Uniaxial compressive stress–strain model for clay brick masonry

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Material stress–strain properties are required in the nonlinear analyses of structures. Stress–strain relationships for some construction materials such as concrete and steel are available in the literature and design codes. However, such relationships are not easily available for one of the most widely used construction material, i.e. masonry. In the present comprehensive experimental study, compressive stress–strain relationships for masonry are determined by testing 84 masonry prism specimens constructed using bricks from four different manufacturers and three mortar grades. By regression analysis of experimental data, analytical expressions are proposed to estimate the modulus of elasticity of masonry. An analytical model is proposed to adequately plot the stress–strain curves for masonry using six control points on the curves. A simplified analytical model is also proposed that can be conveniently used in finite element analysis programs.

Keywords: Brick masonry, compressive strength, mortar, stress–strain relations.

BRICK masonry is a common construction material in India because of its abundance, low cost, good sound and heat insulation properties, and availability of skilled labour. Masonry is extensively used in India as infill walls in reinforced concrete buildings. Analysis and design of buildings with masonry require material properties of masonry; for example, modulus of elasticity of masonry is required in the case of linear static analysis. Stress–strain curves of masonry are required in the case of more detailed nonlinear analyses such as static pushover analysis, which is being used quite frequently nowadays. However, stress–strain curves of masonry are not available easily because of insufficient controlled experimental studies and significant geographical variation in material properties.

Only a few experimental studies worldwide have proposed analytical relationships to plot the stress–strain curves of masonry; however, these relationships are not easy to use because of the requirement of large amount of input data. Limited experimental research has been carried out in India to determine the stress–strain curves of masonry typically constructed in the Indian industry[1,2]. In the present study, extensive experimental testing of brick masonry prisms is performed to obtain the nonlinear stress–strain curves of brick masonry. Further, simple analytical equations are developed by regression analysis of the experimental data to estimate the modulus of elasticity and to plot the stress–strain curves for masonry.

Experimental study

In the present experimental study, 84 specimens of brick masonry prisms (combination of bricks manufactured by four local kilns, and three mortar grades) were tested to obtain stress–strain curves for typical masonry used in the Indian construction industry. Hand-moulded burnt clay solid bricks were used in constructing masonry prisms. The bricks were manufactured by four different manufacturers (designated as M, B, S, and O), and had approximate length, width and height as 230, 110 and 75 mm respectively. Different grades of mortar (cement : lime : sand by volume) used in the study were: 1 : 0 : 6 (weak), 1 : 0 : 3 (strong), and 1 : ½ : 4½ (intermediate). Compressive load was applied in displacement control on five brick high masonry prisms using MTS servo-hydraulic actuator. Approximate height of five-brick high masonry prism with 10 mm thick mortar joints was about 400–410 mm. Epsilon extensometers were used to record the displacement response across three mortar joints as shown in Figure 1 a. Load and displacement measurements were recorded in real time using a computer-based data acquisition system. General test set-up and typical failure modes of masonry prisms are shown in Figure 1.

The compression testing was performed according to ASTM specifications[3], which are quite similar to those given in the Indian masonry code (IS: 1905–1987)[4]. Stress–strain curves obtained from the experimental testing are shown in Figure 2, and a summary of the test results, including prism strength (f'_m), failure strain, and modulus of elasticity of masonry (E_m) is presented in Table 1. Mean compressive strengths of four types of bricks (f_b), and three grades of mortar (f_j) used in the study are also given in Table 1. Mean values of f'_m were found to be 4.1, 7.5 and 6.6 MPa for prisms constructed with weak, strong, and intermediate mortar respectively, and mean ultimate strains at failure were 0.0059, 0.0053 and 0.008 for the three cases respectively. The Indian masonry code[4] specifies some multiplying factors to be used in

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Table 1. Summary of test results on clay brick masonry

Brick type[a]	f_b (MPa)	f_j (MPa)	f'_m (MPa)	Failure strain	E_m (MPa)
Prisms with weak mortar – 1 : 0 : 6 (4 × 7 specimens)					
M	17.7 (0.23)[b]	3.1 (0.22)	4.0 (0.13)	0.0052 (0.53)	2239 (0.30)
B	16.1 (0.08)		2.9 (0.17)	0.0034 (0.45)	1795 (0.17)
O	28.9 (0.23)		5.1 (0.16)	0.0086 (0.15)	2630 (0.14)
S	20.6 (0.17)		4.3 (0.17)	0.0065 (0.14)	2355 (0.19)
Average	20.8 (0.33)		4.1 (0.24)	0.0059 (0.43)	2300 (0.24)
Prisms with strong mortar – 1 : 0 : 3 (4 × 7 specimens)					
M	17.7 (0.23)	20.6 (0.08)	7.4 (0.10)	0.0067 (0.28)	3585 (0.18)
B	16.1 (0.08)		6.5 (0.14)	0.0041 (0.39)	3592 (0.25)
O	28.9 (0.23)		8.5 (0.21)	0.0057 (0.36)	5219 (0.50)
S	20.6 (0.17)		7.6 (0.17)	0.0050 (0.55)	4250 (0.44)
Average	20.8 (0.33)		7.5 (0.18)	0.0053 (0.41)	4200 (0.38)
Prisms with intermediate mortar – 1 : 0.5 : 4.5 (4 × 7 specimens)					
M	17.7 (0.23)	15.2 (0.06)	6.5 (0.19)	0.0102 (0.17)	3542 (0.27)
B	16.1 (0.08)		5.9 (0.23)	0.0062 (0.40)	3509 (0.49)
O	28.9 (0.23)		7.2 (0.24)	0.0092 (0.32)	4712 (0.33)
S	20.6 (0.17)		6.8 (0.23)	0.0066 (0.31)	3325 (0.26)
Average	20.8 (0.33)		6.6 (0.20)	0.0080 (0.34)	3800 (0.35)

[a]M, B, O, S are initials of four different brick manufacturers. Average water absorption of brick specimens was found to be 12.3% with coefficient of variation = 0.13.
[b]Figures in parentheses indicate coefficient of variation.

Figure 1. *a*, Test set-up for compression testing of masonry prism. *b*, *c*, Typical failure modes showing vertical splitting of specimens.
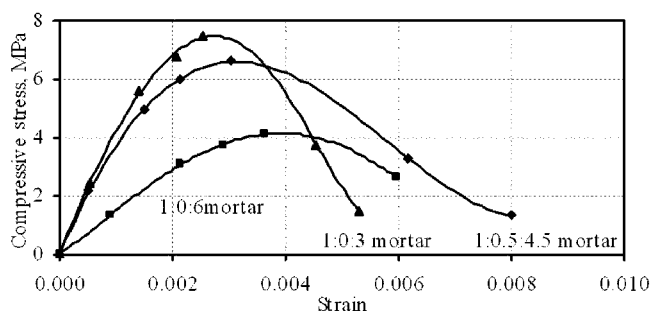
Figure 2. Compressive stress–strain curves for masonry.

correcting the prism strength of masonry for height-to-thickness ratio of prism lower than 5.0. For the present set of tests, this correction factor comes out to be about 0.95. Generally, five-brick high masonry prisms are tested to obtain the prism strength of masonry, and the correction factors are then used to arrive at compressive strength of masonry (or design strength of masonry) from masonry prism strength (ASTM[3]). Therefore, this factor has not been used in the present study, and the results presented here represent the true values as obtained in the tests.

Masonry with weak mortar was found to be less stiffer than that constructed with the other two grades of mortar. Modulus of elasticity is calculated from stress–strain curves by measuring the slope of secant between ordinates corresponding to 5 and 33% of the ultimate strength of the specimens. Experimental results show that E_m can be directly correlated with f'_m, and it was found that E_m

varies between 250 and 1100 times f'_m for the present dataset (Figure 3). A relatively good coefficient of correlation ($C_r = 0.63$) was found between the experimentally observed E_m values and those estimated using the following equation:
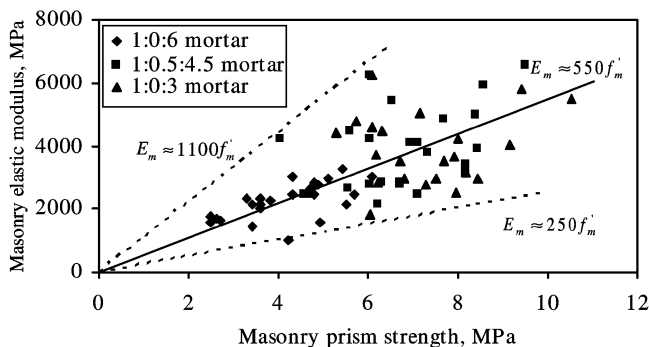
$$E_m \approx 550 f'_m . \tag{1}$$

Therefore, it is recommended that for brick masonry used in the Indian construction industry, E_m may be estimated as an average value of 550 times f'_m.

Performance of masonry with intermediate mortar, which has lime content, was much better than that of masonry with the other two mortar grades (Figure 2). Prism strength of masonry with intermediate mortar was only about 13% lesser than that with strong mortar, while failure strain was about 50% more. A significant improvement in ductility of masonry was observed because of the presence of lime in the mortar, without any considerable reduction in its compressive strength. This shows that using lime in the mortar offers distinct structural advantages.

Control points on stress–strain curves of masonry

Based on the limit states proposed in the literature[5], stress–strain curves of masonry obtained in the present study are idealized using six control points, which correspond to six significant events on stress–strain curves (Figure 4). The control points identified based on the results of the present tests are:

1. $0.33 f'_m$: Point up to which the stress–strain curves remain linear.
2. $0.75 f'_m$: Vertical splitting cracks in bricks start developing at about this stress.
3. $0.90 f'_m$: Vertical splitting cracks in bricks propagate excessively throughout masonry.
4. f'_m: Ultimate stress level in masonry (prism strength).

5. $0.5 f'_m$: Maximum dependable compressive strength of masonry (on descending curve).
6. $0.2 f'_m$: Maximum residual compressive stress in masonry (on descending curve).

Strain values corresponding to these control points are shown in Figure 4. Strain readings corresponding to the control points 5 and 6 could not be recorded in case of prisms constructed with weak mortar, because of brick–mortar bond failure in most of the specimens, and thus abrupt and brittle failure of the specimens. These idealized stress–strain curves can be conveniently used in the analysis and design of masonry structures constructed with bricks and mortar of similar material and strength.

Analytical model for stress–strain curves of masonry

Estimation of prism strength of masonry

It is not always feasible to conduct compression testing of masonry prisms to obtain the actual prism strength, which is the basic structural property for design of masonry. On the other hand, compressive strengths of bricks (f_b) and mortar (f_j) can be easily evaluated by the tests. In such cases, the following relation can be conveniently used to estimate f'_m using f_b and f_j:

$$f'_m = K f_b^{\alpha} f_j^{\beta}, \tag{2}$$

Figure 4. Control points on stress–strain curves of masonry.
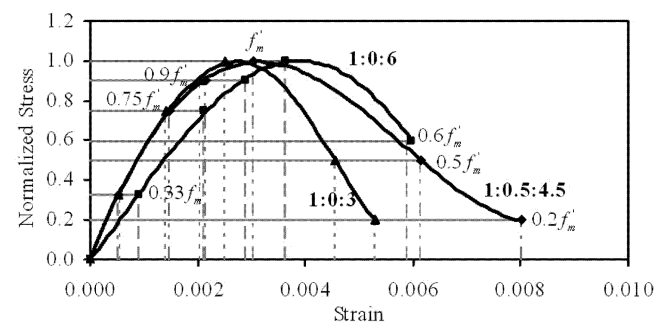
Stress in terms of f'_m	Strain in prism for mortar grades		
	1:0:6	1 : 0 : 3	1 : 0.5 : 4.5
0.00	0.0000	0.0000	0.0000
0.33	0.0009	0.0005	0.0005
0.75	0.0021	0.0014	0.0015
0.90	0.0029	0.0021	0.0021
1.00	0.0036	0.0025	0.0030
0.60	0.0059	–	–
0.50	–	0.0045	0.0062
0.20	–	0.0053	0.0080

Figure 3. Variation of modulus of elasticity of masonry with corresponding compressive strengths.

where K, α and β are constants. In eq. (2), α must be higher than β because f'_m is found to depend more upon the strength of bricks used in masonry. Based on the experimental results of the present study, K, α, and β are obtained as 0.63, 0.49 and 0.32 respectively, by unconstrained regression analysis of eq. (2) using least-square fit method. Therefore, the following equation is proposed for estimating f'_m:

$$f'_m = 0.63 f_b^{0.49} f_j^{0.32}; \qquad (3)$$

Standard error of estimate ≈ 0.48 MPa.

A similar equation with different values of the constants K, α and β was proposed by Dayaratnam[6] for Indian bricks:

$$f'_m = 0.275 f_b^{0.5} f_j^{0.5}; \qquad (4)$$

Standard error of estimate ≈ 2.2 MPa.

Errors in estimation of f'_m would be much higher when eq. (4) is used, because of equal weightage given to the compressive strengths of bricks and mortar. On the contrary, eq. (3), proposed in the present study, predicts realistically the strength that agrees quite well with the experimental observations (Table 2). Therefore, eq. (3) is proposed for estimating f'_m, which is further used here for developing stress–strain curves for masonry.

Estimation of peak strain of masonry

Stress–strain curves obtained in the present study indicate that the ascending part of the curve can be represented by a parabola, which has also been reported earlier[7–10]. The parabolic variation can be expressed in non-dimensional form in terms of stress ratio and strain ratio as:

Table 2. Comparison of masonry prism strength obtained by different sources

Experimental value			Analytical f'_m (MPa) using	
f_b (MPa)	f_j (MPa)	f'_m (MPa)	Present study	Dayaratnam[6]
17.7	3.1	4.0	3.7 (8)	2.0 (96)
16.1		2.9	3.5 (22)	1.9 (49)
28.9		5.1	4.7 (8)	2.6 (96)
20.6		4.3	4.0 (8)	2.2 (96)
17.7	20.6	7.4	6.8 (9)	5.3 (41)
16.1		6.5	6.5 (0)	5.0 (30)
28.9		8.5	8.6 (1)	6.7 (27)
20.6		7.6	7.3 (4)	5.7 (34)
17.7	15.2	6.5	6.2 (6)	4.5 (44)
16.1		5.9	5.9 (0)	4.3 (37)
28.9		7.2	7.8 (9)	5.8 (25)
20.6		6.8	6.6 (3)	4.9 (40)

Figures in parentheses indicate per cent error between experimental values and those obtained from other sources.

$$\frac{f_m}{f'_m} = 2\frac{\varepsilon_m}{\varepsilon'_m} - \left(\frac{\varepsilon_m}{\varepsilon'_m}\right)^2, \qquad (5)$$

where f_m and ε_m are compressive stress and strain in masonry respectively, and ε'_m is the peak strain corresponding to f'_m. Based on the present experimental results, the parabolic curve is further extended in the descending part till f'_m drops to 90%, after which the curve is simplified as a straight line till the residual stress in masonry is reached ($0.2f'_m$). The following equation is proposed in the present study to estimate ε'_m by regression analysis of the experimental data, which provides a good fit to the experimental data:

$$\varepsilon'_m = \frac{0.27 f'_m}{f_j^{0.25} E_m^{0.7}}; \text{ Standard error of estimate} \approx 0.0001.$$
$$(6)$$

Equations (3), (5), and (6) can be easily used to plot the ascending part of the stress–strain curve of the masonry up to $\{\varepsilon'_m, f'_m\}$, and then the parabolic curve can be extended in the descending part up to $0.9f'_m$, as shown in Figure 5. A linear descending part follows the parabolic part till $\{2\varepsilon'_m, 0.2f'_m\}$ for mortar without any lime content, and till $\{2.75\varepsilon'_m, 0.2f'_m\}$ for mortar with lime content. A higher failure strain value is proposed for masonry constructed with lime mortar, because lime is found to introduce flexibility in the masonry.

Simplified tri-linear stress–strain model

Generally, simplified, piece-wise linear stress–strain curves are sufficient as an input in most of the finite element computer analysis programs. Based on the present study, simplified stress–strain curves for masonry consisting of three continuous piece-wise linear segments are proposed here (Figure 6), which can be conveniently
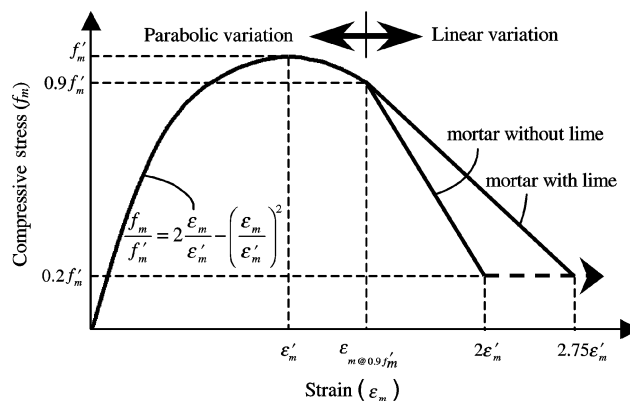
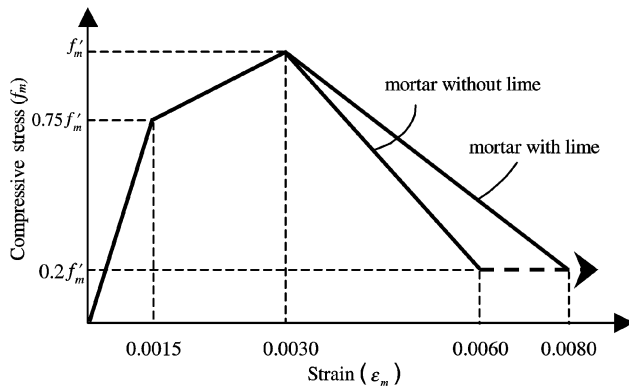**Figure 5.** Analytical model for stress–strain curves of masonry.

Figure 6. Simplified tri-linear stress–strain model for masonry.

specified in computer programs and finite element software for structural analysis. The first segment ends at $0.75f'_m$, and although some nonlinearity was observed in the experimental curves at this stress level, the corresponding strain is considered as 0.0015, assuming the linear behaviour. The second segment ends at f'_m, with corresponding strain as 0.003, followed by the last segment which is continued until the residual stress in masonry ($0.2f'_m$) is reached. Strain corresponding to $0.2f'_m$ is taken as 0.008 for masonry constructed with mortar containing lime, and 0.006 for masonry constructed using mortar without any lime content.

Conclusion

Uniaxial compressive testing of 84 masonry prisms was conducted using four different bricks and three mortar grades. Based on experimental observations, modulus of elasticity of masonry is found to vary between 250 and 1100 times the prism strength of masonry. An average value of 550 times the prism strength is proposed in the present study. The compressive strength of masonry was found to increase with the compressive strength of bricks and mortar. The trend was more prominent in case of masonry constructed with weaker mortar. Therefore, using a mortar grade of higher strength than required may not always produce high-strength masonry. Masonry with lime mortar was found to undergo about 50% more compressive deformation than that constructed using mortar without lime, while the reduction in compressive strength was only about 13% when lime mortar was used. Therefore, adding lime to mortar is a recommended practice in masonry construction.

An analytical model is developed to adequately replicate the experimentally obtained masonry stress–strain curves using six control points. The model is general and quite convenient to use because the only input parameters required are the compressive strengths of the bricks and mortar. Further, a simplified tri-linear stress–strain model for masonry is proposed, which can be easily specified in computer programs and software for analysis of structures with masonry elements. The results discussed in the present study may be valid only for the North Indian brick industry. However, the proposed models are generalized and may be implemented for other regions with slight modifications in input parameters. Further experimental verification may be required for extension of these results for bricks found in other regions, and mortar of different grades.

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