{(n)}(y)\, r_L^{(n)}(y) S_{u_J u_L}(\omega) \qquad (8)$$

and from the dynamic terms (quadruple summation in Eq. (6)), the ones that correspond to the excitation of individual modes:

$$S_D(y,\omega)=\sum_{J=1}^{N}\sum_{L=1}^{N}\sum_{k=1}^{K} [\Phi_k^{(n)}(y)]^2 R_{kJ} R_{kL} |H_k(\omega)|^2 S_{\ddot{u}_J \ddot{u}_L}(\omega) \qquad (9)$$

The root-mean-square (rms) quasi-static and dominant dynamic response can be obtained from the following mean-square expressions:

$$\sigma_{QS}^2(y)=\int_{-\infty}^{+\infty} S_{QS}(y,\omega)d\omega;\;\; \sigma_D^2(y)=\int_{-\infty}^{+\infty} S_D(y,\omega)d\omega \qquad (10)$$

Equations (10) provide information on the corresponding mean maximum quantities, since the mean maximum response is proportional to the rms response. The two spatial variability models are compared through their contribution to the dominant quasi-static and dynamic response of the structures in Sections 3.1 and 3.2.

3.1 Quasi-static response

The mean-square quasi-static response (Eqs. (8) and (10)) is rewritten as:

$$\sigma_{QS}^2(y)=\sum_{J=1}^{N}[r_J^{(n)}(y)]^2\sigma_u^2 + \sum_{J=1}^{N-1}\sum_{L=J+1}^{N} r_J^{(n)}(y)\, r_L^{(n)}(y) D^2(\xi_{JL}) \qquad (11)$$

in which, σ_u is the rms ground displacement:

$$\sigma_u^2=\int_{-\infty}^{+\infty} S_u(\omega)d\omega \qquad (12)$$

and

$$D^2(\xi_{JL}) = \int_{-\infty}^{+\infty} \{S_{u_J u_L}(\omega)+S_{u_L u_J}(\omega)\}d\omega \qquad (13)$$

The quasi-static analysis concentrates on the evaluation of internal forces in the structure (n>0), rather than displacements. It follows from the property of the shape functions:

$$\sum_{J=1}^{N} r_J(y)=1 \quad\rightarrow\quad \sum_{J=1}^{N} r_J^{(n)}(y)=0\;;\; n>0 \qquad (14)$$

Equation (11) is then rewritten as:

$$\sigma_{QS}^2(y) = -\sum_{J=1}^{N-1}\sum_{L=J+1}^{N} r_J^{(n)}(y)\, r_L^{(n)}(y)\,[2\sigma_u^2-D^2(\xi_{JL})] \qquad (15)$$

The expression in the brackets depends only on the seismic ground motion characteristics and represents the mean-square differential ground displacement Δu between the motions of the supports:

$$\sigma_{\Delta u}^2 = 2\sigma_u^2 - D^2(\xi_{JL}) \qquad (16)$$

i.e., the quasi-static internal forces in the structure are controlled by the differential displacements between the supports. As expected, when the seismic motions at the supports are assumed to be the same, the differential support motions are zero, and the seismic excitation does not contribute to the internal forces in the structure (Eq. (15). The rms differential displacements based on models 1 and 2 for support separation distances ranging from 0 to 500m are presented in Fig. 3. Figure 3 indicates that model 1 produces significant differential displacements, as compared to the absolute ground displacement, which is equal to 0.77cm (Eqs. (2b) and (12)). The rms differential displacements of model 2 are considerably lower. It follows then from Fig. 3 and Eq. (15) that model 1 results in higher contributions to the quasi-static internal forces than model 2.

The differences between the differential displacements obtained from the models result from the different behaviour of the spatial variation functions (Fig. 2) in the

low frequency range that controls the displacements (ω<3rad/sec from Eq. (2b)). Model 2 produces full correlation in this frequency range (Eq. (4)). As a result, the displacement time histories described by this model are essentially identical at distances up to 500m (Zerva, 1992b), and the differential displacements are small. On the other hand, model 1 results in partial correlations even at zero frequency for finite separation distances (Eq. (3)). Displacement time histories described by this model exhibit variability in amplitude and frequency content as the separation distance increases (Zerva, 1992a), and, consequently, the differential displacements become significant. It follows then that spatial variability models that yield partial correlations at low frequencies contribute more to the quasi-static internal forces in the structure than models fully correlated in that frequency range.

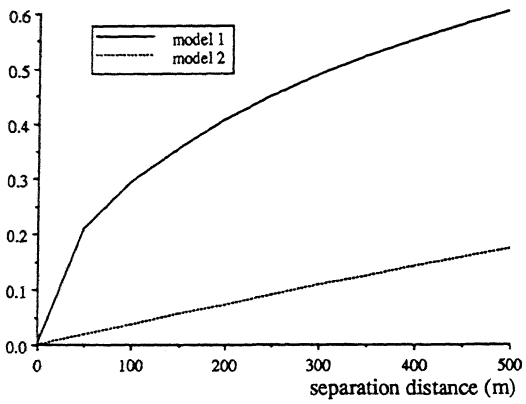

Figure 3. RMS differential ground displacement (in cm)

3.2 Dynamic response

In order to separate the contributions of the absolute acceleration at the supports (Eq. (2a)) from those of the spatial incoherence between support motions, the mean-square dynamic response (Eqs. (9) and (10)), is rewritten as:

$$\sigma_D^2(y) = \sum_{k=1}^{K} \frac{[\Phi_k^{(n)}(y)]^2}{\omega_k^4} \{ \sum_{J=1}^{N} R_{kJ}^2 A^2(\omega_k,\zeta_k) + \sum_{J=1}^{N-1} \sum_{L=J+1}^{N} R_{kJ} R_{kL} A^2(\omega_k,\zeta_k,\xi_{JL}) \} \quad (17)$$

in which, $A(\omega_k,\zeta_k)$ is the rms contribution of the acceleration time history of each support to the dynamic response of the structure:

$$A^2(\omega_k,\zeta_k) = \omega_k^4 \int_{-\infty}^{+\infty} |H_k(\omega)|^2 S_{\ddot{u}}(\omega)\, d\omega \quad (18)$$

$A(\omega_k,\zeta_k)$ is proportional to the mean maximum response of a single oscillator of frequency ω_k and damping ζ_k subjected to the seismic motions described by the Clough-Penzien spectrum; i.e., it is proportional to the ordinates of the conventional response spectrum. $A(\omega_k,\zeta_k,\xi_{JL})$ in Eq. (17) is the rms contribution of the spatial incoherence between the accelerations of two supports J and L to the dynamic response of the structure; the effect of the seismic spatial variability between the support motions is filtered through a single degree of freedom oscillator with frequency ω_k and damping ζ_k:

$$A^2(\omega_k,\zeta_k,\xi_{JL}) = \omega_k^4 \int_{-\infty}^{+\infty} |H_k(\omega)|^2 \cdot \{ S_{\ddot{u}_J\ddot{u}_L}(\omega) + S_{\ddot{u}_L\ddot{u}_J}(\omega) \}\, d\omega \quad (19)$$

Because of its similarities to the conventional response spectrum, and, in addition, because it takes into consideration the differences between the time histories at the two supports, $A(\omega_k,\zeta_k,\xi_{JL})$ is termed "differential response spectrum" in this study. The comparison of the effects of the two spatial variability models on the dynamic response of lifelines reduces then to the comparison of their corresponding differential response spectra, which depend only on the seismic ground motion characteristics and given values for the modal frequency and the damping coefficient.

Figure 4 presents the contribution of the two spatial variability models to the dynamic response of the structure for support separation distances 100, 300 and 500m. The modal frequency varies from 3.14(=π) to 50 rad/sec and the damping coefficient is equal to 5% of critical for all cases. For comparison purposes, the differential response spectra for equal support excitations are also presented in Fig. 4; the differential response spectra in this case are identical for all separation distances.

Figure 4 indicates that the assumption of equal support excitations produces the highest ordinates for the differential response spectra. Equal support excitations, however, do not necessarily induce the highest lifeline response, because they do not contribute to the internal quasi-static forces in the structure, as already mentioned in Section 3.1. For a separation distance of 100m, model 2 produces higher amplitudes for the differential response spectra than model 1; the two models tend to similar values as the modal frequency increases. For the longer separation distances (300 and 500m) both models produce similar results in the lower frequency range with model 2 producing slightly higher amplitudes. For frequencies higher than 12 and 7rad/sec for ξ=300 and 500m, respectively, model 2 produces significantly lower amplitudes for the differential response spectra and, consequently, lower contribution to the dynamic response of lifelines.

When the modal frequency falls within the range of the dominant frequencies of the seismic excitation, the major contribution to the response results from frequencies in the vicinity of the modal frequency under

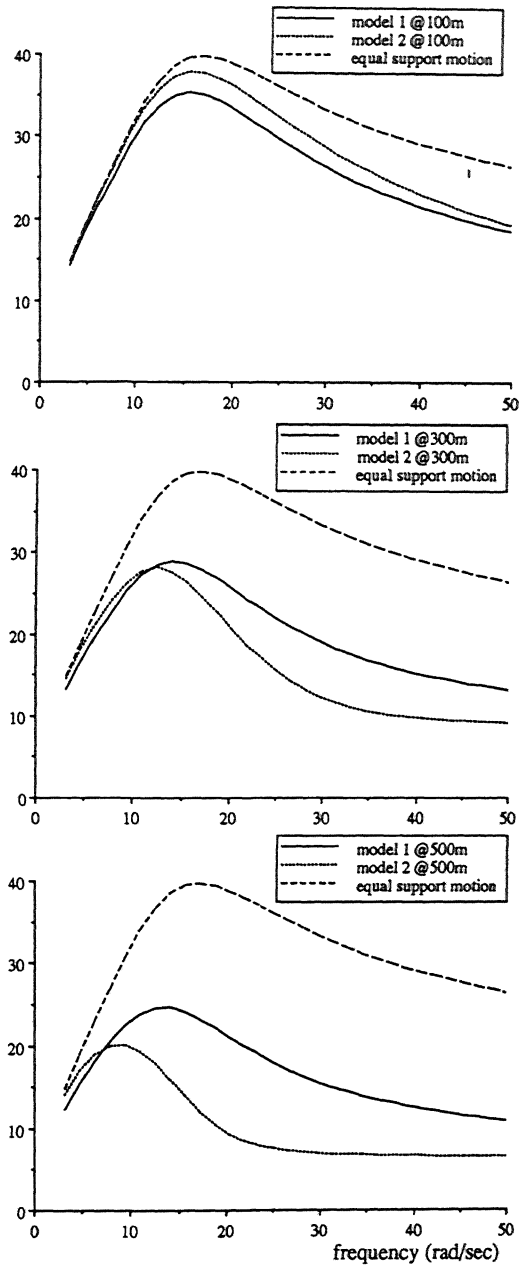

Figure 4. Differential response spectra (in cm/sec^2)

consideration. The seismic excitation for the evaluation of the differential response spectra in the case of equal support motions is twice the PSD of the ground acceleration (Eqs. (1), (5) and (19)). On the other hand, when models 1 or 2 are used, the seismic excitation for the differential response spectra is twice the cross spectral density of the seismic motions (Eq. (19)). The cross spectral density is the product of the PSD (Fig. 1) and the spatial variability (Fig. 2). It appears then that the spatial variability model that produces higher correlations in the vicinity of the modal frequency will also yield higher contributions to the dynamic response of the structure. Indeed, for ω<12rad/sec and ω<7rad/sec, model 2 produces higher correlations than model 1 for ξ=300 and 500m, respectively (Fig. 2), and results in slightly higher amplitudes for the differential response spectra (Fig. 4). For the range of higher frequencies, the exponential decay of model 2 is sharper than that of model 1 (Fig. 2), and its contribution to the dynamic response of the lifeline becomes significantly lower than that of model 1 (Fig. 4). For the shorter separation distance (ξ=100m), model 2 produces higher correlations in the range of dominant frequencies of the acceleration PSD (Fig. 2) and results in higher amplitudes for the differential response spectra than model 1 (Fig. 4). Furthermore, the assumption of equal support motions, which yields the highest correlation (=1) for all frequencies and separation distances, produces the highest contribution to the dynamic response. Figure 4 then indicates that the dynamic response of lifelines is controlled by the exponential decay of the spatial variability models: the higher the correlation in the vicinity of the natural frequency of the mode under consideration, the higher the contribution of the mode to the dynamic response of the structure.

It is noted from Fig. 4 that the frequency range of significant amplitudes for the differential response spectra of model 2 at ξ=300 and 500m is narrow, and the spectra tend to constant values at lower frequencies than either those of model 1 or the ones obtained through the assumption of equal support motions. Response spectra tend to constant values when the modal frequencies increase past the dominant frequencies of the seismic excitation. Indeed, the frequencies at which the differential response spectra for model 2 tend to constant values (approximately 40 and 25rad/sec for separation distances of 300 and 500m, respectively in Fig. 4) coincide with the frequencies at which the spatial variation, and, consequently, the seismic excitation, for model 2 approaches zero (Fig. 2). For the differential response spectra obtained from model 1 it may be observed that, although the amplitudes decrease as the separation distance increases, the frequency content of the spectra is essentially the same for the separation distances analyzed. This is due to the fact that the spatial variability of this model does not exhibit the sharp exponential decay of model 2 (Fig. 2).

4 CLOSURE

This study analyzes the variability that results in the seismic response of lifelines when the spatial variation of the seismic motions at their supports is described by different models. Two spatial variability models commonly used in lifeline earthquake engineering were used in the analysis. The models were compared through their contribution to the dominant quasi-static and dy-

namic response of the structures. Differential displacements and differential response spectra were evaluated for this purpose. The analysis suggested that, although both spatial variability models are valid representations for the spatial incoherence of the seismic ground motions, they induce different lifeline response. Spatial variability models that yield partial correlations at low frequencies produce higher contributions to the quasi-static response of the structures than models fully correlated in that frequency range. It was also found that spatial variability models that decay slowly with separation distance and frequency contribute more to the dynamic response than models that exhibit a sharper exponential decay.

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