DEPENDENCE OF DUCTILITY AND ENERGY DISSIPATION ON LIMITING STRAIN STATES IN SEISMIC DESIGN OF RC COLUMNS

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SUMMARY

In limit state design approach, this balanced axial load depends on limiting strain states defined for reinforcing steel and concrete under combined axial and bending effects. This paper proposes a limiting strain state for steel in the limit state method of design keeping in mind results of a numerical investigation that considers variations in the properties of column sections, namely (a) geometry, (b) location of longitudinal steel, and (c) percentage of longitudinal steel. This proposal is based on two performance pointers, namely section ductility and strain energy stored in the cross-section.

INTRODUCTION

The philosophy of limit state design of reinforced concrete (RC) structures proposes that the structure should safely withstand all possible loads throughout its design life by satisfying certain specified acceptable limit states of collapse and serviceability. The possibility of a structure attaining one of its collapse limit states is determined using the probable variations in both material strengths and loads. To estimate the strength of the structure, strain limiting states are specified on materials by the limit state design methods in the Indian Concrete Code IS:456 [1], New Zealand code NZS 3101 [2] and the strength design method in American code ACI 318-2003 [3] to ensure the desired behaviour. The desirable ductile seismic response of a reinforced concrete (RC) column section is achieved by satisfying the strength and deformation demands during strong earthquake shaking through flexural yielding of longitudinal tension reinforcement. This study investigates the limiting strain in reinforcing steel that will assure this desired limit state of collapse in flexure with a reasonable section ductility and energy dissipation. In particular focus is the use of narrow columns in seismic regions.

LIMITING STRAIN STATES

An RC section reaches limit state of collapse in flexure when either the compressive strain in the extreme fibre of concrete or the tensile strain in the extreme layer of steel reaches the specified limiting value. The IS:456 [1] recommends *0.0035* as the limiting compressive strain in concrete, while the NZS

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3101 [2] and ACI 318-2003 [3] recommend *0.003*. Although the California bridge code CALTRANS [4] specifies the limiting compressive strain as *0.003,* ATC 32 [5] prescribes *0.004* for estimating the design flexural strength in *ductile* columns that undergo subsequent increase in strengths after yielding; for columns other than ductile ones, the limiting compressive strain in ATC 32 [5] is specified as *0.003*. While NZS 3101 [2] and ACI 318-2003 [3] specify the limiting strain in steel as the yield strain $\varepsilon_y = f_y / E_s$ (f_y is the characteristic yield stress of reinforcement steel and E_s is the modulus of elasticity of steel), IS:456 [1] specifies only the minimum value of strain in the extreme layer of steel as $0.002 + (f_y / 1.15E_s)$. All values of strains higher than this are also admissible by the Indian code, irrespective of whether they can be achieved in reality or not.

In addition to the limit states, codes also specify the balanced strain condition as the simultaneous tensile yielding of extreme steel layer and compressive crushing of extreme concrete fibre [3]. To ensure ductile behaviour of RC columns and bridge piers under low axial loads, the ACI 318-2003 [3] and the CALTRANS [4] specify the amount of tension reinforcement in the section to be less than $0.75\rho_b$, where ρ_b is the ratio of reinforcement producing balanced strain condition in the section under flexure without axial loads.

BALANCED SECTION

For flexural limit state design, IS:456 [1] specifies the limiting compressive strain of concrete as *0.0035* (Figure 1). But for the tensile strain in the extreme layer of steel, only a lower bound value of $0.002 + (f_y / 1.15E_s)$ is specified; the tensile strain in reinforcement is permitted to reach any value more than this specified minimum. Therefore, the precedence of flexural yielding of steel over crushing of concrete, i.e. the under-reinforced design of RC columns, is not guaranteed by the design procedure.
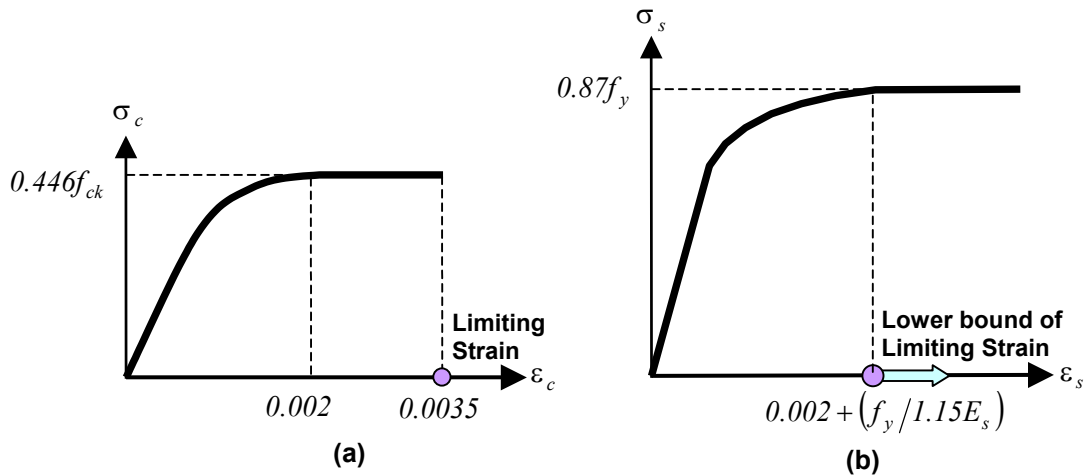

Figure 1: Flexural limit states for (a) concrete, and (b) steel as per IS:456 [1].

In flexural design of an RC column section, the limiting strains in concrete and steel decide the balanced axial load which helps in identifying the range of tension failure under compressive axial load and thereby the possible ductility in the column. The force equilibrium at this balanced strain condition gives the balanced compressive axial load P_b on the section as

$$P_b = C_c + C_s - T \tag{1}$$

where C_c is the compression in concrete, C_s the compression in steel, and T the tension in steel. A balanced axial load factor λ is introduced such that $\lambda = P_b / P_{uz}$ where P_{uz} is the code-specified ultimate

uniaxial compression capacity given by
$$P_{uz} = 0.45f_{ck}bD + 0.75f_yA_{sc} .$$ (2)

In Eq.(2), b and D are the width and depth of the section, f_{ck} the grade of concrete, f_y the yield strength of steel, and A_{sc} the total area of vertical reinforcement. To ensure a large range of tension failure under compressive axial load, P_b needs to be as large as possible and the actual compressive axial load P on the section needs to be less than this during strong shaking.

λ is calculated by the following step-wise procedure:

Step 1: Obtain balanced strain distribution by defining the limiting strains - compressive strain $\varepsilon_{c,max}$ at extreme concrete fibre and the tensile strain $\varepsilon_{st,max}$ at the extreme steel layer (Figure 2).
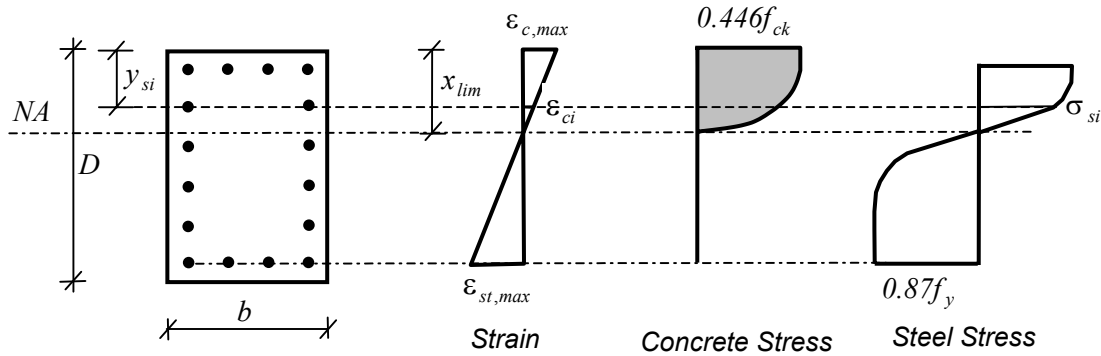

Figure 2: Limit state of collapse in flexure for a balanced section.

Step 2: Obtain C_c, C_s, and T using the design stress-strain curves. Calculate P_{uz} using Eq.(2).
Step 3: Obtain λ from Eq.(1) as
$$\lambda = \frac{C_c + C_s - T}{P_{uz}} .$$ (3)

The variation of λ is studied as a function of section sizes b and D, and distribution of steel. The implications of parameters $\varepsilon_{st,max}$ and λ are discussed in detail in the section below.

NUMERICAL STUDY

Column sections of different widths and depths are considered. The grade of concrete is taken as *M20* and the reinforcement *Fe415*. In all sections, *0.8%* vertical reinforcement is provided, which is the minimum percentage of steel specified for RC columns [1] and is expected to give the largest ductility. For *Fe415* grade of reinforcement, the code-specified minimum value of tensile strain [1] in the extreme layer of steel is *0.0038*. Here, the limiting tensile strain in the extreme layer of steel is assumed as *0.01*; thus, a limited extent of ductile behaviour of reinforcement is allowed beyond the code-specified value of *0.0038*. The limiting compressive strain in the extreme concrete fibre is taken as the code-specified value of *0.0035*.

Three types of vertical steel distributions (Figure 3), namely (a) Case A: steel on all four sides, (b) Case B: equal steel on two faces along width, and (c) Case C: equal steel on the two faces along depth. In the preliminary study, the vertical steel is assumed as a continuous distribution instead of discrete bars. The volumetric ratio of vertical steel is converted to an equivalent linear distribution of steel along the appropriate sides. The code-specified stress-strain distribution of steel with the assumed limiting strain is discretised into seven linear segments (Figure 4). C_s and T are obtained by integrating the stress distribution over the linear segments.
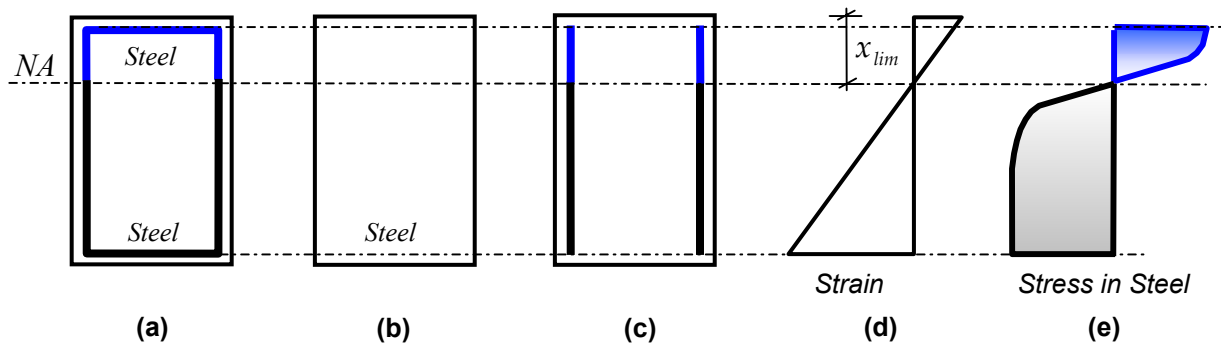
Figure 3: Calculation of forces in steel with (a) Case A: reinforcement on four sides, (b) Case B: reinforcement on two faces along width, and (c) Case C: reinforcement on two faces along depth.
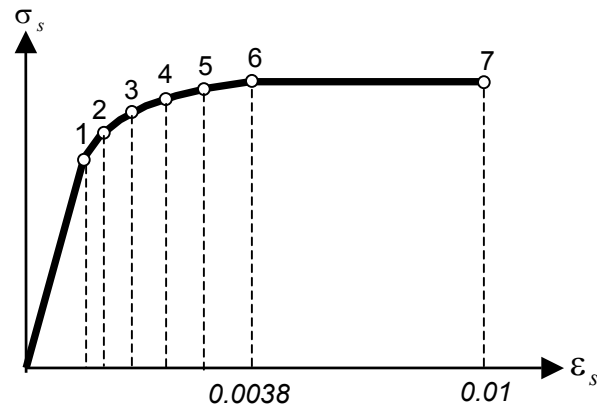

Figure 4: Discretisation of design stress-strain curve for steel.

The variations of λ for the cases considered are shown in Figure 5. The salient results of the effects of distributions of longitudinal steel are:

1. In Case B, the balanced axial load capacity increases rapidly for small depths, but reaches an upper bound value of *0.16* at large depths.
2. In Cases A and C, λ decreases with increasing depth of the section particularly for wide columns.
3. The balanced axial loads obtained in Case B are more than those obtained in Cases A and C.
4. In Case A with *230mm* (narrow) width, the balanced axial load factor λ reaches only *0.04* and that too at higher depths. Thus, the tension failure region of $P_u - M_u$ interaction curve is very small.
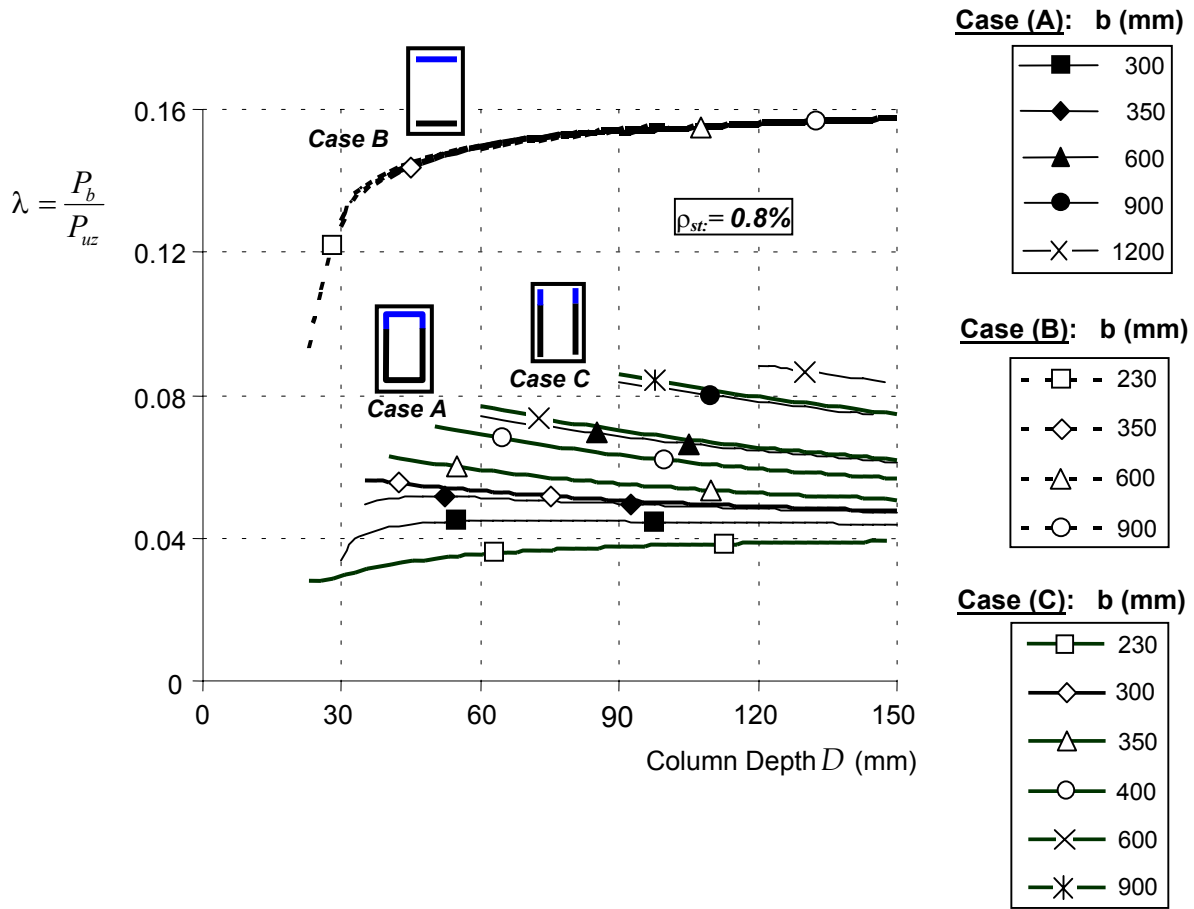
Figure 5: Effect of vertical steel distribution on balanced axial load capacity of column sections.

For a particular cross-sectional area (*300 mm* width and *500 mm* depth) and equal distribution of longitudinal steel on two faces along width (Case B) in the RC column, the variation of λ with the limiting tensile strain $\varepsilon_{st,max}$ in longitudinal steel is shown in Figure 6. The salient observations are:
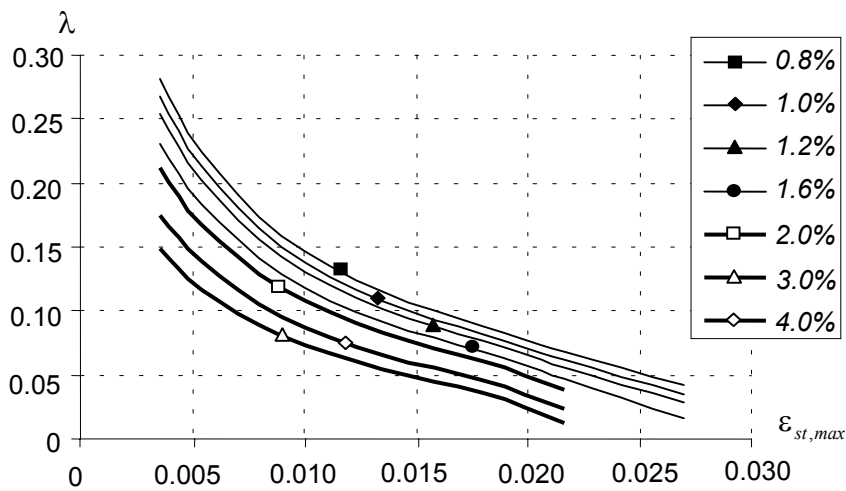

Figure 6: Effect of limiting strain in longitudinal steel on balanced axial load capacity of 300×500mm sections with steel on opposite sides (Case B).

1. The balanced axial load capacity of RC columns decreases with increase in $\varepsilon_{st,max}$.
2. For a given percentage of longitudinal steel and a limiting tensile strain $\varepsilon_{st,max}$ for the longitudinal steel, the axial load on the section needs to be restricted for ensuring tension failure behaviour.

Strain Energy

The strain energy stored at a RC section under flexure is a performance pointer towards the ductile behaviour of the section. The axial force-strain energy curve is obtained as below:

Step 1: Choose depth of neutral axis x; x is varied from $-\infty$ to $+\infty$. In the range $[-5D, +5D]$, a large number of points may be chosen to obtain smooth interaction curves.

Step 2: Identify the region in which x lies, *i.e.,* region AB (Figure 7) if $-\infty < x < -0.5D$, region BC if $-0.5D \le x < x_b$, region CD if $x_b \le x < +0.5D$ and region DE if $+0.5D \le x < +\infty$.
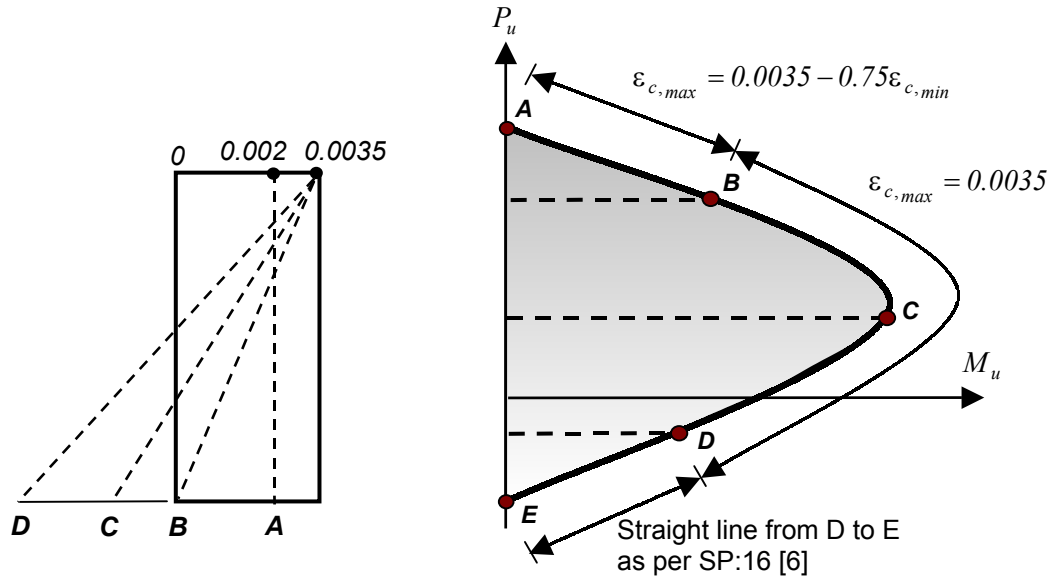

Figure 7: Limiting strains and strain distributions in a RC cross section at different states under combined axial load and bending moment, as per IS:456 [1].

Step 3: The strain energy stored in a concrete fibre at a strain level ε_c is given as $\int \sigma_c b dx$. Then, the total energy stored in concrete over the whole cross-section will be $\int \left(\int \sigma_c b dx \right) d\varepsilon$.

Step 4: The strain energy in a steel bar at a strain of ε_s is computed as the area under the stress-strain curve upto the strain ε_s. The total strain energy is the sum of those in individual bars.

Step 5: The total strain energy stored in the section is the sum of the energies obtained in Steps *4* and *5*.

The axial load-strain energy interaction curve obtained for the RC section of size *300mm×500mm* and longitudinal steel *0.8%* is shown in Figure 8. The energy ratio E_r is obtained by normalizing the energy values with respect to the total energy dissipated in straining the steel and concrete areas to strain levels of *0.01* and *0.0035* respectively. The salient observations are:

1. The strain energy capacity increases with the increasing limiting tensile strain in steel.
2. The strain energy capacity of the section is higher for axial compressive loads below the balance point than for the loads above it, if $\varepsilon_{st,max}$ is larger than *0.0076*.
3. With increasing limiting strain $\varepsilon_{st,max}$ in steel, the tension failure design regime gets reduced.
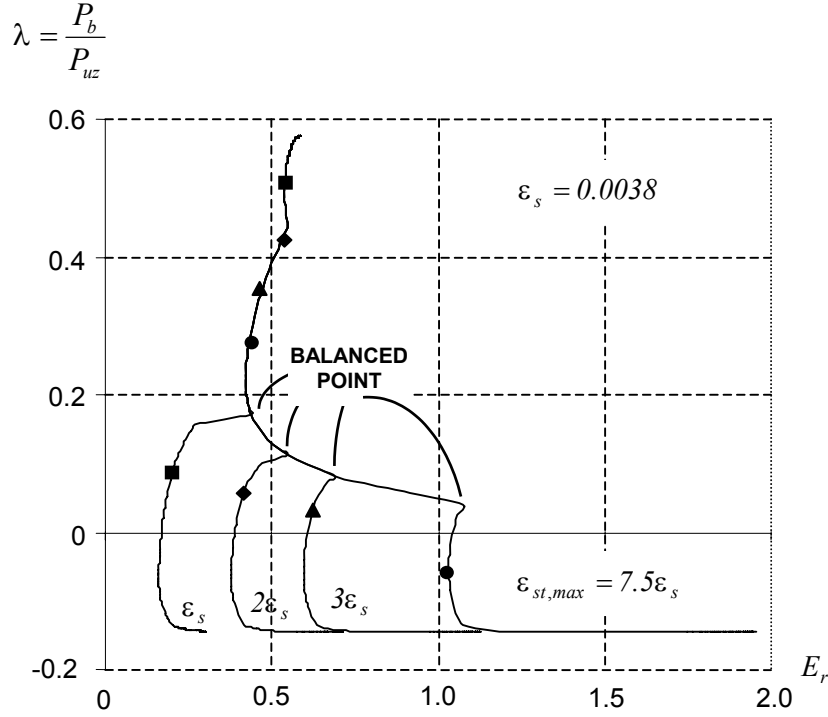
$$\lambda = \frac{P_b}{P_{uz}}$$

Figure 8: Effect of limiting strain in longitudinal steel on the strain energy capacity of column sections.

Strain Ductility

Strain ductility capacity of a RC section is obtained as the ratio of the maximum permissible tensile strain in the longitudinal steel and the tensile strain ε_s in the longitudinal steel specified as $\varepsilon_s = 0.002 + (f_y / 1.15E_s)$. In the numerical study, the limiting strains $\varepsilon_{st,max}$ in longitudinal steel are varied as multiples of ε_s. For the balanced strain condition, the limiting compressive strain in concrete taken as *0.0035* corresponding to each value of $\varepsilon_{st,max}$. The steel ratio ρ_s is obtained as the ratio of the area $A_{st,b}$ of steel required for a strain level of $\varepsilon_{st,max}$ and the area A_{st} of steel required for a strain level of ε_s. The steel area ratio-strain ductility curve is obtained for section of size *300mm×500mm* with longitudinal reinforcement distributed equally on two opposite sides. The salient observations are:

1. With increase in axial load, the strain ductility capacity μ_{st} decreases for the same steel ratio.
2. For the same axial load level, the amount of longitudinal steel required decreases to obtain the desired ductility of the section.
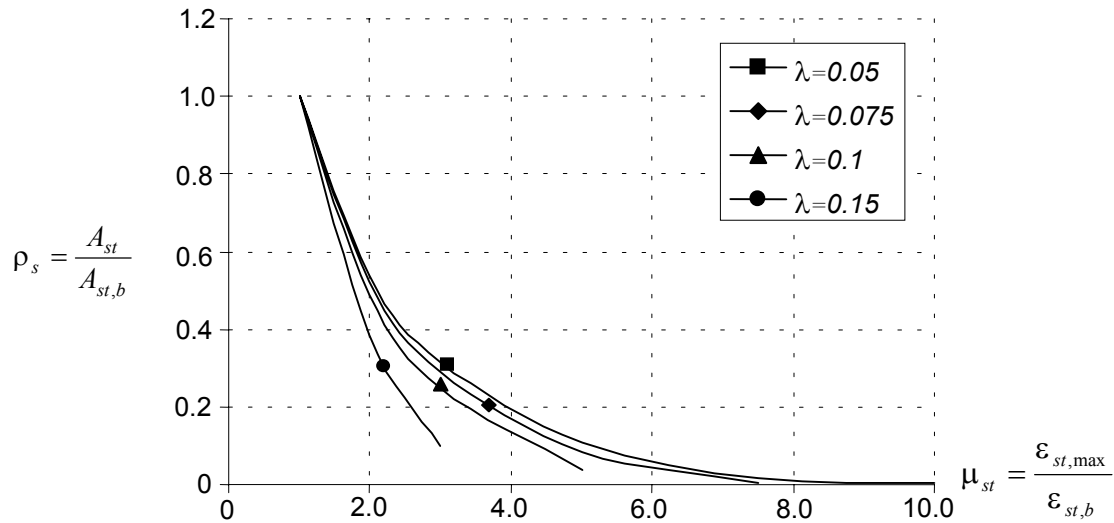
Figure 9: Effect of balanced load factor λ on longitudinal steel ratio of column sections.

CONCLUSIONS

The salient conclusions of the numerical study are:

1. Column sections with concentrated longitudinal steel at the tension and compression faces offer more desirable seismic behaviour than those with distributed steel.
2. The strain energy capacity of a RC column section reflects the effect of limiting tensile strain in longitudinal steel in the tension failure design regime.
3. The amount and distribution of longitudinal steel and the balanced axial load factor of an RC column section determine the ductility capacity of the section.
4. A limiting strain of $\varepsilon_{st,max} = 2\varepsilon_s = 0.0078$ in tension for steel is proposed. This provides reasonably uniform energy dissipation for all levels of axial load.

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