Ultimate flexural strength of reinforced concrete circular hollow sections

Durgesh C. Rai, Kamlesh Kumar and Hemant B. Kaushik

Reinforced concrete circular hollow sections are chiefly used in shaft type staging for supporting overhead water tanks, other tower structures and chimneys. As per current Indian practice such sections are designed using working stress approach, which neither explicitly ensures safety against collapse nor results in economic design. Further, it is necessary that these structures are checked for the ultimate-load conditions in view of large tensile stresses caused by horizontal loads due to earthquake and strong wind conditions, which is also stipulated in IS 456 : 2000. The analysis for ultimate flexural strength involves selecting a position of neutral axis in terms of an angle and calculating the ultimate axial force and ultimate bending moment resisted by the resulting stress envelope of steel and concrete, for which IS 456 : 2000 recommended material models for limit state design method is used. Closed-form expressions are presented for generating envelopes of ultimate flexural resistance in terms of axial force and bending moment interaction curve. For some commonly occurring parameters, these interaction curves are also plotted in non-dimensional form, which can be easily used for analysis and design of thin RC hollow circular sections.

Keywords: *Hollow circular sections, ultimate strength, flexure, P-M interaction.*

Hollow circular sections of reinforced concrete are often used for structural members that are to resist combined action of axial and bending moments. Shaft type staging of elevated water tanks, other tower structures, and chimneys are some examples of structures which rely on strength of thin hollow circular sections for their load resistance ability. These structures are subjected to significantly large lateral loads due to wind or earthquakes, in addition to direct compressive force which causes additional flexural stresses. It is essential that these structural members are checked at the ultimate-load conditions in view of large tensile stresses caused by horizontal loads due to earthquakes and wind, which is also stipulated in IS 456 : 2000[1].

As per current Indian practice, for example, analysis of the shaft type staging is performed in accordance with the provisions of IS 11682 : 1985, which follows the Working Stress Design (WSD) approach, assuming elastic behaviour of materials and use of permissible stresses on cracked section[2]. However, Section B-4.3 requires that such "members subjected to combined direct load and flexure shall be designed by limit state method". Even the earlier version IS 456 : 1978 had similar provision in its Section 46.3, where it required that "Members ... should be further checked for their strength under ultimate load conditions to ensure the desired margin of safety, this check is especially necessary when the bending moment is due to horizontal loads."[3] For solid circular sections, many textbooks (such as by Dayaratnam) have provided necessary expressions to compute flexural strength in presence of axial loads as per Limit State Design (LSD) provisions[4].

Currently there are no Indian codes available for designing hollow circular sections by limit state design method and it has been a long felt need to develop such provisions, especially as Part 2 of Indian chimney code, IS 4998 : 1992[5]. On the other hand, internationally all major reinforced concrete codes have addressed this issue and a detailed comparative study is reported elsewhere[6]. These codes primarily differ in the description of concrete stress-strain curve that is used in developing the strength estimates.

The primary objective of this paper is to provide an ultimate flexure strength analysis for design of RC hollow circular sections, using material models given in IS 456 : 2000 with appropriate partial safety factors. It should be noted that the ultimate strength of the section is further affected by the consideration of shear, ovaling effects (circumferential stresses), thin shell buckling, temperature etc., which is not considered in this paper. A combination of load factors and strength reduction factors appropriate to the particular structure type may be further required for acceptable margin of safety in design. For typical shell radius to thickness ratios (5 to 50) used for such RC cylindrical structures, local buckling of shell is rather improbable and therefore, is not usually considered in strength calculations.

Ultimate flexure strength of circular hollow section

A circular section of shaft of mean radius r and thickness t with an opening subtending an angle 2β at the centre is shown in the Fig. 1. The ultimate flexure strength analysis of the staging section involves the calculation of ultimate direct force P_u and ultimate bending moment M_u that can be resisted by the resulting stress envelope of the steel and concrete . The assumptions made in the analysis are:

Plane sections remain plane after bending which implies that the strain distribution is linear across the section.

2. Thickness of staging wall is thin compared with its diameter.
3. Strength of concrete in tension is assumed zero.
4. The stress-strain curves for concrete and reinforcing steel are as shown in Fig. 1. These are 'essentially' same as those given in IS 456 : 2000 for the factored strength calculations with partial safety factors of 1.5 and 1.15 for concrete and steel, respectively.
5. The reinforcement in compression zone is taken in to account and the yield strength of the reinforcing steel is taken as 0.75 f_y in compression.

The normal straight line stress-strain relationship of WSD is replaced by full elasto-plastic stress-strain relationship for steel and a combination of parabolic and straight line relationship for concrete as suggested in IS 456 : 2000 for LSD. Further, in deriving geometric properties of the section, it is assumed that the shell thickness is small in comparison to its radius and this assumption introduces no appreciable error for typical shells with mean radius to thickness ratio greater than 10. The stress-strain curve is modified to account for partial safety factor for strength of materials. The design stress-strain relationship of concrete consists of parabolic curve rising to a stress of $0.45f_{ck}$ at a strain of 0.002 and then continuing at the same stress to a maximum strain of 0.0035. The design stress-strain curve for steel is simplified by taking initial part of the curve corresponding to a straight line of slope E_s up to a constant value of 0.87 f_y in tension and $0.75f_y$ in compression. The assumed elasto-plastic behaviour with definite yield point is representative of TMT reinforcing bars which are now increasingly used. However, results will differ only marginally with the curves for CTD bars as given in IS 456 : 2000. Above mentioned assumptions are similar to those made by Pinfold while developing P-M

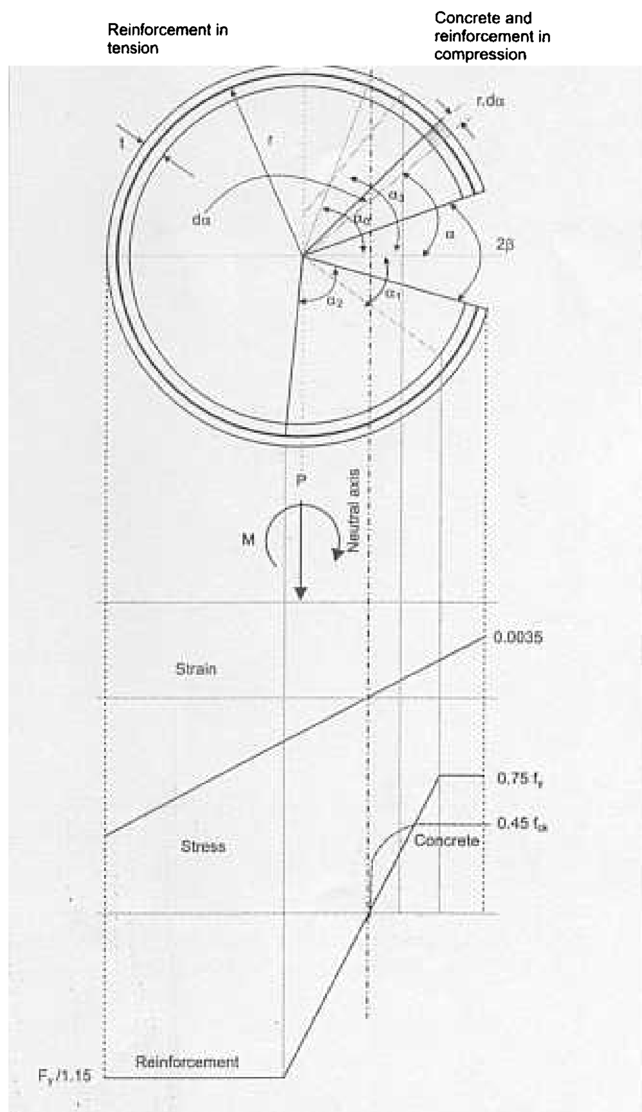

Fig 1 Stress-strain curves for concrete and steel materials used in the analysis as per IS 456:2000

interaction curves using materials models of British concrete code, CP110[7,8].

Similar to elastic analysis, two zones of neutral-axis position are considered, one where it falls inside the section and the other when it is outside on the same side away from the opening.

Case 1: Neutral axis falls inside the section

Assume that the neutral axis is located by angle α_0 and α_1, α_2, and α_3 be the yield point angles: α_1 and α_2 corresponds to yield strains of steel in compression and tension, respectively and α_3 corresponds to the concrete stain of 0.002 at which concrete reaches its peak strength. Consider an element $d\alpha$ at an angle α as shown in *Fig* 1. These angles can be expressed as under

$$\cos\alpha = \left(\frac{\varepsilon}{0.0035}\right)(\cos\beta - \cos\alpha_0) + \cos\alpha_0 \quad (1)$$

$$\cos\alpha_1 = \left(\frac{\sigma_{ysc}}{0.0035E_s}\right)(\cos\beta - \cos\alpha_0) + \cos\alpha_0 \quad (2)$$

$$\cos\alpha_2 = \left(\frac{-\sigma_{yst}}{0.0035E_s}\right)(\cos\beta - \cos\alpha_0) + \cos\alpha_0 \quad (3)$$

$$\cos\alpha_3 = \left(\frac{0.002}{0.0035}\right)(\cos\beta - \cos\alpha_0) + \cos\alpha_0 \quad (4)$$

where E_s is Young's modulus of steel (200 GPa), and ε is strain in concrete in the element $d\alpha, \sigma_{ysc}$ and are yield σ_{ysc} stresses in steel in tension and compression, respectively, with appropriate partial safety factors. Stresses in concrete and reinforcing bars can be given as

$$\sigma_c = \begin{cases} 0.45f_{ck} & \alpha_3 > \alpha > \beta \\ 0.45f_{ck}\left\{1 - \left(\dfrac{0.002-\varepsilon}{0.002}\right)^2\right\} & \alpha_0 > \alpha > \alpha_3 \end{cases} \quad (5)$$

$$\sigma_s = \begin{Bmatrix} \sigma_{ysc} = 0.75f_y & \alpha_1 > \alpha > \beta \\ \varepsilon E_s & \alpha_2 > \alpha > \alpha_1 \\ -\sigma_{yst} = -0.87f_y & \alpha_2 > \alpha > \pi \end{Bmatrix} \quad (6)$$

The expressions for *Mu* and *Pu* are obtained as described in the following:

The area of the segment is $tr\,d\alpha$ and its moment of inertia is $tr^2\cos\alpha\,d\alpha$. Therefore, if ρ be the amount of steel reinforcement in the shell section (expressed as ratio of total cross-sectional area), axial force and moment due to stresses in concrete are

$$dP_{uc} = \sigma_c tr(1-\rho)d\alpha \cong \sigma_c tr\,d\alpha$$

$$dM_{uc} = \sigma_c tr^2\cos\alpha\,d\alpha$$

Similarly, the axial forces and moment due to stresses in steel reinforcements are

$$dP_{us} = \rho\sigma_s tr\,d\alpha$$

$$dM_{us} = \rho\sigma_s tr^2\cos\alpha\,d\alpha \quad (10)$$

Therefore, total axial force P_u can be obtained by summing as follows

$$P_u = P_{uc} + P_{us}$$
$$= 2\int_\beta^{\alpha_0}\sigma_c tr.d\alpha + 2\int_\beta^{\pi}\rho\sigma_s tr.d\alpha \quad (11)$$

The integrals of (11) can be computed using expressions as follows:

$$2\int_\beta^{\alpha_0}\sigma_c tr d\alpha = 2\int_\beta^{\alpha_3}(0.45f_{ck})tr d\alpha + 2\int_{\alpha_3}^{\alpha_0}0.45f_{ck}\left\{1-\left(\frac{0.002-\varepsilon}{0.002}\right)^2\right\}$$

$$= 0.9f_{ck}tr(\alpha_3 - \beta) + \frac{3.15f_{ck}tr}{\cos\beta - \cos\alpha_0}\begin{Bmatrix}\sin\alpha_0 - \sin\alpha_3 - \\ (\alpha_0 - \alpha_3)\cos\alpha_0\end{Bmatrix}$$
$$- \frac{2.78f_{ck}tr}{(\cos\beta - \cos\alpha_0)^2}\left\{0.5\begin{pmatrix}(\alpha_0 - \alpha_3) + \sin\alpha_0\cos\alpha_0 \\ -\sin\alpha_3\cos\alpha_3\end{pmatrix}\right\}$$
$$- \frac{2.78f_{ck}tr}{(\cos\beta - \cos\alpha_0)^2}\left\{2\begin{pmatrix}(\sin\alpha_0\cos\alpha_0 - \sin\alpha_3\cos\alpha_0) + \\ (\alpha_0 - \alpha_3)\cos^2\alpha_0\end{pmatrix}\right\} \quad \ldots\ldots (12)$$

$$2\int_\beta^{\pi}\rho\sigma_s tr.d\alpha = 2\int_\beta^{\alpha_1}\rho\sigma_{ysc}tr.d\alpha + 2\int_{\alpha_1}^{\alpha_2}\rho E_s\varepsilon_s tr.d\alpha +$$
$$2\int^{\pi}\rho(-\sigma_{yst})tr.d\alpha \quad (13)$$

$$= 2\rho\sigma_{ysc}tr(\alpha_1 - \beta) + \frac{0.007\rho E_s tr}{\cos\beta - \cos\alpha_0}\begin{pmatrix}\sin\alpha_2 - \sin\alpha_1 - \\ (\alpha_2 - \alpha_1)\cos\alpha_0\end{pmatrix},$$
$$2\rho\sigma_{yst}tr(\pi - \alpha_2)$$

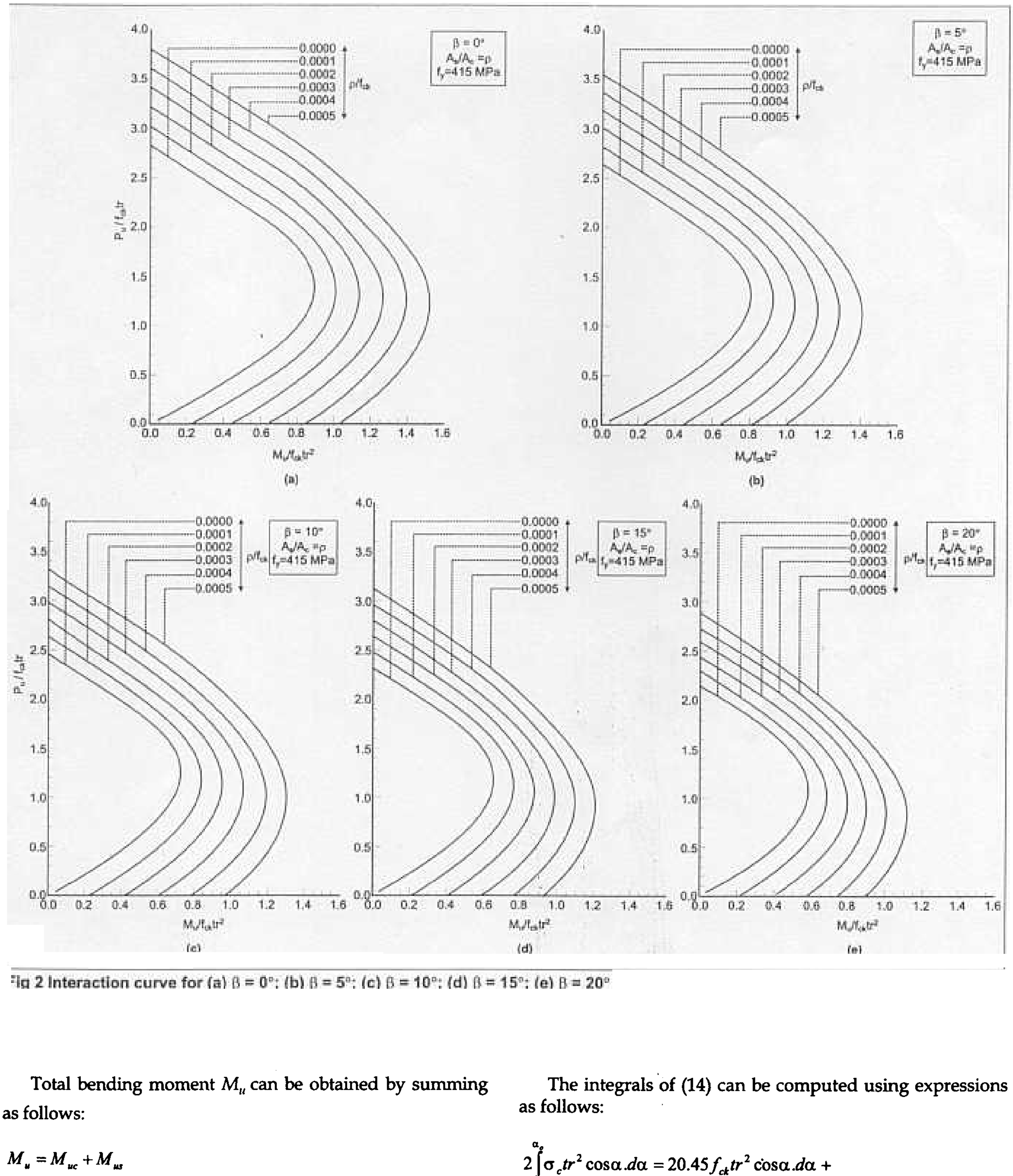

Fig 2 Interaction curve for (a) β = 0°; (b) β = 5°; (c) β = 10°; (d) β = 15°; (e) β = 20°

Total bending moment M_u can be obtained by summing as follows:

$$M_u = M_{uc} + M_{us} = 2\int_{\beta}^{\alpha_0}\sigma_c tr^2\cos\alpha.d\alpha + 2\int_{\beta}^{\pi}\rho\sigma_s tr^2\cos\alpha.d\alpha \tag{14}$$

The integrals of (14) can be computed using expressions as follows:

$$2\int^{\alpha_0}\sigma_c tr^2\cos\alpha.d\alpha = 20.45 f_{ck} tr^2\cos\alpha.d\alpha + 9 f_{ck} tr^2\int_{\alpha_3}\left[7\frac{(\cos\alpha - \cos\alpha_0)}{(\cos\beta - \cos\alpha_0)}\cos\alpha - 6.125\frac{(\cos\alpha - \cos\alpha_0)^2}{(\cos\beta - \cos\alpha_0)^2}\right].d\alpha$$

$$=0.9f_{ck}tr^2(\sin\alpha_3-\sin\beta)+$$
$$-\frac{3.15f_{ck}tr^2}{\cos\beta-\cos\alpha_0}\{0.5((\alpha_0-\alpha_3)+\sin\alpha_0\cos\alpha_0)\}$$
$$-\frac{3.15f_{ck}tr^2}{\cos\beta-\cos\alpha_0}\left\{\begin{array}{l}0.5(\sin\alpha_3\cos\alpha_3)+\\ \cos\alpha_0(\sin\alpha_0-\sin\alpha_3)\end{array}\right\}$$
$$-\frac{2.78f_{ck}tr^2}{(\cos\beta-\cos\alpha_0)^2}\left\{\begin{array}{l}(\sin\alpha_0-\sin\alpha_3)-\\ 0.333(\sin^3\alpha_0-\sin^3\alpha_3)\end{array}\right\}$$
$$\frac{2.78f_{ck}tr^2}{(\cos\beta-\cos\alpha_0)^2}\left\{\cos\alpha_0\left|\begin{array}{l}(\alpha_0-\alpha_3)+\\ \sin\alpha_0\cos\alpha_0-\\ \sin\alpha_3\cos\alpha_3+\\ \cos^2\alpha_0(\sin\alpha_0-\sin\alpha_3)\end{array}\right|\right\} \quad (15)$$

$$2\int_{\beta}^{\pi}\rho\sigma_s tr^2\cos\alpha.d\alpha=2\int_{\beta}^{\alpha_1}\rho\sigma_s tr^2\cos\alpha.d\alpha+$$
$$2\int_{\alpha_1}^{\alpha_2}\rho\sigma_s tr^2\cos\alpha.d\alpha+2\int_{\alpha_2}^{\pi}\rho\sigma_s tr^2\cos\alpha.d\alpha$$

$$=2\rho\sigma_{ysc}tr^2(\sin\alpha_1-\sin\beta)+\frac{0.007\rho E_s tr^2}{\cos\beta-\cos\alpha_0}\times$$
$$\{0.5((\alpha_2-\alpha_1)+\sin\alpha_2\cos\alpha_2-\sin\alpha_1\cos\alpha_1)\}$$
$$-\frac{0.007\rho E_s tr^2}{\cos\beta-\cos\alpha_0}\{0.5\cos\alpha_1(\sin\alpha_2-\sin\alpha_1)\}$$
$$-2\rho\sigma_{yst}tr^2\sin\alpha_2$$

16)

The total moment capacity M_u can be obtained from equations (14) to (16). For a given cross-section, various values of M_u are obtained by varying αo which determines the location of the neutral axis. For a given section the envelope of ultimate resistance can be presented in the form of an interaction plot with $M_u/(tr^2f_{ck})$ as abscissa and P_u / (trf_{ck}) as ordinate.

Case 2: Neutral axis outside the section

Under this condition the entire section is in compression and neutral axis in this case has moved through $\alpha_0 = \beta$ to π and further to produce a uniform concrete stress block. At this stage, the stress in the concrete is $0.45f_{ck}$ and that in the reinforcement is σ_{ysc}. In this case relationship between M_u / (tr^2f_{ck}) and $P_u/(trf_{ck})$ can be assumed to be linear and thus the point given by a uniform compressive stress block is joined by a straight line to the point corresponding to α_0 =180°, for which the *Mu* and *Pu* are obtained as below:

$$=2(\pi-\beta)\,rt\,(0.45f_{ck}+\rho\sigma\ \ ) \quad (17)$$

ınd

$$M_u=-\int_{-\beta}^{+\beta}(0.45f_{ck}+\rho\sigma_{ysc})tr^2\cos\varphi\,d\varphi \text{ where } \varphi<\beta$$

$$=-2(0.45f_{ck}+\rho\sigma_{ysc})tr^2\sin\beta \quad (18)$$

The above equations can be easily programmed for numerical computation of values P_u and M_u which defines the interaction curve for a given thin hollow circular RC section with an opening. Interaction curves for typical values of β are plotted for easy reference in *Fig* 2, for various amount of reinforcement of f_y = 415 MPa. A computer program in FORTRAN language was developed to generate the interaction curves using above equations with very small increments of variable α_0. However, it should be noted that these closed-form equations are immune from typical errors associated with numerical integration. The program calculates moment capacity of hollow circular section for various values of axial loads and given percentage of longitudinal steel in shell. Interaction curves are drawn by plotting several such points for different percentage of longitudinal steel in the section.

To illustrate the use of these interaction curves, consider an example of a chimney section with the following details: mean radius r = 3250 mm, thickness t = 210 mm, axial load P = 7,800 kN, bending moment due to lateral wind load M = 22,000 kNm, f_{ck} = 25 MPa, f_y = 415 MPa and β = 0°. This has been adapted from a solved example in Pinfold but with different material properties and load factors[7]. For load case of 0.9DL+1.5WL, factored loads are: axial load P_u = 0.9 × 7,800 = 7,000 kN, and moment M_u = 1.5 × 22,000 = 33,000 kNm. For these axial load and bending moment, eccentricity, $e = M_u/P_u$

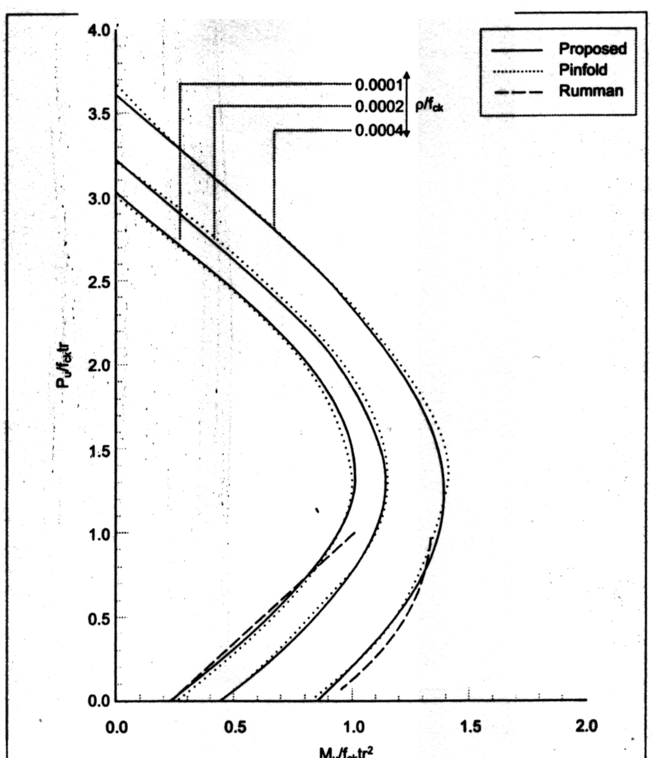

Fig 3 Comparison of proposed interaction curves reported by Pinfold and Rumman for β = 0°

= 4.7, which gives e/r = 1.45, implying presence of significant tensile stresses. Further, for $P_u/f_{ck}tr$ = 0.41, and $M_u/f_{ck}tr^2$ = 0.6, ρ/f_{ck} is read as 0.001 from *Fig 2(a)*, which means that amount of longitudinal reinforcement that need to be provided in the shell section is 0.25 percent.

Validation of interaction curves

For validation purposes, the P-M interaction curves developed and presented above were compared with similar curves available in the literature. Two such studies have been chosen for the comparison: (a) interaction curves developed by Rumman which form the basis of American code ACI 307R for chimneys and (b) the interaction curves developed by Pinfold[7] using material models of CP 110[8,9,10]. As shown in *Fig* 3, interaction curves are plotted for a case with no opening and with three different longitudinal steel reinforcements. It is clear that the proposed curves have a good match with those developed by Pinfold and Rumman. The simplified curves developed by Rumman are applicable for normalized axial load ($P_u/f_{ck}tr$) less than one which are typical for real life designs. The approximate expression of Rumman was modified for concrete cube strength from original cylinder strength and a recommended strength reduction factor of 0.9 was used.

Indian standard IS 11628 : 1985 prescribes a procedure to check safety of circular hollow sections under combined action of axial force and moment, which is based on the working stress design method using a cracked section analysis. Using the code equations, P-M interaction curves can be developed

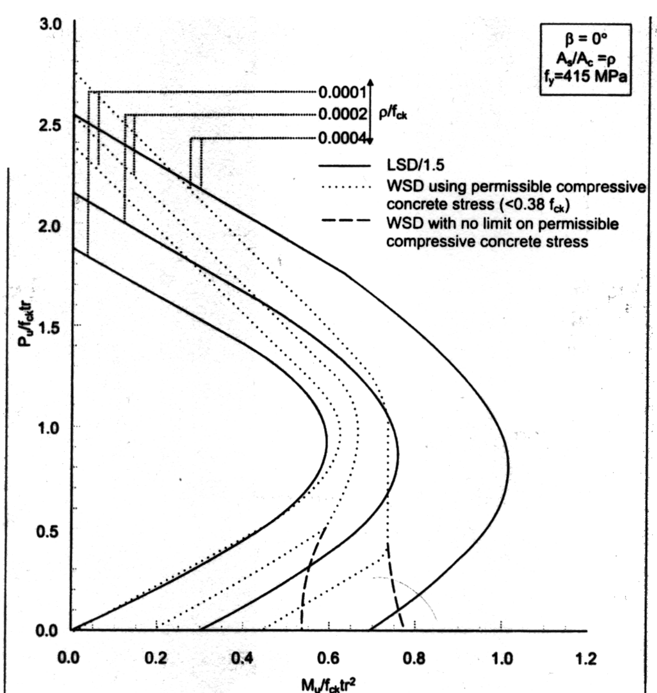

Fig 4 Comprison of proposed LSD and WSD (IS:11682) interaction curves for β = 0°

easily representing the permissible strength of such sections. These curves have been generated and compared in *Fig* 4 with those developed for ultimate (strength) conditions as described above for 'no opening' case (β = 0°). The permissible stresses for concrete and steel were taken as 0.38 f_{ck} and 0.57 f_y, respectively, as specified in the code for load combination of dead and wind loads. Additional curves were also developed in which yielding in steel reinforcement is permitted by not limiting the permissible compressive concrete stress as required by IS 11628 : 1985.

For comparison, the ultimate strength curves of *Fig 2(a)* have been reduced by an average load factor of 1.5 to bring it to working load levels. As expected, the differences in no reinforcement case is negligible for tension controlled region. However, an increased strength for the WSD method is observed in compression controlled region due to higher permissible concrete compressive stress, that is, 0.38 f_{ck} in WSD against 0.3 f_{ck} (= 0.45 f_{ck} /1.5) in LSD. However, for higher reinforcement cases, the WSD curves provide lower strength especially in the tension controlled region because of very low limit on the steel tensile strain. As a result, the advantage of steel reinforcement is not fully utilised in estimating the strength, which, in contrast, is appropriately accounted for in the LSD method.

Conclusion

Thin hollow circular sections typically used for shaft type staging of elevated tanks, other tower structure and chimneys should be designed by ultimate strength (limit state design) method instead of currently used working stress method to ensure higher reliability against collapse and possible reduction of overall cost. Moreover, the IS 456 : 2000 requires members subjected to combined bending moment and axial force to be designed by limit state method using appropriate load factors. This becomes even more necessary when the bending moment is due to horizontal loads, such as earthquakes which can result in significantly large bending moment demands. Ultimate flexural strength analysis of RC circular hollow sections described in this paper can be followed for the purpose of analysis, and for design purposes a suitable combination of load factors and strength reduction factors need to be considered to achieve an acceptable margin of safety. In order to minimise cumbersome and iterative calculations, the flexural strength of typical sections has been presented in form of non-dimensional P-M interaction curves, which will be useful in the design offices. The interaction curves have been verified by comparing with those reported in the literature and developed with working stress design philosophy.

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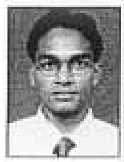

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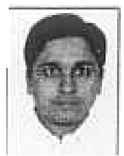

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