Earthquake Engineering, Tenth World Conference © 1992 Balkema, Rotterdam. ISBN 90 5410 060 5

Response spectrum method for incoherent support motions

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ABSTRACT: A new response spectrum method is developed for seismic analysis of linear multi-degree-of-freedom, multiply-supported structures subjected to spatially varying ground motions. Variations of the ground motion due to wave passage, loss of coherency with distance, and variation of local soil conditions are included. The method is based on fundamental principles of random vibration theory and properly accounts for the effects of correlation between the support motions as well as between the modes of the structure.

1 INTRODUCTION

Observations during recent earthquakes, notably the Loma Prieta earthquake of October 17, 1989, have clearly demonstrated that seismic ground motions can vary significantly over distances which are of the same order of magnitude as the dimensions of some extended structures, such as bridges. Three phenomena are responsible for these variations: (1) the difference in the arrival times of seismic waves at different stations, denoted herein as the "wave passage" effect; (2) the loss of coherency of the motion due to reflections and refractions of the waves in the heterogeneous medium of the ground, as well as due to the difference in the manner of superposition of waves arriving from an extended source at various stations, denoted herein as the "incoherence effect"; and (3) the difference in the local soil conditions at each station and the manner in which they influence the amplitude and frequency content of the bedrock motion, denoted herein as the "local" effect. Recent analyses with array recordings have shed light on the nature of these effects and their relative magnitudes (e.g., Bolt et al. 1982, Harichandran and Vanmarcke 1986, Abrahamson et al. 1991).

The effect of differential support motions on the response of extended structures has been of concern for a long time and studies based on time history or random vibrations analyses have been carried out (e.g., Abdel-Ghaffar and Rubin 1982, Zerva 1990). It is known that, under realistic conditions, the differences in the support motions can significantly influence the internal forces generated in the structure. While in most cases the magnitudes of these forces are reduced, there are situations where differential support motions may actually result in larger internal forces. The failure of several bridges during the Loma Prieta earthquake has highlighted the need for a better understanding of this phenomenon, and for the development of practical analysis tools that can accurately account for its effects.

This paper, based on a report by the authors (Der Kiureghian and Neuenhofer, 1991), describes a new response spectrum method for seismic analysis of structures subjected to multiple support excitations, which properly accounts for the wave passage, incoherence, and local effects. The method is based on the principles of random vibration theory and is an extension of the well known CQC method (Der Kiureghian 1981, 1991) to the case of multiple-support excitations. The method is practical, since it employs information that is normally available in seismic design applications, i.e., peak ground displacements and response spectra at each support degree of freedom, a rough estimate of the duration of motion, and a coherency function. An example, demonstrating an application of the method and characterizing the influence of differential support motions on selected responses of a structure, concludes the paper.

2 EQUATIONS OF MOTION

The equations of motion for a discretized, n-degree-of-freedom linear system subjected to m support motions can be written in the matrix form (Clough and Penzien 1975)

$$\begin{bmatrix} \mathbf{M} & \mathbf{M}_c \\ \mathbf{M}_c^{\mathrm{T}} & \mathbf{M}_g \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{x}} \\ \ddot{\mathbf{u}} \end{Bmatrix} + \begin{bmatrix} \mathbf{C} & \mathbf{C}_c \\ \mathbf{C}_c^{\mathrm{T}} & \mathbf{C}_g \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{u}} \end{Bmatrix} + \begin{bmatrix} \mathbf{K} & \mathbf{K}_c \\ \mathbf{K}_c^{\mathrm{T}} & \mathbf{K}_g \end{bmatrix} \begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{F} \end{Bmatrix} \quad (1)$$

where $\mathbf{x}=[x_1,\ldots,x_n]^{\mathrm{T}}$ is the n-vector of (total) displacements at the unconstrained degrees of freedom; $\mathbf{u}=[u_1,\ldots,u_m]^{\mathrm{T}}$ is the m-vector of prescribed support displacements; $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the $n\times n$ mass, damping and stiffness matrices associated with the unconstrained degrees of freedom, respectively; $\mathbf{M}_g$, $\mathbf{C}_g$ and $\mathbf{K}_g$ are the $m\times m$ matrices associated with the support degrees of freedom; $\mathbf{M}_c$, $\mathbf{C}_c$ and $\mathbf{K}_c$ are the $n\times m$ coupling matrices associated with both sets of degrees of freedom; and $\mathbf{F}$ is the m-vector

of reacting forces at the support degrees of freedom. Both $\mathbf{x}$ and $\mathbf{u}$ may contain translational as well as rotational components.

Following conventional procedures, we decompose the response into pseudo-static and dynamic components:

$$\mathbf{x} = \mathbf{x}^s + \mathbf{x}^d \tag{2}$$

where the pseudo-static component, $\mathbf{x}^s$, is the solution to Eq. 1 without the inertia and damping terms

$$\mathbf{x}^s = -\mathbf{K}^{-1}\mathbf{K}_c\,\mathbf{u} = \mathbf{R}\,\mathbf{u} \tag{3}$$

in which $\mathbf{R} = -\mathbf{K}^{-1}\mathbf{K}_c$ is denoted the influence matrix. Substituting Eqs. 2 and 3 in Eq. 1, the dynamic component of the response is obtained in the differential form

$$\begin{aligned}\mathbf{M}\ddot{\mathbf{x}}^d + \mathbf{C}\dot{\mathbf{x}}^d + \mathbf{K}\mathbf{x}^d &= -(\mathbf{MR}+\mathbf{M}_c)\,\ddot{\mathbf{u}} - (\mathbf{CR}+\mathbf{C}_c)\,\dot{\mathbf{u}} \\ &\approx -(\mathbf{MR}+\mathbf{M}_c)\,\ddot{\mathbf{u}}\end{aligned} \tag{4}$$

The right-hand side is approximated by neglecting the damping forces, which are usually much smaller than the inertia forces on the same side. The above reduction is exact when the damping matrix is proportional to the stiffness matrix. It is noted that $\mathbf{M}_c = 0$ if a lumped mass model is used.

To formulate a response spectrum method, it is necessary to employ the normal mode approach. Let $\Phi = [\boldsymbol{\phi}_1 \cdots \boldsymbol{\phi}_n]$, ω_i and ζ_i, $i = 1, \cdots, n$, denote the modal matrix, natural frequencies and modal damping ratios of the structure with its support points fixed. Using the transformation $\mathbf{x}^d = \Phi\,\mathbf{y}$, $\mathbf{y} = [y_1, \cdots, y_n]^{\mathrm{T}}$, in Eq. 4 and employing the orthogonality of the mode shapes (assuming classical damping), the decoupled equations of motion are

$$\ddot{y}_i + 2\zeta_i\omega_i\dot{y}_i + \omega_i^2 y_i = \sum_{k=1}^{m} \beta_{ki}\,\ddot{u}_k(t) \quad i = 1, \cdots, n \tag{5}$$

where the index k denotes the degrees of freedom associated with the prescribed support motions, the subscript i denotes the mode number, and β_{ki} is the modal participation factor given by

$$\beta_{ki} = -\frac{\boldsymbol{\phi}_i^{\mathrm{T}}(\mathbf{M}\mathbf{r}_k + \mathbf{M}_c\mathbf{i}_k)}{\boldsymbol{\phi}_i^{\mathrm{T}}\mathbf{M}\boldsymbol{\phi}_i} \tag{6}$$

in which $\mathbf{r}_k$ is the k-th column of $\mathbf{R}$ and $\mathbf{i}_k$ is the k-th column of an $m \times m$ identity matrix. It is convenient to define a normalized modal response $s_{ki}(t)$, representing the response of a single-degree-of-freedom oscillator of unit mass, frequency ω_i and damping ζ_i, which is subjected to the base motion $u_k(t)$. From Eq. 5, $s_{ki}(t)$ satisfies

$$\ddot{s}_{ki} + 2\zeta_i\omega_i\dot{s}_{ki} + \omega_i^2 s_{ki} = \ddot{u}_k(t) \tag{7}$$

Obviously, $y_i(t) = \sum_{k=1}^{m} \beta_{ki}\,s_{ki}(t)$.

A generic response quantity of interest, $z(t)$ (e.g., a nodal displacement, an internal force, stress or strain component), in general can be expressed as a linear function of the nodal displacements $\mathbf{x}$, i.e.,

$$z(t) = \mathbf{q}^{\mathrm{T}}\mathbf{x}(t) = \mathbf{q}^{\mathrm{T}}[\mathbf{x}^s(t) + \mathbf{x}^d(t)] \tag{8}$$

where $\mathbf{q}$ is a response transfer vector that usually depends on the geometry and stiffness properties of the structure. Substituting for the pseudo-static component of $\mathbf{x}$ from Eq. 3 and for the dynamic component in terms of the normalized modal responses, the generic response $z(t)$ is

$$z(t) = \sum_{k=1}^{m} a_k u_k(t) + \sum_{k=1}^{m}\sum_{i=1}^{n} b_{ki}\,s_{ki}(t) \tag{9}$$

in which

$$a_k = \mathbf{q}^{\mathrm{T}}\mathbf{r}_k \quad k = 1, \ldots, m \tag{10}$$

$$b_{ki} = \mathbf{q}^{\mathrm{T}}\boldsymbol{\phi}_i\beta_{ki} \quad k = 1, \ldots, m; \quad i = 1, \ldots, n \tag{11}$$

are denoted effective influence factors and effective modal participation factors, respectively. It is useful to note that a_k and b_{ki} are functions only of the structural properties, and that $s_{ki}(t)$ is dependent only on the i-th modal frequency and damping ratio and the k-th input motion. The first sum on the right-hand side of Eq. 9 represents the pseudo-static component of the response and the double-sum term represents the dynamic component.

3 THE RESPONSE SPECTRUM METHOD

For the response spectrum method, we assume the following information is available: the set of peak ground displacements $u_{k,\max}$ for all support points; the set of displacement response spectra $D_k(\omega, \zeta)$ (representing the mean peak response of an oscillator of frequency ω and damping ζ to the base acceleration $\ddot{u}_k$) for each support motion; an estimate of the duration of motion τ; and the coherency function $\gamma_{kl}(\omega)$ characterizing the spatial variability of the ground motion in the region. The coherency function is defined as the normalized cross power spectral density (PSD) of the ground acceleration processes at two stations k and l, i.e.,

$$\gamma_{kl}(\omega) = \frac{G_{\ddot{u}_k\ddot{u}_l}(\omega)}{[G_{\ddot{u}_k\ddot{u}_k}(\omega)\,G_{\ddot{u}_l\ddot{u}_l}(\omega)]^{1/2}} \tag{12}$$

where $G_{xy}(\omega)$ denotes the cross-PSD of processes $x(t)$ and $y(t)$. Empirical models of the coherency function are available from array recordings (Harichandran and Vanmarcke 1986, Abrahamson et al. 1991). This function in general is complex valued, with its modulus characterizing the incoherence effect and the phase angle characterizing the wave passage effect. We note that the response spectrum for each support degree of freedom can be different, thus allowing for the local effects at each station.

The new response spectrum method is based on the superposition rule in Eq. 9 and the fundamental principles of stationary random vibration. The details of the derivation are beyond the scope of this paper. It suffices to say that the method is rigorously derived and properly accounts for the effects of incoherence, wave passage, local site conditions, as well as the correlation between the modes of vibration of the structure. The method works best when the significant

segment of the excitation is quasi-stationary and it is several times longer than the fundamental period of the structure. The combination rule for the mean of the absolute maximum response is given as follows:

$$E[\max|z(t)|] = \left[\sum_{k=1}^{m}\sum_{l=1}^{m} a_k a_l \rho_{u_k u_l} u_{k,\max} u_{l,\max} + 2\sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{j=1}^{n} a_k b_{lj} \rho_{u_k s_{lj}} u_{k,\max} D_l(\omega_j,\zeta_j) + \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ki} b_{lj} \rho_{s_{ki} s_{lj}} D_k(\omega_i,\zeta_i) D_l(\omega_j,\zeta_j) \right]^{1/2} \quad (13)$$

In this expression, the double sum represents the square of the pseudo-static response, the quadruple sum represents the square of the dynamic response, and the triple sum represents a coupling term between the pseudo-static and dynamic responses that arises from the covariance between the two components. All terms in this combination rule are readily available, except the three cross-correlation coefficients $\rho_{u_k u_l}$, $\rho_{u_k s_{lj}}$ and $\rho_{s_{ki} s_{lj}}$. These coefficients incorporate the effects of incoherence and wave passage, as well as the correlation between the vibration modes. Interpretation of these coefficients and their evaluation are described in the remainder of this section.

The three cross-correlation coefficients $\rho_{u_k u_l}$, $\rho_{u_k s_{lj}}$ and $\rho_{s_{ki} s_{lj}}$ can be interpreted in terms of two oscillators of frequencies ω_i and ω_j and damping ratios ζ_i and ζ_j, which are subjected to the ground motions $u_k(t)$ and $u_l(t)$, respectively (see Fig. 1). Specifically, $\rho_{u_k u_l}$ denotes the cross-

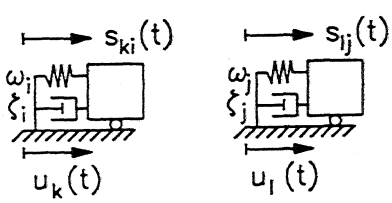

Figure 1. Pair of oscillators

correlation between the two support displacements (at the same time instance), $\rho_{u_k s_{lj}}$ denotes the cross-correlation coefficient between the displacement at support k and the response of the oscillator at support l, and $\rho_{s_{ki} s_{lj}}$ denotes the cross-correlation coefficient between the responses of the two oscillators. Assuming stationary processes and using the definition in Eq. 12, these coefficients can be written in terms of the coherency function and the PSD's of the individual support accelerations as (see Der Kiureghian and Neuenhofer 1991)

$$\rho_{u_k u_l} = \frac{\int_{-\infty}^{\infty} \omega^{-4} \gamma_{kl}(\omega)[G_{\ddot{u}_k\ddot{u}_k}(\omega) G_{\ddot{u}_l\ddot{u}_l}(\omega)]^{1/2} d\omega}{\left[\int_{-\infty}^{\infty} \omega^{-4} G_{\ddot{u}_k\ddot{u}_k}(\omega) d\omega \int_{-\infty}^{\infty} \omega^{-4} G_{\ddot{u}_l\ddot{u}_l}(\omega) d\omega\right]^{1/2}} \quad (14)$$

$$\rho_{u_k s_{lj}} = \frac{\int_{-\infty}^{\infty} \omega^{-2} H_j(-\omega) \gamma_{kl}(\omega)[G_{\ddot{u}_k\ddot{u}_k}(\omega) G_{\ddot{u}_l\ddot{u}_l}(\omega)]^{1/2} d\omega}{\left[\int_{-\infty}^{\infty} \omega^{-4} G_{\ddot{u}_k\ddot{u}_k}(\omega) d\omega \int_{-\infty}^{\infty} |H_j(\omega)|^2 G_{\ddot{u}_l\ddot{u}_l}(\omega) d\omega\right]^{1/2}} \quad (15)$$

$$\rho_{s_{ki} s_{lj}} = \frac{\int_{-\infty}^{\infty} H_i(\omega) H_j(-\omega) \gamma_{kl}(\omega)[G_{\ddot{u}_k\ddot{u}_k}(\omega) G_{\ddot{u}_l\ddot{u}_l}(\omega)]^{1/2} d\omega}{\left[\int_{-\infty}^{\infty} |H_i(\omega)|^2 G_{\ddot{u}_k\ddot{u}_k}(\omega) d\omega \int_{-\infty}^{\infty} |H_j(\omega)|^2 G_{\ddot{u}_l\ddot{u}_l}(\omega) d\omega\right]^{1/2}} \quad (16)$$

in which $H_i(\omega) = [\omega_i^2 - \omega^2 + 2i\zeta_i\omega_i\omega]^{-1}$ are the modal frequency response functions. It is seen that, in addition to the coherency function, it is necessary to know the individual PSD's of ground acceleration at each support degree of freedom. It is quite well known, however, that the PSD of ground acceleration is closely connected to the response spectrum and several approximate relations between the two are available (e.g., Kaul 1978). In the present work, owing to the need to develop the PSD for the entire range of frequencies, a new formula is developed which, omitting the subscript k for the support degree of freedom, is

$$G_{\ddot{u}\ddot{u}}(\omega) = \frac{\omega^{p+2}}{\omega^p + \omega_f^p}\left(\frac{2\zeta\omega}{\pi} + \frac{4}{\pi\tau}\right)\left[\frac{D(\omega,\zeta)}{p_s(\omega)}\right]^2 \quad \omega \geq 0 \quad (17)$$

in which $p_s(\omega)$ is a peak factor function, τ is the duration of the excitation, and p and ω_f are parameters determined such that the area underneath the PSD matches the mean square ground acceleration. It is important to note that the coefficients in Eqs. 14-16 are not overly sensitive to the shapes of the individual PSD's (due to the integrals involved), and hence a rough estimate of the PSD is sufficient to make good approximations of these coefficients.

Extensive numerical analysis has revealed the following properties of the cross-correlation coefficients(see Der Kiureghian and Neuenhofer 1991) : (a) The cross-correlation coefficient $\rho_{u_k u_l}$ is generally large, especially for soft soil conditions, and its contribution to the pseudo-static component of the response cannot be neglected. (b) The cross-correlation coefficient $\rho_{u_k s_{lj}}$ is usually small, except for frequencies below 0.5 Hz. The contribution of this coefficient and, therefore, that of the cross term between the pseudo-static and dynamic components of the response may be neglected without a significant loss of accuracy if the fundamental frequency of the system is greater than about 0.5 Hz. (c) The cross-correlation coefficient $\rho_{s_{ki} s_{lj}}$ depends on the separation between the two frequencies and the soil

conditions at the two stations. For firm soil and in absence of the wave passage effect, the correlation is small for well spaced frequencies (as long as they are not far from the range of dominant input frequencies) and tends to further decay with increasing incoherence. For soft soils, or in the presence of the wave passage effect, the correlation coefficient can be significant even for well spaced modes if there is not a strong incoherence effect.

4 EXAMPLE APPLICATION

As an example application of the new response spectrum method, we consider the two-span continuous beam in Fig. 2, which has uniform mass and stiffness properties and simple supports. The beam is discretized into 20 elements along each $L=50\,m$ span. The system is represented by 38 translational and 41 rotational degrees of freedom, and 3 translational support degrees of freedom (i.e., $m=3$). Since no mass moments of inertia are associated with the rotational degrees of freedom, the latter are condensed out and the system analyzed has $n=38$ degrees of freedom. Only the first four modes of vibration are considered in the analysis.

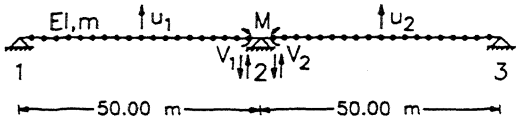

Figure 2. Example structure

Two values of the fundamental period of the beam are considered: $T=1\,s$ and $T=1/4\,s$. These are obtained for $EI/m=2.53\times10^6$ and $40.5\times10^6\ m^4/s^2$, respectively, where EI denotes the flexural rigidity and m denotes the mass per unit length of the beam. The resulting frequencies of the first four modes are listed in Table 1. We refer to the two beams as the "flexible" and the "stiff" beam, respectively. The modal damping ratio is assumed to be 5 percent in all modes.

Table 1. Modal Frequencies of Example Structures

mode	ω_i, rad/s "flexible" beam	"stiff" beam
1	6.28	25.13
2	9.82	39.30
3	25.13	100.57
4	31.80	127.26

Five response quantities are considered for the analysis: the mid-span deflections u_1 and u_2, the bending moment M at the middle support, and the shear forces V_1 and V_2 at the faces of the middle support. Although the beam is symmetric, the two mid-span deflections and the shear forces on both sides of the middle support are considered, since different results are obtained for these pairs of responses due to the wave passage effect.

The motion at each support is assumed to be vertical and specified by identical response spectra consistent with the design spectrum recommended by the Structural Engineering Association of California for $\ddot{u}_{k,\max}=0.5\,g$. The coherency function is assumed to have the form $\gamma_{kl}(\omega)=\exp[-(\alpha\omega d_{kl}/v_s)^2]\exp(i\omega d^L_{kl}/v_{app})$ suggested by Luco and Wong (1986), where α is an incoherence factor, d_{kl} denotes the horizontal distance between stations k and l and d^L_{kl} is its projection along the longitudinal direction of propagation of waves, v_s is the shear wave velocity of the medium, and v_{app} is the surface apparent wave velocity. It is noted that the first exponential term characterizes the effect of incoherence, whereas the second exponential term characterizes the wave passage effect. Five different cases are considered: *Case 1*: Fully coherent (uniform) motions at all three supports, i.e., $\gamma_{kl}(i\omega)=1$; *Case 2*: Only wave passage effect included (i.e., $\alpha=0$) with $v_{app}=400\,m/s$; *Case 3*: Only incoherence effect included (i.e., $v_{app}=\infty$) with $v_s/\alpha=600\,m/s$; *Case 4*: Both wave passage and incoherence effects included with $v_{app}=400\,m/s$ and $v_s/\alpha=600\,m/s$; and *Case 5*: Mutually statistically independent support motions, i.e., $\gamma_{kl}(i\omega)=0$ for $k\neq l$. For cases 2 and 4, it is assumed that the waves propagate in the direction from support 1 to support 3 and that $d^L_{kl}=d_{kl}$. The results for the various cases are compared to determine the relative influences of the wave passage and incoherence effects, as well as to determine the consequences of assuming uniform excitations (Case 1) or independent excitations (Case 5), which are commonly assumed in the current practice.

The normalized mean values of the peak responses are listed in Table 2 for both the "flexible" and "stiff" beams, respectively. The total response for each case, denoted z_t and representing the result obtained from the combination rule in Eq. 13, is listed in the columns 3 and 8. Note that the pair of values for the midspan displacement responses u_1 and u_2, and for the middle support shear forces V_1 and V_2 are identical for Cases 1, 3 and 5 due to the symmetry of the beam and the absence of the wave passage effect. In Cases 2 and 4, these pairs of responses are different, in spite of the symmetry of the beam, due to the directionality of the wave passage effect.

In columns 4 and 9 are listed the ratios of the total response to the total response for Case 1. These ratios indicate the influence of the spatial variability on each of the response quantities. By comparing Cases 2 and 3, it is apparent that the influence of the wave passage effect is greater than that of the incoherence effect for both the "flexi-

Table 2. Mean peak responses of example structures

response $z \times 10^3$ (1)	case (2)	"flexible" beam z_t (3)	$z_t/z_{t,case\ 1}$ (4)	ss/z_t^2 (5)	$s\text{-}d/z_t^2$ (6)	dd/z_t^2 (7)	"stiff" beam z_t (3)	$z_t/z_{t,case\ 1}$ (8)	ss/z_t^2 (9)	$s\text{-}d/z_t^2$ (10)	dd/z_t^2 (11)
$\frac{u_1}{L}$	1	7.86	1	0.950	−0.080	0.130	7.63	1.000	1.006	−0.006	0.001
	2	7.85	0.999	0.948	−0.090	0.142	7.62	0.998	1.006	−0.007	0.000
	3	7.80	0.993	0.960	−0.080	0.120	7.62	0.999	1.006	−0.007	0.001
	4	7.78	0.990	0.962	−0.090	0.128	7.61	0.997	1.006	−0.007	0.001
	5	6.36	0.809	0.938	−0.088	0.150	6.13	0.804	1.006	−0.008	0.001
$\frac{u_2}{L}$	1	7.86	1	0.950	−0.080	0.130	7.63	1.000	1.006	−0.006	0.001
	2	7.71	0.981	0.983	−0.069	0.086	7.62	0.998	1.006	−0.006	0.001
	3	7.80	0.993	0.960	−0.080	0.120	7.62	0.999	1.006	−0.007	0.001
	4	7.73	0.984	0.974	−0.071	0.097	7.61	0.997	1.006	−0.006	0.001
	5	6.36	0.809	0.938	−0.088	0.150	6.13	0.804	1.006	−0.008	0.001
$\frac{LM}{EI}$	1	60.6	1	0	0	1	4.23	1	0	0	1
	2	46.3	0.764	0.001	0.015	0.984	3.60	0.853	0.138	0.269	0.594
	3	51.4	0.848	0.001	0.009	0.990	3.68	0.870	0.184	0.178	0.638
	4	45.4	0.749	0.003	0.022	0.975	4.34	1.027	0.361	0.198	0.441
	5	51.2	0.845	0.302	0.066	0.632	28.5	6.739	0.975	0.015	0.010
$\frac{L^2V_1}{EI}$	1	238	1	0	0	1	16.7	1	0	0	1
	2	212	0.891	0.000	0.004	0.996	11.9	0.714	0.013	0.108	0.880
	3	211	0.884	0.000	0.003	0.997	13.4	0.802	0.014	0.057	0.929
	4	198	0.832	0.000	0.007	0.993	13.6	0.817	0.037	0.088	0.875
	5	180	0.753	0.025	0.026	0.949	31.7	1.902	0.787	0.056	0.157
$\frac{L^2V_2}{EI}$	1	238	1	0	0	1	16.7	1	0	0	1
	2	180	0.753	0.000	0.004	0.996	12.7	0.764	0.011	0.088	0.901
	3	211	0.884	0.000	0.003	0.997	13.4	0.802	0.014	0.057	0.929
	4	181	0.760	0.000	0.004	0.995	13.6	0.814	0.037	0.087	0.876
	5	180	0.753	0.025	0.026	0.949	31.7	1.902	0.787	0.056	0.157

z_t total value of the peak response
$z_{t,case\ 1}$ total value of the peak response for Case 1
ss square of the pseudo-static component of response
$s\text{-}d$ cross term between pseudo-static and dynamic components of response
dd square of the dynamic component of response

ble" and "stiff" beams (i.e., the ratios for Case 2 are smaller than the corresponding ratios for Case 3), except for responses u_1 and V_1 of the "flexible" beam. For Case 4, which includes both the wave passage and the incoherence effects, the combined influence in some cases is smaller (i.e., the ratio is closer to 1.0) than in the individual cases just mentioned. In fact, for response M of the "stiff" beam, the ratio for Case 4 is greater than unity (i.e., the response is amplified due to the spatial variability of the ground motion), whereas in Cases 2 and 3 the ratios are smaller than unity. This is due to the elimination of certain negative cross-correlation terms arising from the wave passage effect by the incoherence effect. All ratios for Cases 2-4 are smaller than unity, except the case just mentioned, indicating that the spatial variability tends to reduce the total response for both systems. This reduction is insignificant for responses u_1 and u_2 (the midspan deflections), but rather significant for the responses M, V_1 and V_2. The former is due to the dominance of the pseudo-static component of the displacement response as described below.

As evident from the response M of the "stiff" beam in Case 4, reduction in the response due to the spatial variability effect is not a general rule. One may expect a large pseudo-static response to be generated in a stiff structure when there is a rapid loss of coherency in the ground motion. Such pseudo-static response may lead to a larger total response compared with the response for uniform support motions (Case 1). This is evident for Case 5 with uncorrelated support motions, where the ratios for the responses M, V_1 and V_2 are greater than unity for the "stiff" beam. Nevertheless, from the results in Table 2, one may conclude that the effect of spatial variability is more likely to reduce the response (compared with the uniform motion case) than to amplify it.

Columns 5-7 and 10-12 of Table 2 list ratios of the three terms inside the square brackets in Eq. 13 with respect to the square of the total response. They indicate the relative contributions of the pseudo-static part (the double-sum term), the cross term between the pseudo-static and dynamic

parts (the triple-sum term), and the dynamic part (the quadruple-sum term) to the square of the response. Note that for Case 1 the pseudo-static and cross terms are zero for the internal force responses M, V_1 and V_2, since rigid body motions of the beam do not generate internal forces.

From the results in the columns 5-7 and 10-12 of Table 2, the displacement responses u_1 and u_2 are found to be dominated by the pseudo-static part, especially for the "stiff" beam. It is interesting to note that for these responses the cross term between the pseudo-static and dynamic parts is negative. The bending moment, M, and shear force responses, V_1 and V_2, are primarily dominated by the dynamic part of the response, except in Case 5 for the "stiff" beam. The cross term between the pseudo-static and dynamic parts for these responses is positive and in many cases is greater than the contribution of the pseudo-static component alone. However, in most cases the contribution of the cross term can be neglected with little effect on the total response.

The results for the above example show that the influence of spatial variability of the ground motion on the response of a multiply-supported structure can be significant. In the present case, this influence results in a reduction of the peak response, in some instances by almost 30 percent. However, this trend cannot be generalized since for stiffer structures and in cases of rapid loss of coherence, the response can be amplified due to the increased contribution of the pseudo-static part of the response.

5 SUMMARY AND CONCLUSIONS

A new response spectrum method is developed for seismic analysis of linear multi-degree-of-freedom, multiply-supported structures subjected to spatially varying ground motions. Variations of ground motion due to wave passage, loss of coherency with distance, and variation of local soil conditions are considered. The former two effects are modeled in terms of a coherency function, whereas the local soil effect is considered in terms of its influence on the response spectral shape at each individual support point. The method is based on fundamental principles of random vibration theory and properly accounts for the effects of correlation between the support motions as well as between the modal responses of the structure. The combination rule explicitly accounts for the contributions of the pseudo-static and dynamic components of the response, as well as for their covariance.

An example application is considered, which demonstrates the influence of the spatial variability of the ground motion on selected responses of the structure, and examines the relative contributions of the pseudo-static, dynamic, and their covariance terms to the total response. It is found that in most cases the spatial variability tends to reduce the response (in relation to the case with uniform support motions), often by a significant amount (e.g., close to 30 percent). However, this rule cannot be generalized since, under certain conditions (i.e., stiff structures and rapid loss of coherency), the response may actually amplify due to an increase in the pseudo-static component of the response.

6 ACKNOWLEDGMENT

This study was supported by the National Science Foundation Grant No. BCS-9011112 with Dr. S-C. Liu as Program Director. The second author was supported in part by the German Academic Exchange Service (DAAD) during his studies at the University of California at Berkeley. This support is gratefully acknowledged.

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