Earthquake Engineering, Tenth World Conference © 1992 Balkema, Rotterdam. ISBN 90 5410 060 5

An analytical evaluation of the ductility of reinforced concrete members

H. Nakamura, J. Niwa & T. Tanabe
Nagoya University, Japan

ABSTRACT: The finite element analysis considering both geometrical and material nonlinearities was performed for reinforced concrete(RC) members. The effects on the ductility of axial stress, web reinforcement ratio, longitudinal reinforcement ratio and shear span to beam depth ratio were investigated analytically. An equation to evaluate ductility was proposed based on the results of the analysis. It was, then, compared with as many experimental results as can be collected and was proved that the proposed equation can estimate the ductility for RC members accurately far better than any proposed models in the past.

1 INTRODUCTION

The seismic design of RC structures considers it essential that the seismic energy be absorbed by ductility after the reinforcement yielding in structures. Therefore, prediction of the ductility should be as accurate as possible. Many experimental studies were carried out to this object and several evaluation methods of the ductility are proposed on the basis of the results of experiments of RC members in Japan. The method to evaluate ductility correctly, however, have not ever been established, since the ultimate behavior of RC structures can be divided into a number of ways and it involves many factors in regard to material nonlinearities besides the factors of structural dimensions. Moreover, the proposed method based on experimental results are almost obtained from a small number of data and they do not have wide application for RC members.

In view of these deficiencies, to obtain applicable results for all RC members, it is pointed that the ductility should be evaluated analytically. An Analytical study will make it possible to combine the experimental results obtained from RC members having different dimensions and will give us valuable information in establishing a generalized method to evaluate ductility.

The purpose of this paper is propose an equation to evaluate the ductility correctly for all RC members. This equation was obtained from the results of the developed analysis, which was able to define the behavior of RC structures up to the ultimate state, and was assured its reliability by the vast comparison with the experimental data.

2 CALCULATION METHOD

Calculation method adopts the finite element analysis based on the finite displacement theory for RC members in which layered beam element is used to consider the material nonlinearities.

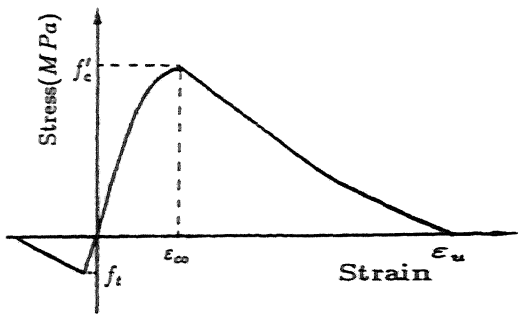

Figure 1: Stress-Strain Relation for Concrete

2.1 Stiffness equation

The beam-column finite element formulation was derived considering the finite displacement for RC members. The incremental stiffness equation used in the analysis is as follows.

$$([K]+[K_g])^{(n)}\{\Delta d\}^{(n+1)} = \{\Delta F\}^{(n+1)} + \{F_r\}^{(n)} \quad (1)$$

where $[K]$ denotes the stiffness matrix, $[K_g]$ denotes the geometric matrix and $\{F_r\}$ denotes the unbalanced force vector which is introduced when equilibrium in the previous load step is not satisfied strictly.

2.2 Material modeling

Stress-strain relation for concrete used is shown in Fig.1. In the zone of compression, the relation is represented by a second degree parabola and a linear falling branch. The slope for the falling branch is determined by *Kent* and *Park* model. In zone of tension, stress increase linearly with a proportionality constant of $2f'_c/\varepsilon_{c0}$ to tensile strength. And after that, it decreases linearly to the strain of 0.002.

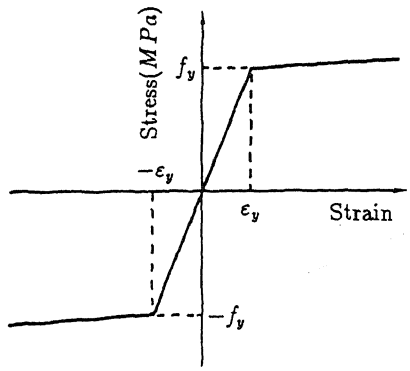

Figure 2: Stress-Strain Relation for Reinforcement

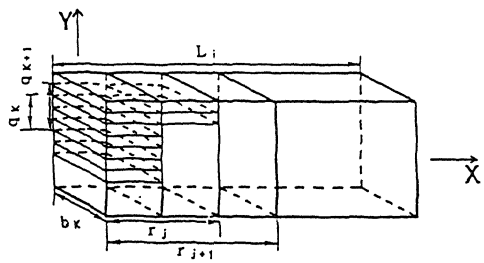

Figure 3: Layered Beam Element

Stress-strain relation for reinforcement is shown in Fig.2. It is assumed that stress is proportional to strain with initial stiffness to the yielding point, and increases linearly with a slope one hundredth of initial stiffness after that in both tension and compression zone.

2.3 Layered beam formulation

A computer program was developed according to the previous discussions. Since the stress and Young's modulus vary with the depth of the element, an element is subdivided into a number of layers. That is, a layered beam element shown in Fig.3 was used. Integration to obtain the matrices and unbalance force can be represented by the sum of the amount in each subdivided element in which the material nonlinearity is assumed. Young's modulus used in the stress-strain relation for each layer is the tangent modulus. Material nonlinearity and cracking of concrete are thus considered accurately.

In the analysis, the stiffness equations are solved iteratively to compute displacement increment with *Newton – Raphson* method. The analysis was performed by updating stiffness matrix at every load step within each step until the norm of unbalanced force relative to applied force becomes stationary.

3 CHARACTERISTIC POINT FOR DUCTILITY OF REINFORCED CONCRETE MEMBERS

It is well known that RC members subjected to cyclic loads often lose the restoring force rapidly with diagonal shear crack having "X" shape after yielding of longitudinal reinforcement in a cycle of loadings. This is the special characteristic of the failure of the cyclic loading test, which is different from the one of monotonous loadings. The ductility of RC member is generally defined by the displacement corresponding to the characteristic point where the restoring force decrease rapidly, and before the characteristic point the decrease of restoring force is small. To evaluate the ductility analytically, we must define a characteristic point, which can be thought to correspond to such a point in experiment, in analysis.

The numerical results were compared with the results obtained from cyclic loading tests in which axial stress (σ_0), web reinforcement ratio (ρ_w) and shear span to depth ratio (A/D) were changed. The analytical results, then, showed that the lateral displacement, which the restoring force decreases rapidly in the cyclic loading tests, correspond to the lateral displacement of the maximum moment point in the analysis under monotonous loading. The maximum moment is the sum of the moment derived from the lateral load and the moment introduced by the $P-\Delta$ effect. Fig.4 shows an example obtained from analysis in which the web reinforcement ratio was changed. The solid and the broken lines correspond to the results of experiment and analysis, respectively. The experimental results are the skeleton curve under cyclic loadings and analytical results are the curve obtained from monotonous loading. The maximum moment points obtained by analysis are shown with the mark "●" and the characteristic points in experiment are shown with the mark "▲". It is seen from the figure that both points are in good agreement for every web reinforcement ratio. One reason for the fact is considered that concrete in compression zone moves into the falling branch in stress-strain relation after the maximum moment point is reached. In other words, beyond this point, resistance of concrete decrease in every cycle and load carrying capacity of member decreases, the concrete cover spalls and finally shear deformation increase rapidly in a cycle if the cyclic loading is carried out in this range.

Hence, we define the analytical ductility of RC members subjected to cyclic loadings by Eq(2), i.e., the ductility in the analysis is so defined as the ratio of the lateral displacement corresponding to the maximum moment point ($\delta_{M_{max}}$) and to the yielding of the reinforcement in a member (δ_y).

$$\mu = \delta_{M_{max}}/\delta_y \tag{2}$$

Once the ductility ratio is defined analytically, the effects on the ductility of variable factors can be investigated numerically.

4 EFFECTS ON THE DUCTILITY OF VARIABLE FACTORS

The parametric analysis, in which the factors of axial stress(σ_0), web reinforcement ratio(ρ_w), longitudinal reinforcement ratio(P_t) and shear span to depth ratio(A/D) are changed, are performed to get information to evaluate ductility. It is generally reported

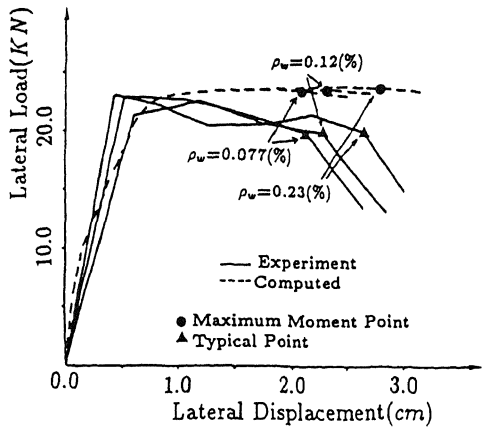

Figure 4: Effect of Web Reinforcement Ratio on the Load-Displacement Relation

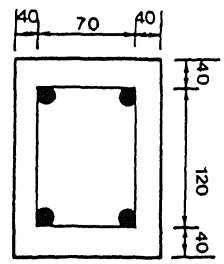

Figure 5: Dimension of Analytical Model ($unit; mm$)

that these factors are closely related to the ductility of RC members. In the analysis, the axial stress range from $-2(MPa)$ to $8(MPa)$, the web reinforcement ratios from 0.0 to 1.0%, the longitudinal reinforcement ratios from 0.3 to 1.5% and shear span to depth ratios from 1 to 6 were used. The range of these factors are decided considering the test specimens of experiments performed in the past.

A model used in analysis was a RC member having a cross section of 20 × 15(cm) and a beam depth of 16(cm), which is shown in Fig.5. The reason why the model used is only one is that the equation to evaluate ductility will be represented in non-dimensional form and be compared with many experimental results in which the factors were varied widely. The material properties used are given in Table.1.

Table.1 Material Properties

Reinforcement		Concrete		
$f_y(MPa)$	$\varepsilon_y(\mu)$	$f'_c(MPa)$	$f_t(MPa)$	$\varepsilon_{co}(\mu)$
400	2500	30	3	2000

Since it seems that the axial stress is the most important factor for ductility, the effects of the web reinforcement ratio(ρ_w), longitudinal reinforcement ratio (P_t) and shear span-depth ratio(A/D) are investigated in terms of axial stress(σ_0). That is, the effects

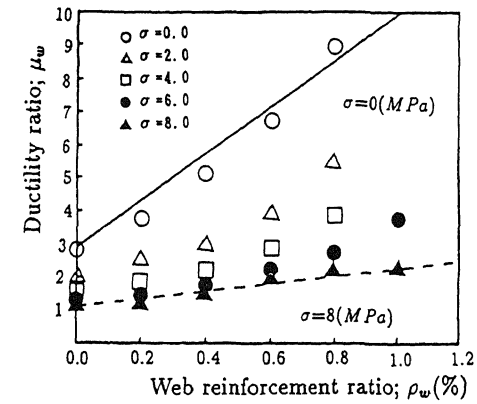

Figure 6: Relation of Web Reinforcement Ratio and Ductility Ratio

on the ductility (μ) of each factor are investigated always in combination with the axial stress, "$\rho_w - \sigma_0 - \mu$ relation" for example.

The relationship between web reinforcement ratio and axial stress ratio(σ_0/f'_c) is formulated first, and the effects of P_t and A/D are given as the coefficient ($\beta_{P_t} = \mu/\mu_{P_t=1.0}, \beta_{AD} = \mu/\mu_{AD=4}$) of μ_w. Therefore, the following discussion is based on these form.

4.1 Effect of web reinforcement ratio and axial stress

The analysis where web reinforcement ratio(ρ_w) and axial stress (σ_0) changed are performed under the condition that the longitudinal reinforcement ratio is 1.0% and shear span ratio is 4. The effect of web reinforcement ratio is taken into consideration by varying the slope of a falling branch for concrete using *Kent* and *Park* model.

The results of analysis are shown in Fig.6. The ductility ratio increased in proportion to the increase of web reinforcement ratio, which is identified with experimental results. Moreover, it can be understood from the figure that the ductility ratio becomes smaller relatively and the increase rate of ductility for web reinforcement ratio also become small accordingly as the axial stress increases. The following equation is obtained assuming that the relation between the web reinforcement ratio and ductility ratio is linear for each axial stress level. In this equation, the effect of axial stress is considered by the ratio of axial stress and compressive strength of concrete (f'_c) in order to make the equation in non-dimensional form.

$$\begin{aligned} \mu_w &= a + b\rho_w \qquad (3) \\ a &= 2.9e^{-10/3(\sigma_0/f'_c)} \\ b &= 7.0e^{-7.0(\sigma_0/f'_c)} \end{aligned}$$

The example of Eq(3) for σ_0 of 0 and $8(MPa)$ are shown in Fig.6 by a broken and solid lines, respectively. It is noted that since the equation is formulated from the results of analysis in which ρ_w is varied from 0.0 to 1.0%, the range of application may be below 1.0%.

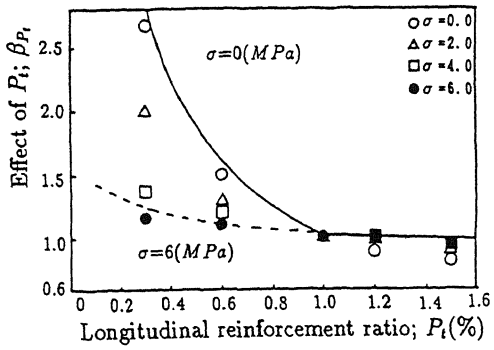

Figure 7: Relation of Longitudinal Reinforcement Ratio and Ductility Ratio

4.2 Effect of longitudinal reinforcement ratio and axial stress

The analysis where longitudinal reinforcement ratio (P_t) and axial stress changed are performed keeping ρ_w as 0.2% and A/D as 4. The results are shown in Fig.7. In this figure, β_{P_t} represents the ratio of ductility for any P_t to the ductility for P_t of 1.0%. The ductility increased with the decrease of the longitudinal reinforcement ratio. Especially, a remarkable effect of longitudinal reinforcement ratio can be seen when P_t is less than 1.0% and axial stress is small. This results are already confirmed by *Machida's* experiment. The effect on ductility of P_t, however, is less remarkable with the increase of the axial stress. The effect of longitudinal reinforcement and axial stress is formulated as follows.

$$\begin{aligned} \beta_{P_t} &= a(P_t)^b \\ a &= 1.03 \\ b &= -0.85e^{-9.0(\sigma_0/f'_c)} \end{aligned} \tag{4}$$

Here, β_{P_t} is a coefficient to explain the influence of P_t for ductility. Although Eq(4) is obtained from method of least squares of the deviation, the equation overestimates the results of analysis in a smaller P_t and underestimates in a larger P_t due to the character of the function. Therefore, the range of β_{P_t} is set as.

$$1.0 \leq \beta_{P_t} \leq 2.8 \tag{5}$$

The range may differ according to the axial stress state, but it is formed independently of σ_0 considering the simplification of the equation.

4.3 Effect of shear span ratio and axial stress

The effects of shear span ratio and axial stress on ductility are investigated under the condition that longitudinal reinforcement ratio is 1.0% and web reinforcement ratio is 0.2%. The shear span ratio, then, is varied to six cases from1.0 to 6.0 and axial stress is varied to five cases from 0 to 8(MPa). Fig.8 shows the effects of shear span ratio on ductility using β_{AD} at the different axial stress state. It is shown that the

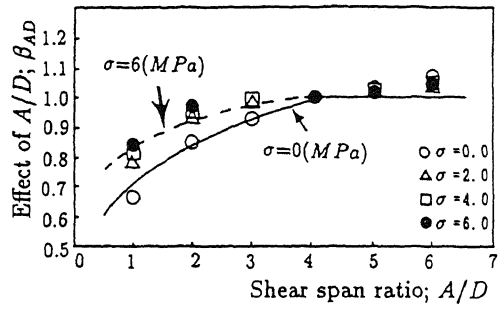

Figure 8: Relation of Shear Span-Depth Ratio and Ductility Ratio

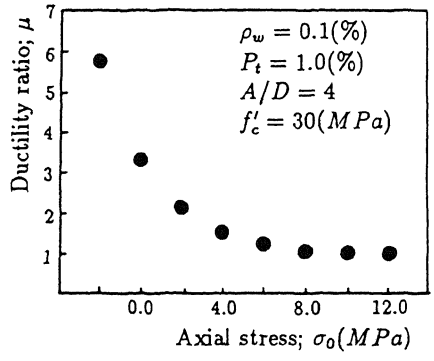

Figure 9: Relation of Axial Stress and Ductility Ratio

ductility decrease rapidly in proportion as the shear span ratio decrease. And the increase of the axial stress reduces the effect of shear span ratio on ductility, which is the same to the case of P_t. Finally, Eq(6), which represents the coefficient of μ_w, is obtained as the effects of shear span ratio on ductility.

$$\begin{aligned} \beta_{AD} &= a(A/D)^b \\ a &= 5/7e^{8/7(\sigma_0/f'_c)} \\ b &= 7/30e^{-4.2(\sigma_0/f'_c)} \end{aligned} \tag{6}$$

However, we add the limit for β_{AD} in order to reduce the error of method of least squares.

$$\beta_{AD} \leq 1.0 \tag{7}$$

4.4 Effect of strength of concrete

Since strength of concrete (f'_c) is given constant values ($30MPa$) in the analysis, the effect was not investigated directly. The effects of variable factors, however, are evaluated with the ratio of axial stress and strength of concrete (σ_0/f'_c) in previous equations. Therefore, the effect of strength of concrete may be considered indirectly in these equations. The ductility are estimated smaller relatively for smaller values of f'_c, if the axial stress is same. And the ductility is estimated larger for a larger f'_c. These facts are similar to the experimental results.

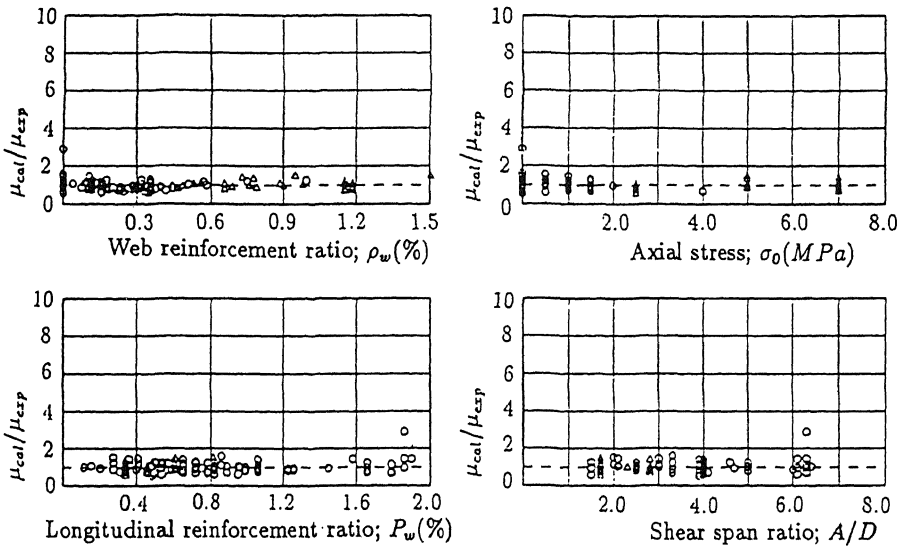

Figure 10: Results of the Proposed Equation

4.5 Effect of axial stress

The effect of axial stress is already mentioned in the effects of ρ_w, P_t, A/D. That is, the effect of axial stress on ductility is predominant and the effect of other factors disappear accordingly as axial stress increases. It is reported that the axial stress is the most influential factor in the experiment for RC members such as columns in buildings acting on large axial force. The analytical results confirmed this fact. On the contrary, for RC members such as pier in which the axial force is small, the effects of the factors other than the axial stress must be evaluated correctly.

The results when the axial stress is changed from $-2(MPa)$ to $12(MPa)$ with constant values of $A/D = 4$, $P_t = 1.0\%$, $\rho_w = 0.1\%$ are shown in Fig.9. The ductility become smaller with increasing axial stress. It is noted that the longitudinal reinforcement in compression yield earlier than in tension for the axial stress over $10(MPa)$ ($\sigma_0/f'_c = 1/3$) and the ductility increase greatly under tensile axial force in the analysis. Although such cases are ignored in the proposed equations, some detailed investigation may be necessary.

5 PROPOSED EQUATION TO EVALUATE DUCTILITY

5.1 Proposition of equation to evaluate ductility

The effect on the ductility of variable factors for RC members are evaluated quantatively based on the results of analysis described in the previous sections. We propose the equation to evaluate the ductility ratio, which is formed to combine a series of equations to estimate ductility as a term of ductility factors.

$$\begin{aligned} \mu &= \mu_w \cdot \beta_{P_t} \cdot \beta_{AD} \\ \mu_w &= a + b\rho_w \end{aligned} \tag{8}$$

$$\begin{aligned} a &= 2.9e^{-10/3(\sigma_0/f'_c)} \\ b &= 7.0e^{-7.0(\sigma_0/f'_c)} \\ \beta_{P_t} &= a(P_t)^b \\ a &= 1.03 \\ b &= -0.85e^{-9.0(\sigma_0/f'_c)} \\ & 1.0 \le \beta_{P_t} \le 2.8 \\ \beta_{AD} &= a(A/D)^b \\ a &= 5/7e^{8/7(\sigma_0/f'_c)} \\ b &= 7/30e^{-4.2(\sigma_0/f'_c)} \\ & \beta_{AD} \le 1.0 \end{aligned}$$

Every term of Eq(8) are represented as a function of axial stress ratio (σ_0/f'_c), since it is considered that the axial stress is the most effective factor for ductility in this study. The proposed equation was formulated such that μ_w is the function of ρ_w and σ_0/f'_c alone and the effects of longitudinal reinforcement ratio (P_t) and the shear span ratio (A/D) being given by β_{P_t} and β_{AD} as the coefficient of μ_w. The equation can explain easily the effect on ductility of each factor. This is a feature of the proposed equation.

5.2 Reliability of proposed equation

The proposed equation is verified by comparison with as many experimental results in Japan as can be collected. The total number of experimental data is 170, and 50 of these data which correspond to RC members such as column in buildings, are characterized with the larger values for σ_0 and ρ_w and smaller values for A/D in comparison with the rest of data for RC members such as pier. The range of experiment for each factor is as follows: $0.0 \le \sigma_0 \le 7.0(MPa)$, $0.0 \le \rho_w \le 1.5(\%)$, $0.12 \le P_t \le 1.9(\%)$, $1.5 \le A/D \le 6.25$, $15.6 \le f'_c \le 55.2(MPa)$. All experimental results were obtained from specimens which were subjected to more than three cycles at every loading

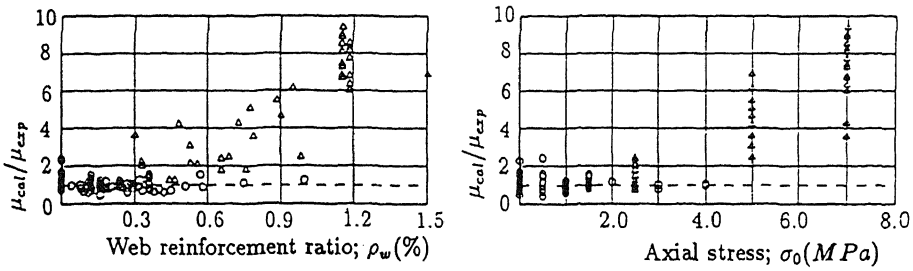

Figure 11: Results of the Machida's Equation

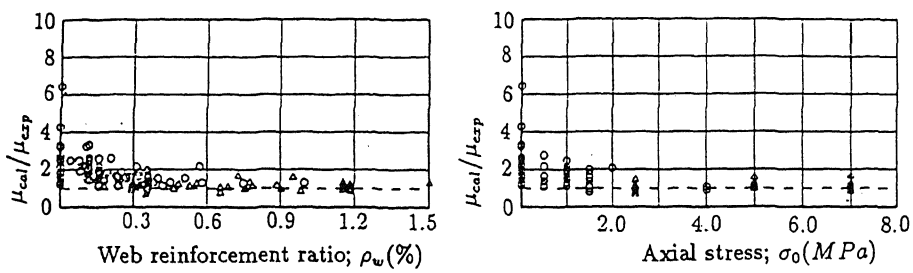

Figure 12: Results of the Arakawa's Equation

cycles. Note that the data used are in great numbers and the range of each factor is also wide.

The results obtained from the proposed equation are shown in Fig.10. On the other hand, Fig.11 and Fig.12 show the results obtained from the proposed equations by *Machida* and by *Arakawa* respectively in Japan which were based on their own experimental results. In these figures, the experimental results for the column in buildings are marked with △ and marked with ○ for the pier. *Machida* et al. formulated the equation from the experimental results aimed at pier and the equation used in Fig.12 is proposed by *Arakawa* et al. for column in buildings. It is understood that the equations which were obtained from the experimental results can not estimate the ductility correctly except for the range where the experiment covers. The ductility equations proposed in the past are based on a small number of data and they do not have wide applicability for RC members. On the other hand, it may be seen that the proposed equation can estimate the ductility more accurately than the other equations for wide range of RC members.

6 CONCLUSION

The ductility of RC members was evaluated analytically using the ductility ratio which is defined as the ratio of the lateral displacement corresponding to the maximum moment point and to the one for the yielding of the reinforcement in a member. The effects on ductility of variable factors were investigated from the results of the parametric analysis. If the axial stress is large, the effect of axial stress is most influential factor and the effect of the other factors do not appear clearly. On the contrary, the effects of the factors other than the axial stress must be evaluated correctly when the RC members are subjected to small axial stress.

The proposed ductility equation was assured its reliability by the vast comparison with the experimental data. It can estimate the ductility of RC members accurately far better than any proposed models in the past and has wide applicability.

REFERENCES

Nakamura,H., Niwa.J. and Tanabe,T. 1991. Analytical study on the ultimate deformation of RC columns. Trans. of JSCE. Vol.420: 63-76.

Kent,D.C. and Park,R. 1971. Flexural members with confined concrete. Proc. of ASCE. Vol.97. No.ST7: 1969-1990.

Machida,A., Mutsuyoshi,H. and Toyoda,K. 1987. Evaluation of ductility of reinforced concrete members. Proc. of JSCE. No.378: 203-212 (in Japanese).

Arakawa,T., Arai,Y., Ohkubo,S. and Egashira,K. 1983. Cyclic behavior and evaluation of inelastic deformation capacity of reinforced concrete columns. Trans. of JCI. Vol.5: 433-440.