Earthquake Engineering, Tenth World Conference © 1994 Balkema, Rotterdam. ISBN 90 5410 060 5

Computational mechanics and earthquake engineering

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ABSTRACT: Computational methods capable of dealing with non linear, dynamic, soil-pore fluid interaction problems are becoming an essential tool for safe design of structures subjected to Earthquakes.
The oversimplified methods involving assumptions of linear elastic behaviour or limit analysis are incapable of giving realistic answers in near failure situations.
The alternative of conducting expensive centrifuge model tests is possible but suffers severe limitations when a phreatic surface exists.
This paper reviews the current achievements and indicates some further direction of research necessary for more economic computations.

1. INTRODUCTION

The study of the interaction of the soil skeleton and the interstitial fluid (generally water) is at the very heart of soil mechanics. With the introduction of the concept of "effective stress" in the early days of soil mechanics it became possible to characterize the full behaviour of saturated soil deformations in terms of:

(1) A constitutive law written in terms of effective stresses in an incremental form

$$d\underset{\sim}{\sigma}' = \underset{\sim}{D}\, d\underset{\sim}{\varepsilon} \tag{1}$$

where in general $\underset{\sim}{D}$ depends on the history and direction of stress increment.

(2) The pore pressure p.
The two extreme cases of "fully drained" and "undrained" soils are merely particular cases of a general situation of coupling in dynamic analysiswhere the development of pore pressure is governed by the flow characteristics of the fluid as well as by the volumetric soil deformations.
The general questions describing the coupling of the soil and fluid phases were originally proposed by Biot [1941,1956] for fully saturated soil using as variables the displacement of the soil ($\underset{\sim}{u}$) and the fluid ($\underset{\sim}{w}$), and these are still today the basis from which computations can start. However it is convenient, as shown by Zienkiewicz and Bettess [1982] and Zienkiewicz and Shiomi [1984], to use for low frequency phenomena, such are associated with earthquakes a formulation based only on soil displacements and pore pressures ($\underset{\sim}{u}$-p).

Two serious sources of non-linearity arise severely limiting, and in many cases of extreme nature, totally invalidating linearized computations.
The first is of course the <u>non linear nature of the constitutive law</u> (viz Eqn.1). This not only confines the stresses to well known maximum limits - such can be approximated by the well known Mohr-Coulomb condition but introduces volumetric changes in the soil skeleton which are dependent on the history of loading and which change substantially the pore pressures. These volumetric changes are most frequently of a densification type resulting in a pore pressure increase which reduces the strength. Occasionally this strength can decrease to near zero values resulting in the phenomenon of <u>liquefaction</u> which has lead to many disasters under extreme earthquake loading.
The second non linearity occurs due to partial saturation of the pores with water. Such partial saturation occurs always above the phreatic line which bounds the fully saturated domain and, due to capillarity effects permits the development of negative (below atmospheric) pressures in a portion of the soil domain. To some extent, as we shall show latter, such negative pressures are beneficial providing some cohesion near the surface of the semi saturated soil.
The study of the effects of partial saturation of course involves the movement

of air in the pores necessary to determine the air pressure in the volume occupied by air. This complicates the total phenomena by introducing a third phase and full studies of this have been made recently by Zienkiewicz el al. [1991]However it suffices frequently to assume a free air access and to put this pressure as the zero datum.
The phenomenon of semi saturation can of course develop in zones not connected with the exterior if for instance extension of the fluid phase occurs in the course of dynamic action. Now "cavitation", or release of air from the fluid occurs when water pressures fall below atmospheric values and again semi saturation is possible permitting capillary action and negative water pressures.
The non linearities outlined above invalidate simplified analysis methods which neglect the historical development of straining and hence leave only two rational approaches for the study of important soil structures subject to earthquake action.
These are in the authors opinion:

(a) The use of full, coupled, and non linear analysis by finite element computer codes which include realistic constitutive relations for the soil matrix

and

(b) The use of scale models in centrifuge tests.

The first of above is the subject of the present paper and its success is dependent on :

(i) Adequate constitutive models describing the behaviour of soil and its permeability.

(ii) Suitable pore pressure - saturation relations.

(iii) Efficient numerical modelling.

The second, i.e., the scale model approach is probably more expensive and suffers from several scaling difficulties. In particular it appears that the effects of semi-saturation can not be effectively modeled and thus realistic studies of dam and embankment failures in which a free surface exists can not be yet performed.
Hopefully as confidence is gained in the use of computational models their use will become more widespread - and indeed when their performance is verified some simplified - but realistic- procedures may well be developed. At the present time however the use of full analysisprocedures is still essential.

2. THE COMPUTATIONAL MODEL

2.1 General remark

The last decade has seen many developments using finite element discretization to solve the dynamic problems of the soil-water "ensemble". This has resulted in several codes currently capable of solving the essential interaction problems. Among these we should mention:

DYNAFLOW of J.H.Prèvost (1982)
developed at Princeton University.

DIANA
developed jointly by O.C. Zienkiewicz and T. Shiomi with a consortium of Japanese industries. See Kawai [1985]

GEFDYN
developed by D.Aubry at Ecole Centrale in Paris (Aubry et al. 1991)

and

SWANDYNE
developed by the authors at the Institute for Numerical Methods in Engineering, Swansea University, UK.

In principle all such codes can deal with all the basic interaction problems but of course there are differences in the numerical techniques and in the constitutive models implemented.
To focus ideas we shall in this paper refer to the methods developed in SWANDYNE which are fully described by Zienkiewicz el al. [1990 a,b].
In what follows we shall briefly discuss:

- -The mathematical formulation.
- -The numerical treatment.
- -The constitutive relations.

and

- -The verification procedures.

2.2 The mathematical model

For low frequency dynamics we shall concentrate on the $\underset{\sim}{u}$-p form of governing equations which are valid for all earthquake engineering problems of practice. This has the advantage over the $\underset{\sim}{u}$-$\underset{\sim}{w}$ forms occasionaly used in allowing consolidation and indeed steady state forms to be modelled without alteration of the code.
However to deal with the frequently encountered phenomena of semi saturation we shall base the equations on he assumption that the pore saturation is Sw and that:

$$S_w \leq 1 \quad \text{when } p = p_w \leq 0 \qquad (2)$$

In this implicitly we assume that the air pressure in the pores is atmospheric (Zero) and that for negative pressures a unique relation

$$S_w = S_w(p) \qquad p \le 0$$
$$S_w = 1 \qquad p > 0 \qquad (3)$$

exists when pressures are negative.
Such relations have been established by many authors who also show for such semi saturated behaviour a unique dependance of relative permeability on the pressure.
In Fig.1 for instance we illustrate typical relations established by Safai and Pinder [1979].(Fig. 1)

The governing equations can be now written for momentum conservation as

$$\sigma_{ij,j} - \rho\, \ddot{u}_i + \rho\, b_i = 0 \qquad (4)a$$

where the total sttress σ_{ij} is representted in terms of effective stress σ_{ij} as

$$\sigma_{ij} = \sigma'_{ij} - \delta_{ij}\, S_w\, p \qquad (4)b$$

The strain increments are defined as

$$d\varepsilon_{ij} = \frac{1}{2}\left(du_{i,j} + du_{j,i}\right) \qquad (4)c$$

and an incremental constitutive law (viz eqn.1) is assumed in the form

$$d\sigma'_{ij} = D_{ijkl}\, d\varepsilon_{kl} \qquad (4)d$$

In above the essential variables are u_i and p and with the knowledge of a suitable constitutive law (4d) and the stress definition (4c) the values of the effective stress can be established. Remark that we have simplified the equations as compared with those of Zienkiewicz et al.[1990] by assuming the effective area α to be unity and that ρ the total density is independent of Sw.

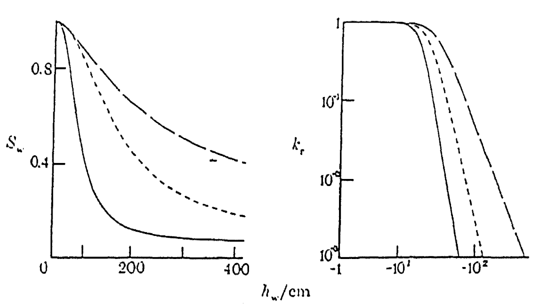


Fig. 1 Typical relations between pore pressure head $h_w = p_w/\gamma_w$, saturation s_w and relative permeability.

The second, <u>continuity of flow,</u> governing equation is given as

$$\dot{\varepsilon}_{ii} + \frac{\dot{p}}{Q^*} - (k_{ij} \cdot p_{,j})_{,i} + (k_{ij} \rho_f S_w b_j)_{,i} = 0 \qquad (5)a$$

where the parameter Q^* is defined as

$$\frac{1}{Q^*} = C_s + \frac{n\, S_w}{K_f} + (1-n)\, S_w\, (S_w + C_s/n)\, /\, K_s \qquad (5)b$$

and where $C_s = dS_w/dp$ which can be established from such curves as those in Fig.1 ,and K_f and K_s are the bulk moduli of the fluid and the solid respectively, n is the porosity, and k_{ij} are the permeability coefficients.
For positive pressures, i.e. fully saturated condition we have

$$1/Q^* = n/K_f + (1-n)/K_s \qquad (5)c$$

It should be remarked that the solution process proceeds in an identical manner for both saturated and partially saturated conditions though of course relationship (5b) ensures much smaller pressure changes in the latter case.
The traction boundary conditions are of course augmented by appropiated boundary conditions on flow.

2.3 Computational model

Here space does not permit the discussion of finite element discretization procedures or the time step algorithms. Both can be found in the detailed papers or in the texts by Zienkiewicz and Taylor [1989,1991].

Clearly many different approximation elements can be used as well as many alternative time stepping procedures and many variants are incorporated in such codes as SWANDYNE. However some general pooints need to be noted which are pertinentt to all approximations.

(1) Only certain combinations of shape functions for $\underset{\sim}{u}$ and p variables are permissible if the full range of

variables is to be included. In particular as the permeability and compressibility of fluid tends to zero the solution tends to an incompressible one for which locking can occur as discussed by Zienkiewicz and Taylor [1991] . Here new possibilities are being investigated to sidestep the difficulties (Zienkiewicz et al. 1992).

(2) Explicit or explicit-implicit procedures have severe limitations on the magnitude of the time step permissible

2.4 Constitutive relations.

The constitutive relations which always involve plastic (irreversible) deformations can be expressed in the form of Eqn. 1 or 4d for almost all geomechanical models -of which today we have a great variety.

It is however imperative that the model used includes the following characteristics:

(i) It must, in the terms of effective stresses, shoow failure (or continuing deformation) when a residual angle of friction is reached. (This is known as the critical state)

(ii) It must be history dependent and show an accumulation of negative volumetric strain which resultso in pore presure increases and hence strength degradation.

In addition it should of course reproduce as accurately as possible stress-strain paths observed in laboratory experiments and do this with a relatively small number of parameters. One such model described by Pastor and Zienkiewicz [1986] and Pastor et al.[1990] is capable of excellent

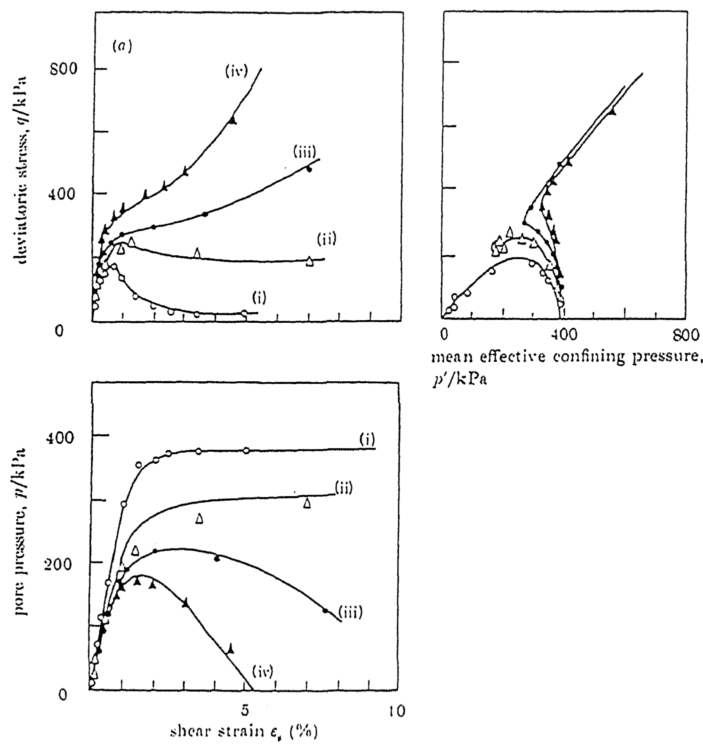


Fig. 2 Comparison of constitutive model by Pastor et al. (1990, 1986) with experients.

(a) Static triaxial tests Dr = 29 %

Dr = 44 %

Dr = 47 %

Dr = 64 %

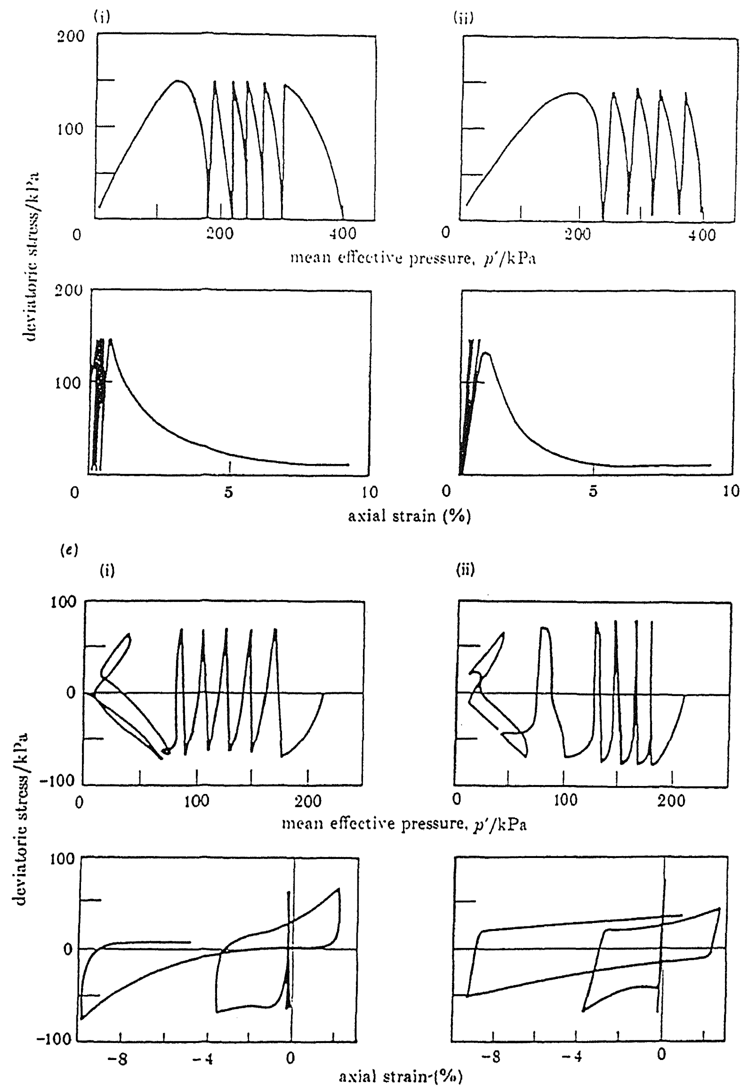


Fig. 2(cont)
(b) Cyclic loading triaxial tests on loose sand

(i) Experiments (ii) Constitutive model

performance in a wide range of tests as shown in Fig.2 where comparison with experiment is indicated. (Fig. 2)

Here modelling is accomplishable with only 8 parameters. We suspect however that with other models similar results can be obtained and that indeed not all the parameters are strictly necessary for obtaining realistic predictions.

3. VERIFICATION PROCEDURES

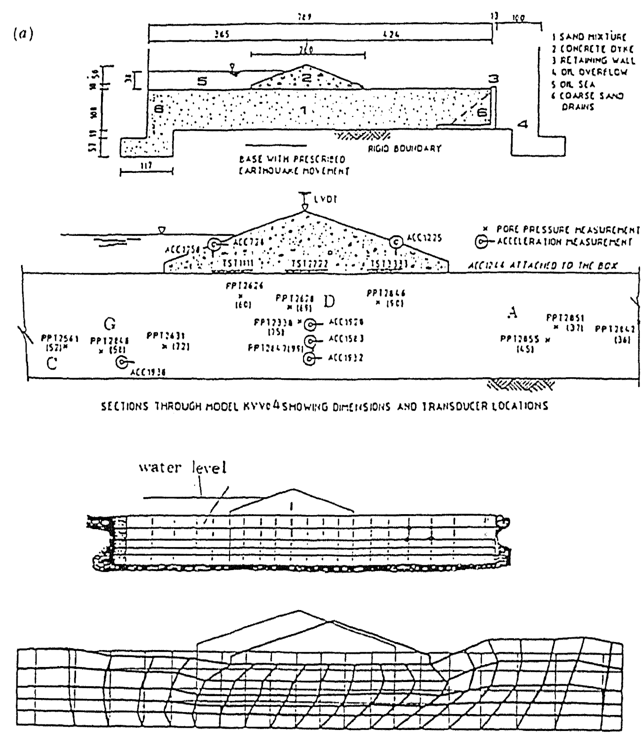
It is of course desirable to verify the prediction of computation. Unfortunately earthquakes of prototype scale do not come to order. Thus only two avenues are present

(a) Comparison with centrifuge modelling in the scale of the centrifuge to avoid difficulties of scale effects already mentioned

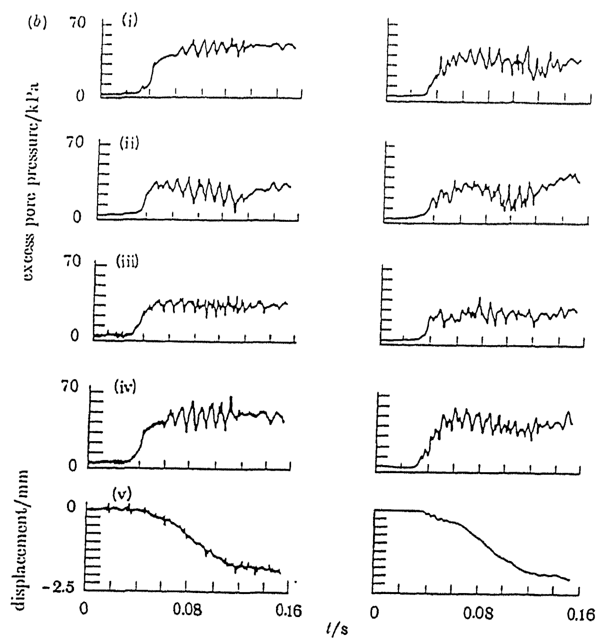
and

(b) Reconstruction of past, real, full scale events using guessed or at least imprecisedly known data.

In Fig.3 we show such a comparison of computation with tests conducted at the



(a) Centrifuge and finite element models of a dyke.



(b) Pore pressure and desplacement computed (left) and Measured (right)

Fig. 3 Comparison of centrifuge tests and computation with the SWANDYNE code.

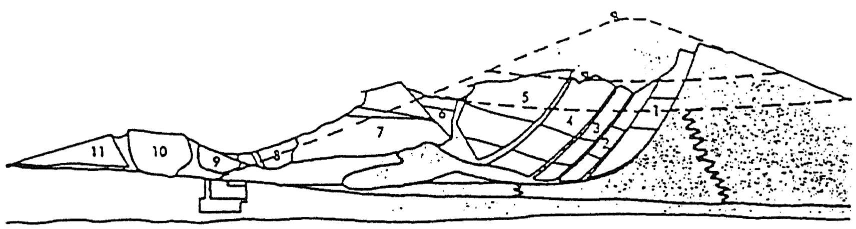
University of Cambridge by K.V.Venter in 1987.(Fig. 3)

Many similar tests have been made by the authors and others but seldom have alternative computational codes and constitutive models been tested on a specific set of centrifuge results. It is of interest to remark that currently a project named VELACS (Verification of Liquefaction Analysis by Centrifuge studies) is under way with the sponsorship of the National Science Foundation (USA). Here detailed material studies followed by computational analysis are preceeding the centrifuge tests and results will be reported in the fall of 1993.

Similar studies are under way sponsored by the EPOCH project in Europe. It is hoped that the result will be convincing for practitioners.
For the problem of reconstructing past events again many studies are reported in the literature. In particular tthe problem of the dramatic failure of the San Fernando dam in 1971 has caused much interest and discussion. Seed et al. [1975] and Seed [1979] illustrates this failure.(Fig. 4)

In Fig.5 we show details of an analysis using the SWANDYNE code by Zienkiewicz and Xie (1991).(Fig. 5)

The predicted deformations are remarkably



Cross-section through embankment after earthquake

Fig. 4 Failure and reconstruction of original conditions of lower San Fernando dam after 1971 earthquake according to Seed [1979].

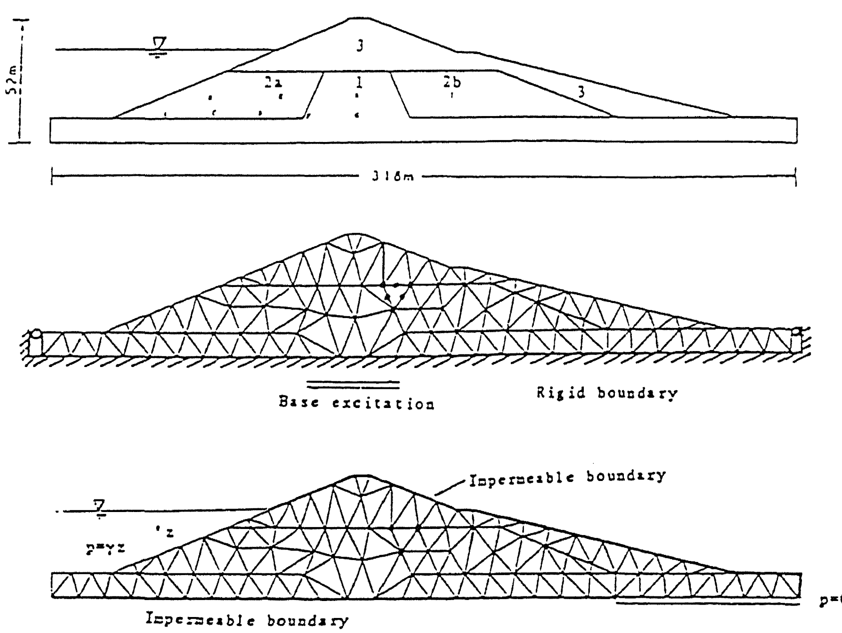


Fig. 5 Analysis of the San Fernando dam failure

(a) Idealization of material zones, boundary conditions and finite element mesh

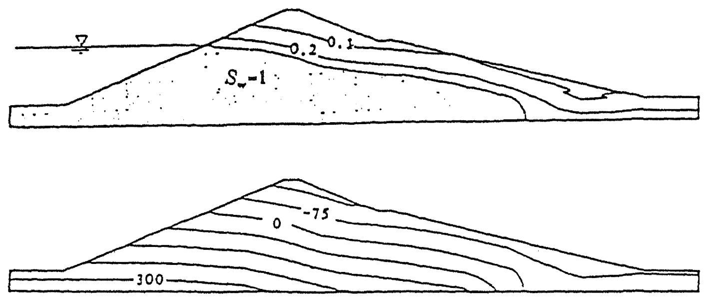


Fig. 5 (cont.)

(b) Initial pressures and saturation.

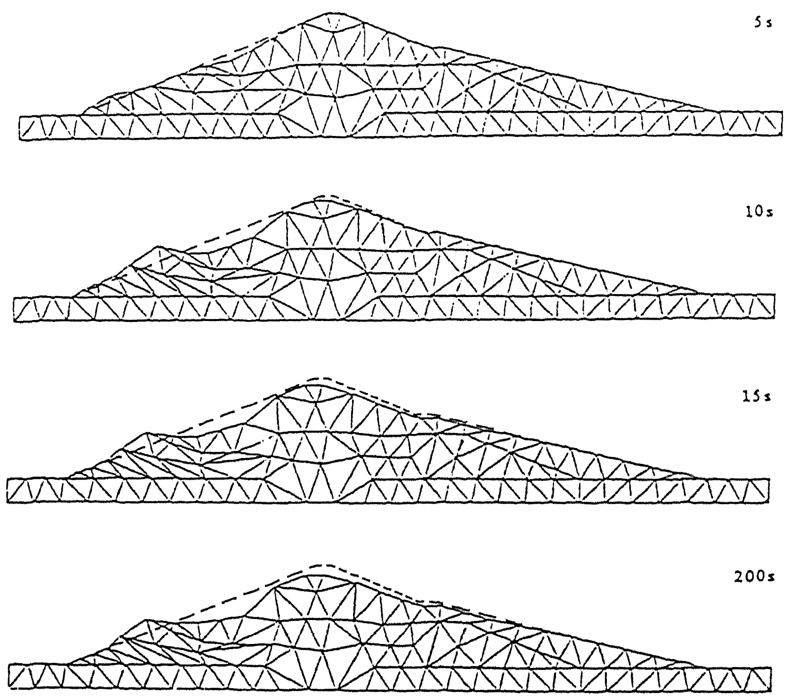


Fig. 5 (cont.)

(c) Deformed shape at various times.

(5, 10, 15 sec. (end of earthquake) and 200 sec.)

close to the ones of failure though of course details of the actual slips are not trully modelled.

4. THE FUTURE

The current state of the art of modelling appears adequate to predict reasonably the nature of failure or movement of important structures. However research must not stop here and much more needs to be done to make computations simpler and to improve the modelling.

From the numerical viewpoint the need is apparent to

(a) Improve the robustness of the codes in dealing with incompressibility.
(b) Add the possibility of adaptive refinement to capture localization

phenomena. (viz Pastor et al. 1992)

(c) Increase code efficiency.

From the physical viewpoint the greatest need is to

(i) Simplify constitutive relations to the point of necessarity - isolating the properties essential in causing failure phenomena - and importantly making parameter identification easier for the user.

Work is currently being done in such phases by the authors and many others and we hope that in the future the current research tools will become standard practice.

5. ACKNOWLEDGEMENTS

The authors are greateful to the EEC-EPOCH project CT91.0046 for supporting much of present research.

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