Seismic design of RC columns and wall sections: Part I – Consistent limit state design philosophy

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The philosophy of flexural limit state design of reinforced concrete sections as per the current Indian Standard is examined. A modification is proposed in the prescription of strain limit states to make the design procedure consistent with this philosophy. The new design P-M interaction curves are drawn as per this modification — for example, sections of columns and structural walls without boundary elements. The flexural capacity and curvature ductility of wall sections are over-estimated as per the provisions of the current Indian Standard; the extents of deviations depend on the sectional characteristics and interaction of various parameters.

Keywords: *Concrete, columns, walls, curvature, ductility, flexure, limit state design, seismic design*

The *Indian standard code of practice for plain and reinforced concrete*[1] considers six different limit states of collapses, namely *(a)* limit state of collapse in flexure, in shear, in compression, and in torsion and *(b)* limit states of serviceability in deflection, and in cracking. This study discusses only the limit state of collapse in flexure. In limit state design for flexure, reinforced concrete (RC) sections are said to have failed when either or both of the following limiting states are reached:

(i) longitudinal reinforcing steel reaching prescribed strain in tension, and

(ii) compression concrete reaching prescribed strain in combined axial bending compression.

These limiting states can be interpreted in different ways. This paper elaborates on these interpretations and highlights their implications.

The central theme of the companion papers is to arrive at RC columns and wall sections that need to resist seismic shaking in a ductile manner. Thus, the observations and conclusions of these companion papers are valid for such RC members particularly in seismic zones III, IV and V.

Limit state of collapse in flexure

IS 456 : 2000

The limit state design for flexure as enumerated in the code (Clause 38.1) is based on the following assumptions.

(i) Plane sections normal to the axis remain plane after bending.

(ii) The maximum strain in concrete at the outermost compression fibre in bending is 0.0035.

(iii) The relationship between the compressive stress distribution and the strain in concrete may be assumed to be rectangle, trapezoid, parabola or any other shape that results in prediction of strength in substantial agreement with test results. An acceptable stress-strain curve is given in *Fig* 1*(a)*. For design purposes, the compressive strength of concrete in bending is assumed to be 0.67 times the cube characteristic strength. A partial safety factor,

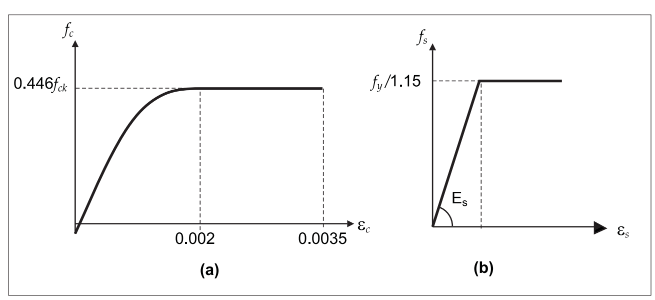

Fig 1 Code-specified stress-strain properties as per IS 456 : 2000[1] for obtaining the design flexural strength of RC wall section *(a)* concrete and *(b)* steel

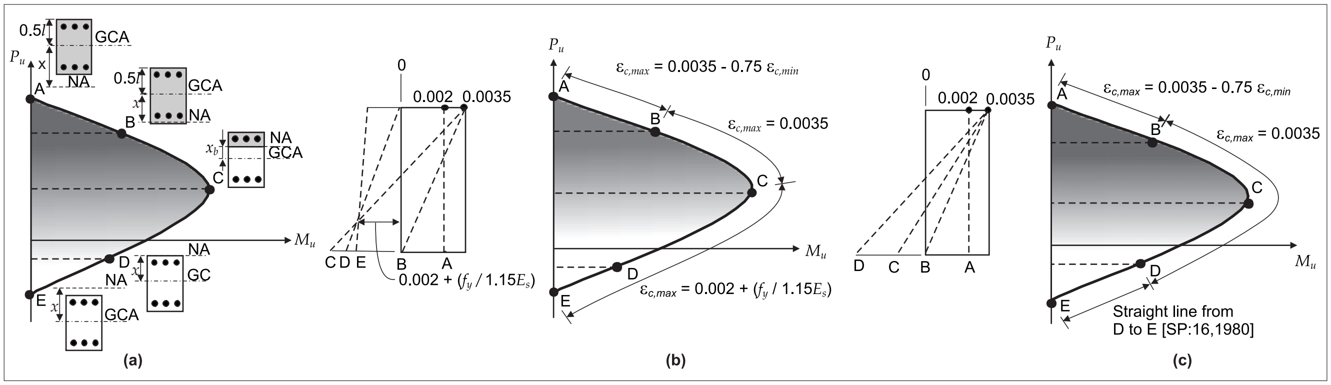

Fig 2 ***(a)*** **General states of a RC cross section under combined axial load and bending moment** ***(b)*** **limiting strains and strain distribution in a RC cross-section at different stages under combined axial load and bending moment, as per the proposed approach** ***(c)*** **limiting strains and strain distributions in a RC cross section at different states under combined axial load and bending moment, as per IS 456 : 2000**

γ_m, of 1.5 is to be applied on the characteristic strength of concrete in addition to the 0.67 factor stated above.

(iv) The tensile strength of concrete is ignored.

(v) The stresses in the reinforcement are derived from representative stress-strain curves. A typical curve is given in *Fig* 1*(b)*. For design purposes, the partial safety factor, γ_m, of 1.15 is to be applied on the characteristic strength of steel.

(vi) The maximum strain in tension reinforcement in the section at failure is to be not less than 0.002 + (f_y/1.15 E_s).

For limit state of collapse in compression, in addition to the above six assumptions, two additional assumptions are also stated in the code Clause 39.1. These are:

(vii) The limiting compressive strain in concrete in axial compression is 0.002.

(viii) The limiting compressive strain, $\varepsilon_{c,max}$, at the highly compressed extreme fibre in concrete subjected to combined axial compression and bending and when there is no tension on the section is

$$\varepsilon_{c,max} = 0.0035 - 0.75\varepsilon_{c,min} \quad ...(1)$$

where,

$\varepsilon_{c,min}$ = the strain in the least compressed fibre.

There are shortcomings in some of the above assumptions which are discussed.

(i) The limit state for collapse in flexure on the tension side is not adequately specified. Currently, a minimum strain in the extreme layer of steel on the tension side is specified in assumption *(vi)*. This implies that when tension steel is placed in layers, the steel in the interior layer may not reach the design stress of (f_y/1.15E_s) Also, by not specifying a limiting maximum strain for steel, it is not clear as to when the limit state of failure of steel is said to have been reached.

(ii) Currently, the limiting maximum compressive strain in concrete of 0.0035 is enforced to be valid for all types of designed sections, that is, for under-reinforced sections as well as for over-reinforced sections. The Indian Standard Explanatory Handbook[2] for IS 456 : 1978 and other popular textbooks of RC design[3,4,5] consider that limit state of collapse in flexure is attained only when the extreme fibre of concrete reaches the strain of 0.0035, irrespective of whether the section is under-reinforced or over-reinforced. In highly under-reinforced sections, constraining the maximum compressive strain in concrete to 0.0035 imposes unduly large curvatures owing to conditions of compatibility and equilibrium. Such large curvatures

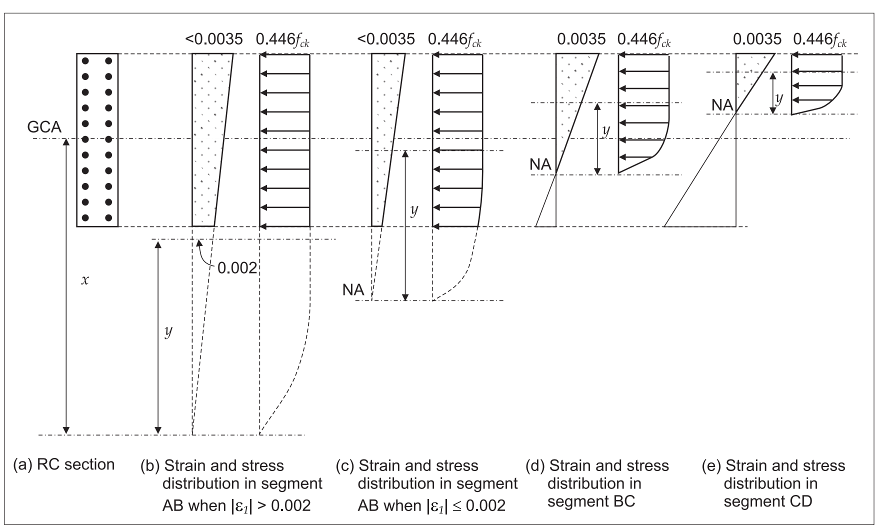

Fig 3 Progressive movement of neutral axis into the section, associated strain and stress variations, and the limiting strain in concrete as per IS 456 : 2000 approach

Table 1: Anomalies in flexural strength values reported in SP 16

Section size, mm	Longitudinal reinforcement: Bars	p_t, percent	Distribution	Bar diameter as per SP:16, mm	$M_u/f_{ck}bD^2$ at $P_u=0$: IS 456:2000 philosophy: Actual no. of bars (1)	No. of bars as per SP:16 (2)	Values reported in Handbook Charts of SP:16 (3)	Error $\frac{((3)-(1))}{(1)}\times 100$
300×300	12Y12	1.51	Equal on four	9.291	0.095	0.094	0.095	0
	12Y16	2.67	opposite sides	12.361	0.154	0.152	0.155	0.65
	12Y20	4.18		15.488	0.219	0.218	0.225	2.74
450×450	28Y12	1.56	Equal on four	14.192	0.106	0.106	0.108	1.89
	28Y16	2.77	opposite sides	18.881	0.175	0.173	0.178	1.71
	28Y20	4.34		23.658	0.257	0.254	0.250	-2.72
650×650	36Y12	0.96	Equal on four	16.069	0.073	0.073	0.078	6.85
	36Y16	1.70	opposite sides	21.383	0.121	0.120	0.123	1.65
	36Y20	2.67		26.849	0.179	0.178	0.183	2.23
	36Y25	4.18		33.571	0.264	0.262	0.270	2.27
750×750	48Y12	0.96	Equal on four	18.541	0.074	0.074	0.081	9.46
	48Y16	1.71	opposite sides	24.673	0.124	0.122	0.130	4.84
	48Y20	2.68		30.979	0.183	0.181	0.182	-0.55
	48Y25	4.19		38.735	0.270	0.266	0.258	-4.44

may not be practically attainable in the commonly built RC sections.

(*iii*) Further, the ultimate moment capacity, M_u, of RC sections is incorrectly estimated in all under-reinforced sections; the error will depend on the section geometry, the amount of reinforcing steel and the distribution of steel within the cross-section.

International codes of practice

In seismic design of RC sections, the following strategies are adopted by international codes of practice in the limit state design process to achieve ductile behaviour:

(*i*) a limit state of tensile strain in steel is specified for longitudinal reinforcement (the strain in steel has to be at least this value)

(*ii*) an upper limit is placed on the amount of longitudinal steel

(*iii*) an upper limit is placed on depth of neutral axis (NA) of the section.

The limit states of strain as per the various codes of practice are the main focus of the companion paper[6] and hence not discussed here. On the other hand, with regards to the upper limit on the axial force in the RC section, ACI-318[7] restricts the axial compressive stress in flexural members to $0.1\ f_c' A_g$, where f_c' is the specified compressive strength of concrete in MPa and A_g the gross section area in mm^2. In the same code, a minimum tensile strain of 0.005 is required in the tensile steel when the section is a tension-controlled one (that is, one in which the steel reaches the limit state and not the extreme fibre of concrete). This strain limit in the tension steel indirectly restricts the tensile steel to 75 percent of the area required for balanced sections[7,8]. The EuroCode EC2[9] also recommends an upper limit on the axial load of $0.35 A_c f_{cd}$ (f_{cd} being the design concrete strength) in RC columns that are required to posses high ductility. NZS 3101[10] prescribes a maximum neutral axis depth of 75 percent of the balanced depth in the beam section during the development of nominal flexural resistance. The American bridge design specifications of AASHTO[11] recommends a restriction on the axial compression in bridge columns of $0.2\ f_c' A_g$ since ductility capacity is reduced with increased compressive axial force.

Proposed limit state design of collapse in flexure

The following are the proposed assumptions and considerations in identifying the limit state of collapse in flexure.

(*i*) Plane sections normal to the axis before bending remain plane after bending, that is, the strain variation across the section is linear

(*ii*) In over-reinforced and balanced sections subjected to bending, the limit state of collapse in flexure is said to be reached when the strain in the extreme compression fibre of concrete reaches the limiting value 0.0035

(*iii*) The stress-strain curve of concrete in compression to be used in design is given in *Fig* 1*(a)*, where the flexural compressive strength is taken as 0.67 times the characteristic cube compressive strength, f_{ck}. The design flexural compressive strength of concrete is obtained as $0.67 f_{ck}$ divided by partial safety factor, γ_m, for concrete strength of 1.5.

(*iv*) The tensile strength of concrete is ignored.

(*v*) The stress-strain curves of mild steel and HYSD bars in tension and compression to be used in design are the same as given in IS 456 : 2000, in which f_y is the characteristic strength of steel. The design strength of steel is obtained as f_y divided by partial safety factor for steel of 1.15.

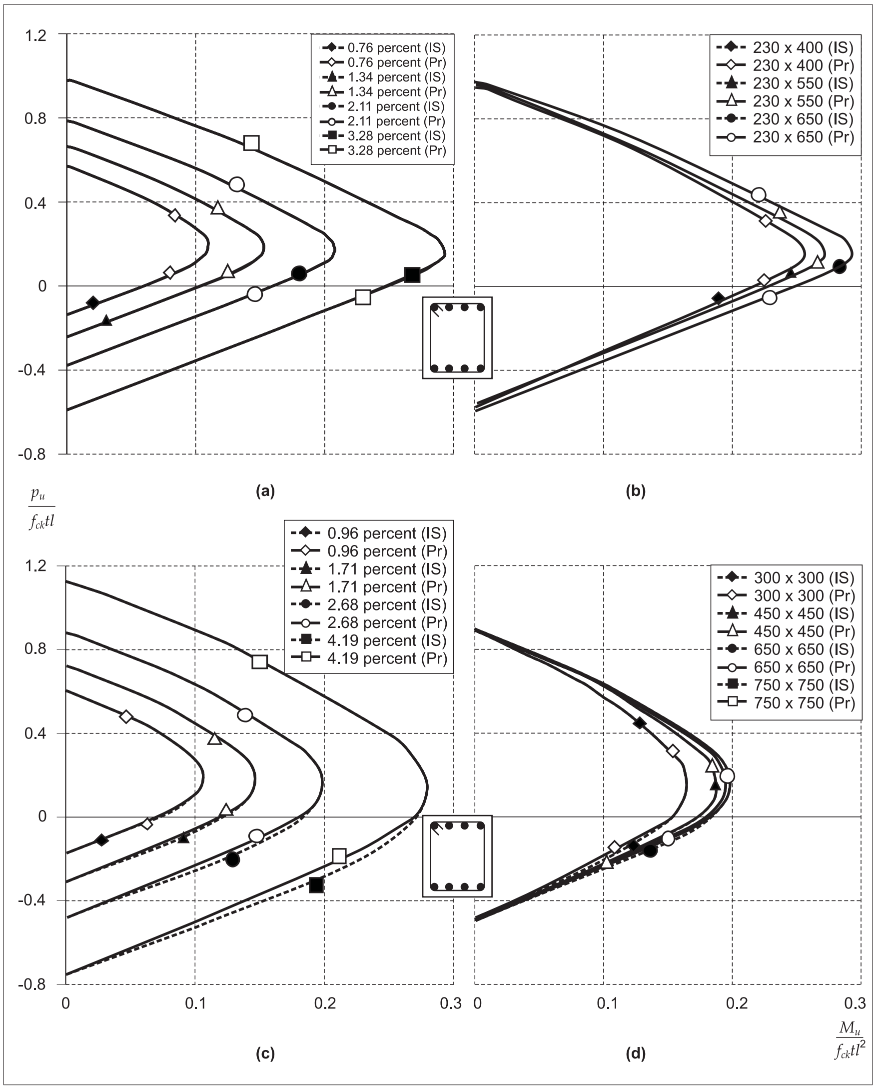

Fig 4 P_u - M_u **curves as per IS 456 : 2000 and proposed approaches** ***(a)*** **230 x 650 mm for all percentages of steel** ***(b)*** **230 x 400 mm for 3.2 percentage steel, 230 x 550 mm for 3.11 percentage steel and 230 x 650 mm for 3.28 percentage steel** ***(c)*** **750 x 750mm for all percentages of steel** ***(d)*** **300 x 300 mm, 450 x 450 mm, 550 x 550 mm and 750 x 750 mm for 2.7 percentage steel**

(*vi*) In under-reinforced sections subjected to bending, the limit state of collapse is said to be reached in flexure when the strain in the extreme layer of steel on tension side reaches the strain value of 0.002+ (f_y/1.15E_s).

For identifying the limit state of collapse in combined flexure and compression, the following additional assumptions and considerations are proposed.

(*vii*) Under uniform compression, the limit state of collapse is said to be reached when the strain in concrete reaches the limiting value 0.002.

(*viii*) Under combined flexure and compression if no tension is developed in the cross-section, the strain, $\varepsilon_{c,max}$, in the concrete fibre with maximum compression is related to the strain, $\varepsilon_{cmin,}$ in the concrete fibre with minimum compression by the Equation (1).

Most of these proposed assumptions are the same as in the current concrete code IS 456 : 2000. Some ambiguities are removed by re-drafting the text. But, in item (*vi*), a significant departure has been made. While 0.002 + (f_y/1.15E_s) is specified as the minimum strain in the extreme layer of steel in tension in the current code, the same strain is taken as the limiting maximum value in the extreme layer of steel in the proposed method. Thus, a limiting value for tensile strain in steel is specified. This value may be small for being the upper limit for tension steel. An alternate and more uniform specification for specifying the limiting state for tension steel for different cross-sections (for example, for both shallow beams and deep wall sections) can be through a limiting curvature of the section. Depending on the structural members under consideration, different values of limiting curvature may be required. This aspect needs further investigation.

The design P_u - M_u interaction curve of a typical RC section consists of four segments, namely the pure compression region AB with compression failure, the high moment region BC with compression failure, the high moment region CD with tension failure, and the pure tension region DE with tension failure, *Fig* 2*(a)*. For points on segment AB, the whole cross section is under compression. The failure is due to compressive strain of concrete reaching the limiting value, $\varepsilon_{c,max}$, given by Equation (1). The position of the neutral axis (NA) changes from a very large distance outside the section to the edge of the section on the tension face, that is, $-\infty < x \leq -0.5\,l$, where x is the distance of the NA from the geometric centroidal axis (GCA) of the section. For points on segment BC, the tension zone moves into the cross section, varying from the edge of the tension face to the balanced depth, that is, $-0.5l \leq x \leq x_b$. The failure is still due to compressive strain in concrete reaching the limiting value $\varepsilon_{c,max}$, which is now a fixed value of 0.0035 and not dependent on the strain at the tension face.

For points on segment CD, as per the proposed method, the failure is due to tensile strain in steel reaching the limiting value of 0.002 + (f_y/1.15E_s), *Fig* 2*(b)*. However, IS 456 : 2000

Table 2: Size, reinforcement details and capacities of column sections as per IS 456:2000 and proposed approaches.

Section size, mm	*Longitudinal reinforcement*			*$M_u/f_{ck}bD^2$ at P_u=0*			*Curvature, ϕ_{max}*			*Curvature ductility, μ_ϕ*		
	Bars	*p_t, percent*	*Distribution*	*IS 456 : 2000*	*Proposed*	*Error, percent w.r.t. proposed*	*IS 456:2000 (10^{-4}/mm)*	*Proposed (10^{-7}/mm)*	*Ratio*	*IS 456:2000*	*Proposed*	*Ratio*
230×400	6Y12	0.74		0.054	0.053	1.89	1.2	7.529	159	15.8	1.75	9.0
	6Y16	1.30	Equal on two	0.093	0.092	1.09	1.9	7.532	252	15.8	1.75	9.0
	6Y20	2.05	opposite sides	0.142	0.141	0.71	2.0	7.535	265	16.5	1.74	9.5
	6Y25	3.20		0.216	0.215	0.47	2.1	7.539	278	17.5	1.73	10.0
230×550	8Y12	0.71		0.056	0.055	1.82	3.1	7.455	416	37.7	1.79	21.0
	8Y16	1.26	Equal on two	0.096	0.095	1.05	3.3	7.458	442	40.9	1.79	23.0
	8Y20	1.99	opposite sides	0.148	0.147	0.68	1.8	7.461	241	21.9	1.78	12.3
	8Y25	3.11		0.227	0.226	0.44	1.9	7.465	255	23.1	1.75	13.2
230×650	10Y12	0.76		0.059	0.058	1.72	1.8	7.455	241	27.0	1.75	15.4
	10Y16	1.34	Equal on two	0.104	0.103	0.97	1.9	7.458	255	28.2	1.75	16.1
	10Y20	2.11	opposite sides	0.161	0.160	0.63	2.0	7.611	263	24.3	1.69	15.2
	10Y25	3.28		0.249	0.248	0.40	2.1	7.465	281	31.6	1.79	17.7
300×300	12Y12	1.51		0.095	0.093	2.15	2.9	7.681	378	17.8	1.71	10.4
	12Y16	2.67	Equal on four	0.154	0.151	1.99	3.3	7.684	430	19.1	1.69	11.3
	12Y20	4.18	opposite sides	0.219	0.216	1.39	1.8	7.687	234	10.2	1.69	6.0
	12Y25	6.55		0.309	0.306	0.98	1.9	7.691	247	10.7	1.68	6.4
450×450	28Y12	1.56		0.106	0.102	3.92	2.5	7.604	329	25.3	1.77	14.3
	28Y16	2.77	Equal on four	0.175	0.172	1.74	3.4	7.607	447	31.9	1.77	18.0
	28Y20	4.34	opposite sides	0.257	0.256	0.39	2.8	7.611	368	26.9	1.76	15.3
	28Y25	6.79		0.373	0.372	0.27	3.0	7.614	394	28.9	1.75	16.5
650×650	36Y12	0.96		0.073	0.068	7.35	1.8	7.455	241	26.9	1.81	14.9
	36Y16	1.70	Equal on four	0.121	0.117	3.42	1.9	7.458	255	28.3	1.80	15.7
	36Y20	2.67	opposite sides	0.179	0.175	2.29	2.0	7.461	268	29.6	1.79	16.5
	36Y25	4.18		0.264	0.262	0.76	2.1	7.465	281	31.6	1.79	17.7
750×750	48Y12	0.96		0.074	0.068	8.82	2.0	7.312	274	35.4	1.82	19.5
	48Y16	1.71	Equal on four	0.124	0.119	4.20	2.1	7.314	287	37.4	1.812	0.7
	48Y20	2.68	opposite sides	0.183	0.179	2.24	2.3	7.318	314	39.5	1.81	21.9
	48Y25	4.19		0.270	0.268	0.76	2.5	7.321	341	42.6	1.81	23.5

still insists on the failure being by the compressive strain in concrete reaching the limiting value of 0.0035, *Fig* 2*(c)*. The neutral axis reaches the edge of the compression face at point D, that is, the range of x for points on segment CD is $x_b \leq x \leq 0.5l$. For points on segment DE, the whole cross section is under tension. The depth of NA, x is in the range $0.5l \leq x \leq \infty$. IS 456:2000 does not prescribe any strain distribution for this region. But in the proposed method, the failure is said to occur when tensile strain in extreme layer of steel reaches the limiting value of $0.002+(f_y/1.15E_s)$.

The above prescription of limiting states ensures consistency in the failure criterion. In segments AB and BC, the failure is due to compressive strain in concrete reaching the limiting value, and in segments CD and DE, it is due to tensile strain in steel reaching the limiting value, *Fig* 2*(b)*. The demarcation between these two failure modes is point C, the balanced point, where failure occurs by both compressive strain in concrete and tensile strain in steel reaching the corresponding limiting values simultaneously. This is in contrast to the current code interpretation, where even in segment CD, the failure is due to compressive strain in concrete reaching the limiting value of 0.0035, *Fig* 2*(c)*.

P_u – M_u and P_u – ϕ_{max} curves for general rectangular section

Based on the existing limit state design philosophy[1] and the proposed philosophy, expressions for the design axial load, P_u the design bending moment, M_u, and the maximum achievable curvature, ϕ_{max}, were derived for RC sections.

IS 456:2000 approach

The variation in strain across the cross-section in these four regions is shown in *Fig* 3.

For segment AB,

$$P_{u,AB} = P_c + \sum_{i=1}^{nw} A_{si}\sigma_{si} \qquad ...(2)$$

$$M_{u,AB} = M_c + \sum_{i=1}^{nw} A_{si}\sigma_{si}\left(0.5l - y_{si}\right) \text{ and} \qquad ...(3)$$

$$\phi_{max,\,AB} = \frac{|\varepsilon_1|}{(-0.5l - x)} \qquad ...(4)$$

where,

Y_{si} = location of the steel bar from the edge of compression face of the section

A_{si} = its area

x = depth of neutral axis from the geometric centroidal axis

l = length of the wall

t = thickness of the wall

f_{ck} = grade of concrete.

$$P_c = \begin{cases} -0.446 f_{ck} lt & for|\varepsilon_1| \geq 0.002 \\ -0.446 f_{ck}(0.5l - y)t + f_c t(0.5l + y) & for|\varepsilon_1| < 0.002 \end{cases} \qquad ...(5)$$

Table 3: Size, reinforcement details and capacities of wall sections as per IS 456:2000 and proposed approaches

Section size, mm	Longitudinal reinforcement: Diameter, mm	Longitudinal reinforcement: Spacing, mm	Longitudinal reinforcement: p_t, percent	$M_u/f_{ck}bD^2$ at $P_u=0$: IS 456:2000	$M_u/f_{ck}bD^2$ at $P_u=0$: Proposed	$M_u/f_{ck}bD^2$ at $P_u=0$: Error, percent w.r.t. proposed	Curvature, ϕ_{max}: IS 456:2000 (10^{-4}/mm)	Curvature, ϕ_{max}: Proposed (10^{-7}/mm)	Curvature, ϕ_{max}: Ratio	Curvature ductility, μ_ϕ: IS 456 : 2000	Curvature ductility, μ_ϕ: Proposed	Curvature ductility, μ_ϕ: Ratio
230×1500	12	280	0.39	0.031	0.027	14.82	2.105	6.851	307	77.5	1.85	42
	12	200	0.52	0.042	0.037	13.51	2.105	6.851	307	77.5	1.85	42
	12	140	0.72	0.052	0.048	8.33	2.105	6.851	307	77.5	1.85	42
	12	100	0.98	0.066	0.062	6.45	2.105	6.851	307	77.5	1.85	42
	16	280	0.69	0.054	0.049	10.20	2.235	6.854	326	82.1	1.85	44
	16	200	0.93	0.066	0.061	8.20	2.235	6.854	326	82.1	1.85	44
	16	140	1.28	0.084	0.079	6.33	2.235	6.854	326	82.1	1.85	44
	16	100	1.74	0.106	0.102	3.92	2.235	6.854	326	82.1	1.85	44
	20	280	1.09	0.077	0.072	6.94	2.376	6.856	347	87.1	1.85	64
	20	200	1.46	0.096	0.091	5.50	2.376	6.856	347	87.1	1.85	64
	20	140	2.00	0.120	0.116	3.45	2.376	6.856	347	87.1	1.85	64
	20	100	2.73	0.151	0.149	1.34	2.376	6.856	347	87.1	1.85	64
230×2500	12	280	0.35	0.027	0.024	12.50	2.014	6.286	320	115.5	1.86	62
	12	200	0.51	0.039	0.035	11.43	2.014	6.286	320	115.5	1.86	62
	12	140	0.71	0.051	0.046	10.87	2.014	6.286	320	115.5	1.86	62
	12	100	0.98	0.067	0.062	8.07	2.014	6.286	320	115.5	1.86	62
	16	280	0.63	0.045	0.041	9.76	2.369	6.288	377	148.0	1.86	79
	16	200	0.90	0.064	0.059	8.48	2.369	6.288	377	148.0	1.86	79
	16	140	1.25	0.082	0.077	6.49	2.369	6.288	377	148.0	1.86	79
	16	100	1.74	0.106	0.102	3.92	2.369	6.288	377	148.0	1.86	79
	20	280	0.98	0.064	0.060	6.67	2.534	6.289	403	158.0	1.86	85
	20	200	1.42	0.092	0.087	5.75	2.534	6.289	403	158.0	1.86	85
	20	140	1.97	0.117	0.113	3.54	2.534	6.289	403	158.0	1.86	85
	20	100	2.73	0.151	0.148	2.03	2.534	6.289	403	158.0	1.86	85
230×3500	12	280	0.36	0.029	0.026	11.54	2.358	5.806	406	208.2	1.87	111
	12	200	0.51	0.039	0.035	11.43	2.358	5.806	406	208.2	1.87	111
	12	140	0.70	0.050	0.046	8.70	2.358	5.806	406	208.2	1.87	111
	12	100	0.98	0.067	0.062	8.07	2.358	5.806	406	208.2	1.87	111
	16	280	0.65	0.048	0.044	9.09	2.521	5.808	434	222.3	1.87	119
	16	200	0.89	0.063	0.058	8.62	2.521	5.808	434	222.3	1.87	119
	16	140	1.24	0.081	0.076	6.58	2.521	5.808	434	222.3	1.87	119
	16	100	1.74	0.106	0.102	3.92	2.521	5.808	434	222.3	1.87	119
	20	280	1.01	0.070	0.065	7.69	2.709	5.809	466	238.7	1.87	127
	20	200	1.40	0.091	0.086	5.81	2.709	5.809	466	238.7	1.87	127
	20	140	1.95	0.115	0.111	3.61	2.709	5.809	466	238.7	1.87	127
	20	100	2.73	0.151	0.148	2.03	2.709	5.809	466	238.7	1.87	127

$$M_c = \begin{Bmatrix} -0.446 f_{ck}[0.5l] & for\left|\varepsilon_1\right| \geq 0.002 \\ -0.446 f_{ck} t(0.5l - y)[0.5(0.5l + y)] - f_c t(0.5l + y)[x_p - y] & \\ & for\left|\varepsilon_1\right| < 0.002 \end{Bmatrix} \qquad ...(6)$$

$$y = x + 0.002\left(\frac{0.5l - x}{\left|\varepsilon_1\right|}\right) \qquad ...(7)$$

where,

σ_{si} = Stress in steel corresponding to strain, e_{si}, from *Fig* 1*(b)*

$$\varepsilon_{si} = \text{Strain in steel} = \varepsilon_1\left(\frac{0.5l - x - y_{si}}{0.5l - x}\right) \qquad ...(8)$$

$$\varepsilon_1 = -0.002\left[\frac{-0.5l - x}{\left(\frac{0.5l}{7} - x\right)}\right] \qquad (9)$$

For the segment BC,

$$P_{u,BC} = -0.361 f_{ck}(0.5l - x)t + \sum_{i=1}^{nw} \sigma_{si} A_{si} \qquad ...(10)$$

$$M_{u,BC} = -0.361 f_{ck} t(0.5l - x)[0.416(0.5l - x)] + \sum_{i=1}^{nw} \sigma_{si} A_{si}(0.5l - y_{si}) \qquad ...(11)$$

and

$$\phi_{\max,BC} = \frac{0.0035}{(0.5l - x)} \qquad ...(12)$$

where,

σ_{si} = stress in steel corresponding to strain, ε_{si}, from *Fig* 1*(b)*

ε_{si} = strain in steel $= 0.0035\left(\frac{0.5l - x - y_{si}}{0.5l - x}\right)$ ...(13)

For segment CD,

$$P_{u,CD} = -0.361 f_{ck}(0.5l - x)t + \sum_{i=1}^{nw} \sigma_{si} A_{si} \qquad ...(14)$$

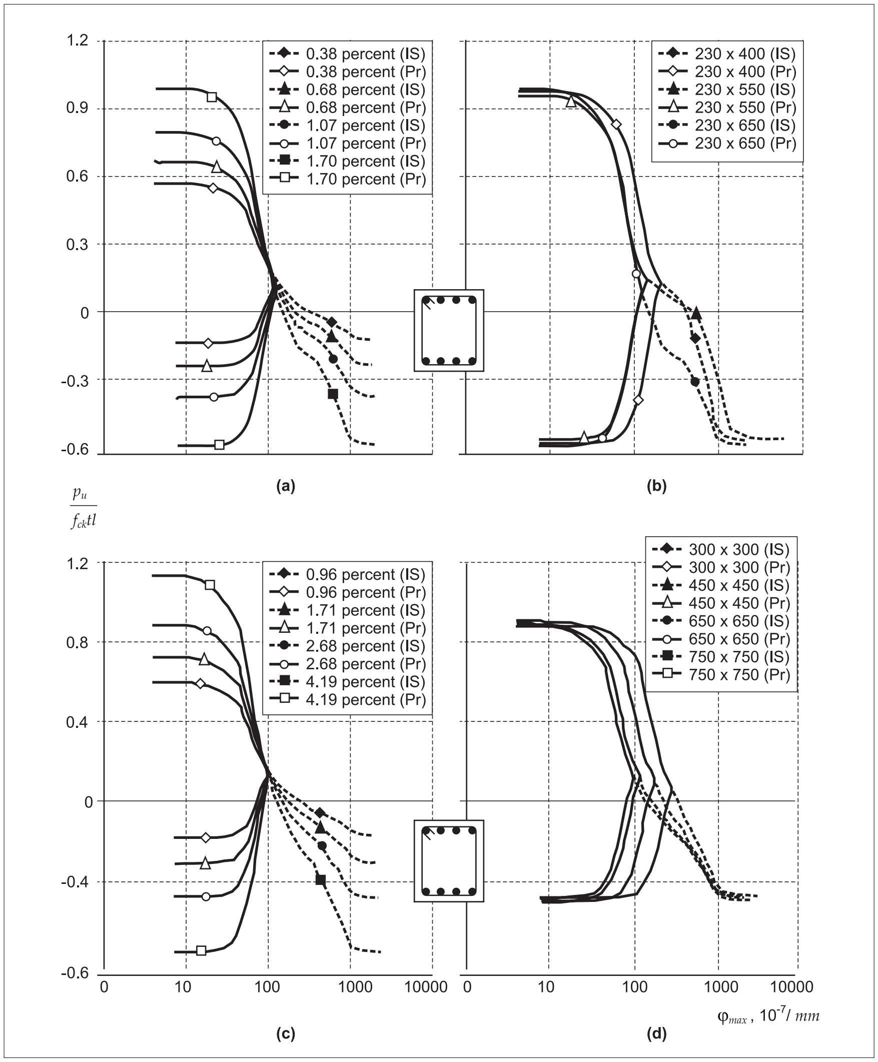

Fig 5 P_u - ϕ_{max} curves as per IS 456 : 2000 and proposed approaches *(a)* 230 x 650 mm for all percentages of steel *(b)* 230 x 400 mm, 230 x 550 mm and 230 x 650 mm for 3.3 percentage steel *(c)* 230 x 750 mm for all percentages of steel, and *(d)* 300 x 300 mm, 450 x 450 mm, 550 x 550 mm and 750 x 750 mm for 2.7 percentage steel

The expression for $\phi_{max,CD}$ was not applicable at point D (when the neutral axis lies on the compression edge of the section).

For segment DE,

$$(P_u)_{DE} = \sum_{i=1}^{nw} \sigma_{si} A_{si} \quad ...(18)$$

and

$$(M_u)_{DE} = \sum_{i=1}^{nw} \sigma_{si} A_{si} (0.5l - y_{si}) \quad ...(19)$$

For this segment, the maximum curvature could be determined based on the considerations of IS 456 : 2000.

Proposed approach

For segments AB and BC, the expressions were same as given earlier. In addition to the terms used earlier, ε_y refers to the yield strain of steel.

For segment CD,

$$P_{u,CD} = P_c + \sum_{i=1}^{nw} \sigma_{si} A_{si} \quad ...(20)$$

$$M_{u,CD} = M_c + \sum_{i=1}^{nw} \sigma_{si} A_{si} (0.5l - y_{si}) \quad ...(21)$$

and

$$\phi_{max,CD} = \frac{\varepsilon_y}{y_{sn} - 0.5l + x} \quad ...(22)$$

$$M_{u,CD} = -0.361 f_{ck} t (0.5l - x)[0.416(0.5l - x)] + \sum_{i=1}^{n} \sigma_{si} A_{si} (0.5l - y_{si}) \quad ...(15)$$

$$\phi_{max,CD,} = \frac{0.0035}{(0.5l - x)} \quad ...(16)$$

where,

σ_{si} = stress in steel corresponding to strain, ε_{si}, from *Fig* 1*(b)*

ε_{si} = strain in steel $= 0.0035 \left(\frac{0.5l - x - y_{si}}{0.5l - x} \right)$ ...(17)

where,

$$P_c = \begin{cases} -0.446 f_{ck} (0.5l - y)t - \frac{2}{3}(0.446 f_{ck})(y - x)t & for |\varepsilon_1| > 0.002 \\ f_c (0.5l - x)t & for |\varepsilon_1| > 0.002 \end{cases} \quad ...(23)$$

$$M_c = \begin{cases} -0.446 f_{ck} t \left[(0.5l - y) 0.5 (y + 0.5l) + \frac{2}{3}(y - x)\left(\frac{3}{8}x + \frac{5}{8}y \right) \right] \\ \qquad for |\varepsilon_1| > 0.002 \\ f_c t (0.5l - x)(x + y_a - x_p) \qquad for |\varepsilon_1| \le 0.002 \end{cases} \quad ...(24)$$

$$y = x + 0.002\left(\frac{0.5l - x}{|\varepsilon_1|}\right) \quad ...(25)$$

$$y_a = x + 0.002\left(\frac{0.5l - x}{|\varepsilon_1|}\right) \quad ...(26)$$

where,

σ_{si} = stress in steel corresponding to strain, ε_{si}, from *Fig* 1*(b)*

$$\varepsilon_{si} = \text{strain in steel} = \varepsilon_y\left(\frac{0.5l - x - y_{si}}{y_{sn} - 0.5l + x}\right) \quad ...(27)$$

$$\varepsilon_1 = \varepsilon_y\left(\frac{0.5l - x}{y_{sn} - 0.5l + x}\right) \quad ...(28)$$

For segment DE,

$$P_{u,DE} = \sum_{i=1}^{nw} \sigma_{si} A_{si} \quad ...(29)$$

$$M_{u,DE} = \sum_{i=1}^{nw} \sigma_{si} A_{si}(0.5l - y_{si}) \quad ...(30)$$

$$\phi_{\max,DE} = \frac{\varepsilon_y}{y_{sn} - 0.5l + x} \quad ...(31)$$

where,

$$\varepsilon_{si} = \varepsilon_y\left(\frac{x - 0.5l + y_{si}}{y_{sn} - 0.5l + x}\right) \quad ...(32)$$

σ_{si} = stress in steel corresponding to strain, ε_{si}

$$\varepsilon_{si} = \text{strain in steel} = \varepsilon_y\left(\frac{0.5l - x - y_{si}}{y_{sn} - 0.5l + x}\right) \quad ...(33)$$

Generating P_u – M_u and P_u – ϕ_{max} curves

The following step-wise procedure was adopted to generate the $P_u - M_u$ interaction diagrams and $P_u - \phi_{max}$ curves.

Step 1

The depth of neutral axis x was chosen. x was varied from $-\infty$ to $+\infty$. When x was in the range $[-5l, +5l]$, a large number of values of x were chosen to obtain the smooth $P_u - M_u$ and $P_u - \phi_{max}$ curves.

Step 2

The region in which x lies was identified, that is, $-\infty < x < -0.5l$ for segment AB, $-0.5l \le x < x_b$ for segment BC, $x_b \le x < +0.5l$ for segment CD and $+0.5l \le x < +\infty$ for segment DE.

Step 3

P_u, M_u and ϕ_{max} were calculated.

Step 4

P_u with $f_{ck}lt$ and M_u with $f_{ck}lt^2$ were normalised. This gave a point on the $P_u - M_u$ interaction curve and the P_u–ϕ_{max} curve.

Step 5

The value of x was changed and the procedure was repeated from Step 2, until the $P_u - M_u$ and P_u–ϕ_{max} curves were generated for all the regions AB, BC, CD and DE.

Numerical comparison

Example isolated RC column and wall sections were considered. The normalised P_u –M_u and P_u–ϕ_{max} curves for each of these sections were calculated as per the IS 456:2000 and proposed approaches, and compared.

Column sections

Seven rectangular RC column sections of different dimensions, percentages of longitudinal steel and the distribution of longitudinal steel were considered, *Table* 1. For all sections, the grades of concrete and steel used were M20 and *Fe* 415 respectively, and the clear cover to longitudinal reinforcement was 40 mm[1].

For the column sections with vertical steel distributed uniformly on four sides, *Table* 1 shows the flexural capacities by three approaches, namely

- calculations using steel as shown in *Tables* 1 and 2
- calculations using 20 bars (irrespective of their diameter to achieve the said percentage of steel) distributed all around the specimen as stated in section 3.2.3.3 of RC Design Handbook[2]
- directly reading values from the charts in the RC Design Handbook[2].

In general, the charts in the Handbook overestimate the flexural capacities when compared with values obtained using IS 456:2000 design philosophy. In the results reported in this paper, the numerical computations were performed by the first approach considering the steel to be uniformly spaced on the perimeter. Even though the number of bars chosen in this approach did not comply with the minimum distance between bars, this approach was adopted to distribute the steel along the perimeter and seems more rational than that considered in the Handbook, wherein bars up to 38.7 mm diameter are also used to accommodate the chosen percentage of steel in 20ϕ bars.

The salient observations on the flexural capacities at zero axial load, *Table* 2, are as given.

(*i*) The current code philosophy overestimates the flexural capacity at zero axial load over the proposed philosophy for all the sections, *Fig* 4. The deviation depends on the sectional parameters, namely dimensions and the reinforcement. For the sections considered, it is upto 8.82 percent larger as per the current code.

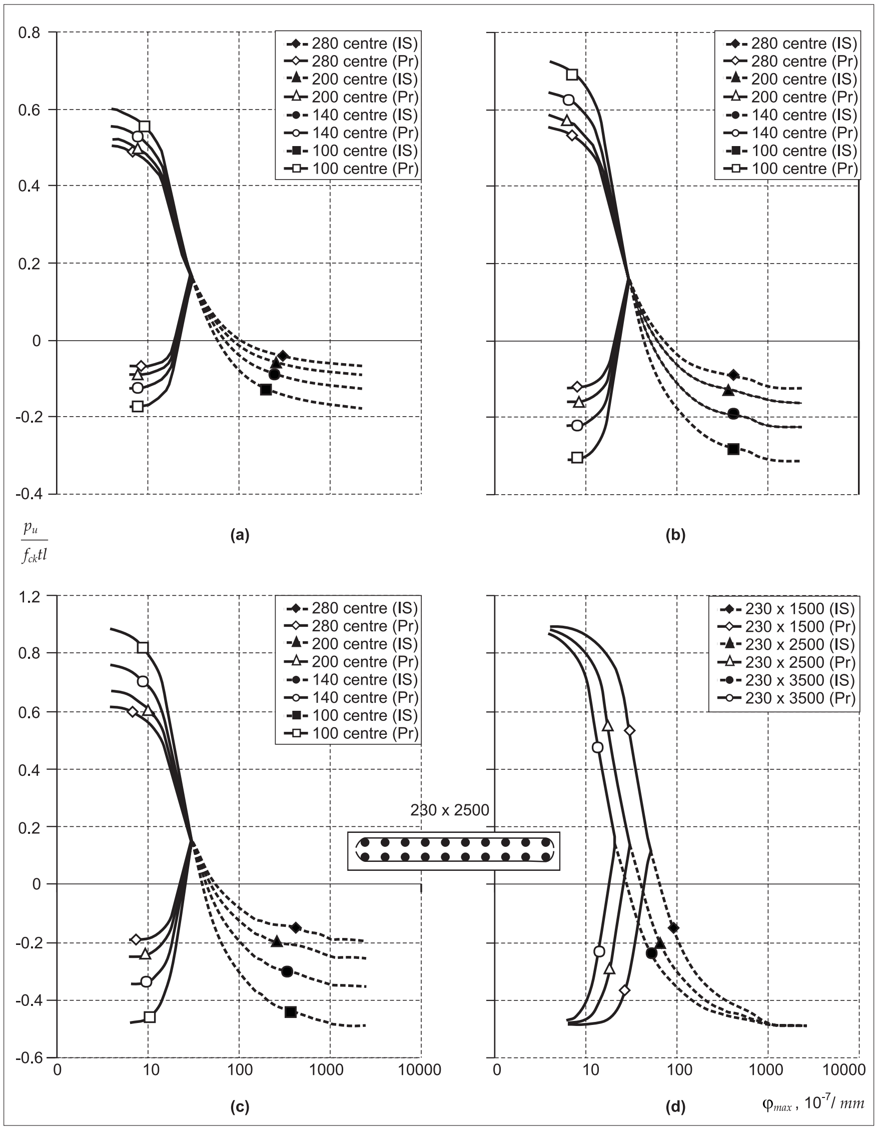

Fig 6 P_u **-** ϕ_{max} **curves for walls without boundary elements as per IS 456 : 2000 and proposed approaches** ***(a)*** **230 x 2500 mm with 12 mm diameter bars** ***(b)*** **230 x 2500 mm with 16 mm diameter bars** ***(c)*** **230 x 2500 mm with 20 mm diameter bars, and** ***(d)*** **for all the sections with Y20,100 on centres**

(*ii*) The deviation in flexural capacity increases with decreasing percentage of vertical steel for the same section.

(*iii*) For the same percentage and configuration of longitudinal steel, the deviation increases with the depth of the section.

(*iv*) Significant deviation in flexural capacity is observed for the section 750 mm × 750 mm, which has many layers of steel in contrast to the other sections.

The salient observations on the curvature capacity, ϕ_{max}, are as given below:

(*i*) ϕ_{max} estimates from the current code philosophy are two orders of magnitude, *Table* 2, higher than that as per the proposed design philosophy, as shown in *Fig* 5.

(*ii*) with the proposed design philosophy, the curvature ductility, μ_ϕ, remains almost constant, *Table* 2, for the different sections. But the curvature ductilities as per the current code philosophy are very high for all the sections (varying from 16 to 43).

Wall sections

Rectangular wall sections without boundary elements were designed as per the code-specified minimum steel ratio and the maximum spacing requirements[11].

The salient observations on the values of ϕ_{max} and μ_ϕ are as follows.

(*i*) The current code philosophy estimates ϕ_{max} by two to three orders higher, *Fig* 6, than that by the proposed design philosophy.

(*ii*) For the same depth of section, ϕ_{max} , is not affected by the amount of vertical steel as per the proposed design philosophy. Varying the depth of the section does alter ϕ_{max}.

(*iii*) With the proposed design philosophy, the curvature ductility, μ_ϕ , remains almost constant for the different sections in the usual range of compressive axial loads 0 to $0.4P_u/f_{ck}tl$. The current code philosophy estimates curvature ductilities for various sections that are unrealistically large (varying from 78 to 239).

The salient observation on the flexural capacities, *Table* 3, is that for the sections considered, the flexural capacity at small axial loads based on the current code philosophy is overestimated up to 14.82 percent, *Fig* 7, over that based on the proposed philosophy. This error depends on the dimensions and reinforcement in the section, and increases with decrease in percentage of vertical steel. The overestimation of flexural capacity by the current IS 456 design philosophy is more for deep RC sections. For ductile seismic behaviour, RC wall sections of large depths in tall RC buildings with shear walls and RC bridges with large piers should be designed with low axial forces levels but large moments. In such cases, the large error in flexural capacity estimates as per the IS 456 philosophy is unacceptable.

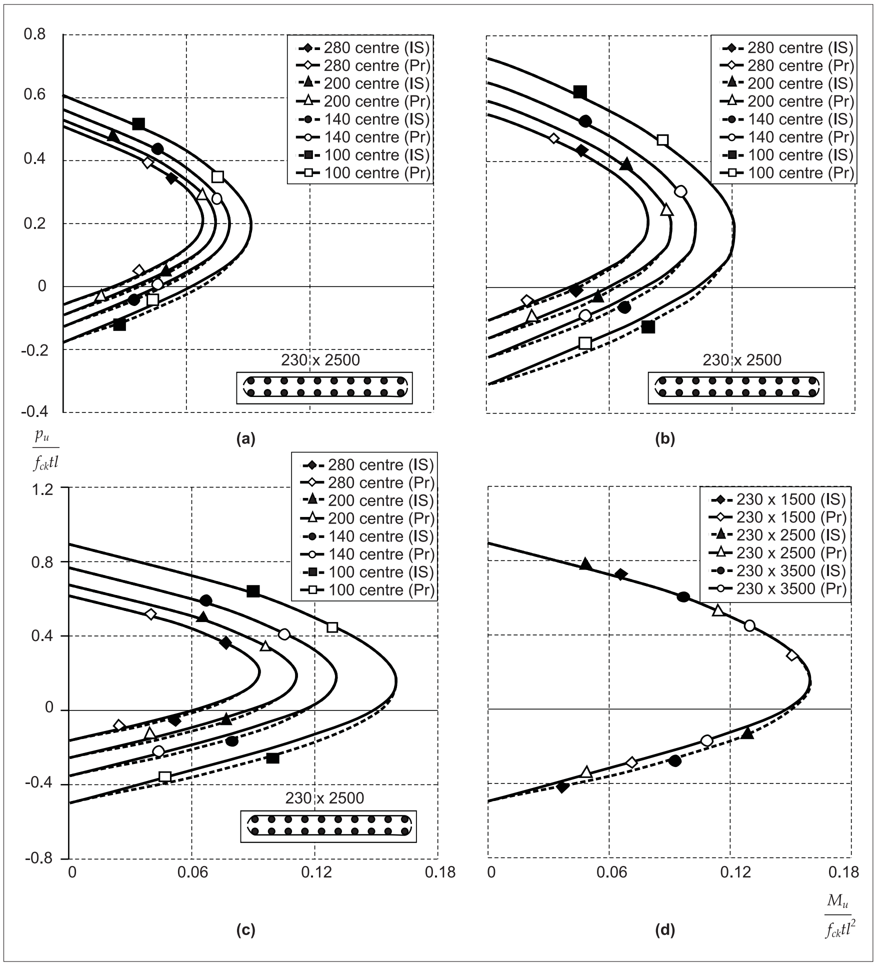

Fig 7 P_u - M_u **curves for wall sections without boundary elements as per IS 456 : 2000 and proposed approaches** ***(a)*** **with 12 mm diameter bars** ***(b)*** **with 16 mm diameter bars** ***(c)*** **with 20 mm diameter bars** ***(d)*** **for Y20,100 on centres**

Conclusions

The current code philosophy as given in IS 456 : 2000 and IS 13920 : 1993 (with no limiting strain in steel in tension) does not completely describe the flexural limit states. This limit state method of design for flexure seems to suggest that RC sections reach unrealistically large ultimate curvatures and curvature ductilities at the ultimate state, in addition to overestimating the flexural capacity at small axial loads. If seismic design has to be conducted with emphasis on curvature ductility and energy dissipation, it is essential to specify a limit state for steel strain in tension. To overcome the shortcoming of the current concrete codes, a limiting strain value for steel is proposed, wherein steel is said to have reached its limit state when the extreme layer of steel on the tension side reaches a strain of $0.002+(f_y/1.15E_s)$. Over the range of low axial load values, this limiting strain for steel results in almost uniform design curvature and design curvature ductility for a wide range of RC sections. Thus, it provides a consistent philosophy of flexural limit state design of RC sections.

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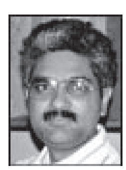

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