

# Hierarchical DWT Based Optimal Diversity Power Allocation for Video Transmission in MIMO Wireless Systems with Quantized Feedback

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**Abstract**—In this work we propose novel algorithms for singular mode diversity order based optimal power allocation for video transmission in Multiple-Input Multiple-Output (MIMO) wireless systems. We motivate a discrete wavelet transform (DWT) and hierarchical block motion estimation algorithm (HBMA) based spatio-temporal video decomposition that is naturally suited to the ordering and diversity properties of the MIMO channel singular modes for video layer allocation. Based on this paradigm, we propose a framework for the MIMO singular value decomposition (SVD) based optimal power allocation, specifically for the context of MIMO video transmission. Further, we consider a practical MIMO scenario with channel state information (CSI) available only at the receiver and employ a codebook based limited feedback technique to feedback the index of the quantized beamforming vector. This significantly reduces the communication overhead on the reverse link of the wireless system. We demonstrate that the proposed hierarchical video decomposition based optimal diversity power allocation employing limited CSI feedback can be formulated as a constrained optimization problem. Subsequently, it is demonstrated that the solution to this problem can be computed by solving an iterative sequence of convex cost minimization problems. A closed form expression for the optimal power allocation is presented for the convex optimization problem at each iterative step. Simulation results demonstrate significant performance improvement of the proposed optimal power allocation schemes over the suboptimal equal power allocation scheme.

## I. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) wireless communication technology has generated significant interest due to the high data throughput of such systems resulting from the spatial-multiplexing of multiple information streams in parallel without the necessity of additional bandwidth. Further, MIMO systems can also yield diversity gains, thus resulting in a robust and reliable wireless link, significantly reducing its susceptibility to a deep fade. MIMO technology is therefore a key component in achieving high data rates for broadband wireless access in the upcoming 4G cellular standards such as WiMAX and LTE. Reliable video transmission with quality of service (QoS) guarantees such as low latency and jitter is a salient feature of several 4G wireless services such as video

conferencing, surveillance and interactive gaming amongst others. However, video transmission in MIMO wireless systems is challenging, particularly due to the time varying nature of wireless channels combined with the lack of channel state information (CSI) at the transmitter.

Employing the singular value decomposition (SVD) of the MIMO channel matrix  $\mathbf{H}$ , a MIMO system can be decomposed as a set of parallel spatial channels corresponding to the singular modes. To maximize the throughput across the MIMO channel, one needs to optimally allocate the limited power resources across the different spatial channels, followed by transmit beamforming. Hence, to achieve high quality video transmission in a MIMO system, it is critical to optimally distribute the limited power resources amongst the singular modes of the MIMO channel. Water filling algorithm based optimal schemes [1] have been proposed for power allocation in MIMO systems, but these solutions are for generic data transmission and are not specifically optimized for video transmission. The scheme in [2] utilizes a quality layers based unequal power allocation scheme for JPEG transmission in MIMO systems based on a heuristic algorithm. Other empirical schemes for unequal power distribution between the channel singular modes have been studied for video transmission in works such as [3], but these schemes are adhoc and depend on a scaling parameter. Hence, there is a significant dearth of precise analytical schemes especially suited for MIMO video transmission.

Moreover, in a practical MIMO wireless scenario, obtaining perfect CSI at the transmitter is unrealistic. In 4G wireless systems, a limited number of bits are assigned for feedback on the reverse link. In such scenarios, it is key to design an optimal quantized feedback based MIMO power allocation scheme to maximize the video transmission quality. Authors in [4] propose an algorithm for subcarrier power allocation and antenna selection for scalable video delivery in MIMO-OFDM systems based on an order-feedback mechanism which is limited in scope as it results in a loss of coding gain. A novel feedback scheme for MIMO systems which yields a significant performance enhancement is codebook based feedback of the

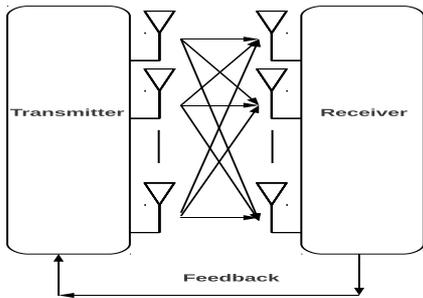


Fig. 1. A schematic of MIMO system with feedback

quantized MIMO beamforming vectors [5]. After each channel estimation epoch at the receiver, the indices of quantized vectors belonging to the codebook and closest to the transmit beamforming vectors in the mean-squared inner product (MSIP) norm are fed back to the transmitter. Such a scheme has the dual advantage of requiring only a few feedback bits on the reverse channel, while achieving a performance very close to that of perfect CSI at the transmitter. An added advantage is that such a fixed codebook of beamforming vectors can be constructed offline, thus greatly limiting the computational complexity of the scheme.

Hence, motivated by the above discussion, we propose a framework for optimal MIMO transmit power allocation employing codebook based limited feedback in the context of video transmission. For this purpose, we consider hierarchical spatio-temporal layering of the video frames similar to the one described in [6] followed by MIMO SVD based substream allocation. We develop an analytical framework for diversity-order dependent power allocation in MIMO systems based on the average video distortion minimization criterion. The proposed framework is robust, since it also takes into consideration the distortion in the transmission due to multi-stream interference arising out of the limited CSI feedback. We formulate the above optimization paradigm as a constrained cost optimization. Following this, we demonstrate that the optimal power allocation can be computed by solving an iterative sequence of video distortion based constrained convex objective minimization problems. We provide a closed form expression for the optimal power allocation at each iterative step above based on the solution of a polynomial root computation. Simulation results for video transmission in quantized feedback based MIMO wireless systems employing several video sequences demonstrate that the proposed schemes outperform equal power allocation in terms of both PSNR and visual quality.

The subsequent sections of this paper are organized as follows. We begin with a description of the MIMO system model and quantized beamforming vector codebook construction in the next section. The expression for the mode received SNR for this paradigm of quantized feedback based transmit beamforming is presented subsequently. In section III we describe a singular mode diversity order based hierarchical

video transmission (DHVT) scheme. The distortion can be further reduced through optimal power allocation based video transmission (OPVT) and we derive an optimization framework towards achieving this objective in section IV. It is demonstrated therein that the optimal power allocation for the MIMO substreams can be computed as the solution of an iterative series of convex objective minimization problems. Simulation results are presented in section V to illustrate the performance gains of the proposed DHVT and OPVT schemes over the suboptimal single layer video transmission (SLVT), which does not leverage the diversity properties of the MIMO system.

## II. SYSTEM MODEL

A schematic of a MIMO wireless system with multiple transmit and receive antennas is shown in Fig.1. We consider a MIMO system with  $t$  transmit antennas and  $r$  receive antennas, modeled as,

$$\mathbf{y}(k) = \mathbf{H}\mathbf{x}(k) + \eta(k),$$

where  $\mathbf{x}(k) \in \mathbb{C}^{t \times 1}$  and  $\mathbf{y}(k) \in \mathbb{C}^{r \times 1}$  are the transmitted and received signal vectors respectively. The element  $h_{ij}$ ,  $1 \leq i \leq r, 1 \leq j \leq t$  of the channel matrix  $\mathbf{H} \in \mathbb{C}^{r \times t}$  is the flat-fading channel coefficient between the  $j^{\text{th}}$  transmit antenna and  $i^{\text{th}}$  receive antenna. These complex coefficients  $h_{ij}$  are IID Rayleigh distributed. The vector  $\eta \sim \mathcal{CN}(0, \sigma_\eta^2 \mathbf{I}_r)$  is complex additive spatio-temporally white Gaussian noise with covariance matrix  $E(\eta\eta^H) = \sigma_\eta^2 \mathbf{I}_r$ . The singular value decomposition (SVD) of the channel matrix  $\mathbf{H}$  defined above is given as  $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{W}^H$  where  $\mathbf{U} \in \mathbb{C}^{r \times r}$  and  $\mathbf{W} \in \mathbb{C}^{t \times t}$  are unitary matrices and  $\mathbf{\Sigma} \in \mathbb{R}^{r \times t}$  is a diagonal matrix, whose elements are the non-negative ordered singular values  $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_m \geq 0$  of  $\mathbf{H}$ , where  $m \triangleq \min(r, t)$ .

It has been demonstrated in literature [7] that the capacity of the MIMO system can be maximized by performing transmit beamforming along the columns of the right singular matrix i.e.  $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_m]$  combined with optimal power allocation amongst the singular modes. The resulting MIMO system model for transmit beamforming, corresponding to beamforming the symbols  $\tilde{\mathbf{x}}(k) = [\tilde{x}_1(k), \tilde{x}_2(k), \dots, \tilde{x}_m(k)]^T$  along the right singular vectors can be described as,

$$\mathbf{y}(k) = \mathbf{H}\mathbf{W}\tilde{\mathbf{x}}(k) + \eta(k), \quad (1)$$

where  $E\{\tilde{x}_k^H \tilde{x}_k\} = P_k$ , the power allocated to the  $k^{\text{th}}$  singular mode. For symbol detection, the receiver employs receive beamforming with the left singular matrix  $\mathbf{U}$  as  $\tilde{\mathbf{y}}(k) = \mathbf{U}^H \mathbf{y}(k)$  to decompose the received vectors  $\mathbf{y}(k)$  into  $m$  parallel substreams corresponding to the transmit streams  $\tilde{x}_j, 1 \leq j \leq m$ .

However, in a practical MIMO wireless system, the transmitter does not possess knowledge of the channel state information (CSI)  $\mathbf{W}, \mathbf{\Sigma}$  since the channel is estimated at the receiver. Hence, the channel estimate has to be relayed from the receiver to transmitter, termed as *feedback* in the context of

wireless communications. Further, exact feedback of the coefficients of each of the transmit beamforming vectors  $\mathbf{w}_j$  incurs a high bit-rate overhead on the reverse link. Thus, quantized beamforming vector based limited feedback schemes, where the receiver quantizes each vector  $\mathbf{w}_j, 1 \leq j \leq m$  as,

$$\hat{\mathbf{w}}_j = \mathcal{Q}(\mathbf{w}_j) \in \mathcal{C}, 1 \leq j \leq m,$$

have gained immense popularity. The quantized vectors  $\hat{\mathbf{w}}_j$  belong to the codebook  $\mathcal{C}$  and the receiver feeds back the index of the quantized beamforming vector, thereby greatly limiting the number of feedback bits required. In the section below we describe the algorithm for the transmit beamforming vector codebook construction. In the discussion that follows, the inner product between two vectors  $\mathbf{v}_i, \mathbf{v}_j$ , denoted by  $\langle \mathbf{v}_i, \mathbf{v}_j \rangle$  is defined as,

$$\langle \mathbf{v}_i, \mathbf{v}_j \rangle \triangleq \mathbf{v}_i^H \mathbf{v}_j.$$

### A. Beamforming Vector Codebook

We employ the mean-squared inner product (MSIP) based nearest-neighbor scheme elaborated in [5] for codebook construction and quantization of the transmit beamforming vectors  $\mathbf{w}_j$ . Let  $B$  denote the number of feedback bits per singular mode on the reverse link. Employing the scheme in the work above, we construct a vector codebook  $\mathcal{C}$  of  $N = 2^B$  unit-norm quantized vectors. The vector quantizer  $\mathcal{Q}(\cdot)$  is designed such that it maximizes the mean squared inner-product (MSIP)  $E |\langle \mathbf{v}, \mathcal{Q}(\mathbf{v}) \rangle|^2$ . This approach has been naturally shown to result in maximizing the gains resulting from transmit beamforming employing the quantized beamforming vectors. This codebook construction scheme is based on the Lloyd's algorithm for optimal quantizer design and iteratively computes the Voronoi regions  $R_i$  and the codebook vectors  $\mathbf{c}_i \in \mathcal{C}, 1 \leq i \leq N$ .

*Nearest Neighbourhood Computation:* Let the codebook  $\mathcal{C}^{(k)}$  at the  $k^{th}$  iteration comprise of the  $N$  quantizer beamforming vectors  $\{\mathbf{c}_1^{(k)}, \mathbf{c}_2^{(k)}, \dots, \mathbf{c}_N^{(k)}\}$ . The  $N$  Voronoi regions  $R_i^{(k)}$  corresponding to the vectors  $\mathbf{c}_i^{(k)}, 1 \leq i \leq N$  at the  $k^{th}$  iteration are computed as,

$$\left\{ \mathbf{v} \in \mathbb{C}^{t \times 1} : \|\mathbf{v}\| = 1 \text{ and } \left| \langle \mathbf{v}, \mathbf{c}_i^{(k)} \rangle \right| \geq \left| \langle \mathbf{v}, \mathbf{c}_j^{(k)} \rangle \right| \forall j \neq i \right\}.$$

*Centroid Computation:* For the given set of Voronoi regions  $R_i^{(k)}, 1 \leq k \leq N$  at the end of the  $k^{(k)}$  iteration above, we compute the optimum quantization vectors in the  $(k+1)^{th}$  iteration as follows. Consider the matrix  $\mathbf{G}_i^{(k)}$  defined as  $\mathbf{G}_i^{(k)} = E(\mathbf{v}\mathbf{v}^H), \mathbf{v} \in R_i$ . The quantization vectors  $\mathbf{c}_i^{(k+1)}$  can be computed as the solution to,

$$\mathbf{G}_i^{(k)} \mathbf{c}_i^{(k+1)} = \lambda_{\max} \mathbf{c}_i^{(k+1)}, \left\| \mathbf{c}_i^{(k+1)} \right\| = 1,$$

where  $\lambda_{\max}$  is the largest eigenvalue of  $\mathbf{G}_i^{(k)}$ . In other words,  $\mathbf{c}_i^{(k+1)}$  is the principal unit-norm eigenvector of  $\mathbf{G}_i^{(k)}$ . This process is iterated until  $E(|\langle \mathbf{v}, \mathcal{Q}(\mathbf{v}) \rangle|^2)$  converges. In practice, this algorithm for codebook computation can be implemented very efficiently by choosing a large ensemble of vectors  $\tilde{\mathbf{v}}_i \in \tilde{\mathbf{V}}, 1 \leq i \leq J$ , where  $J$  is a large number and the

vectors  $\tilde{\mathbf{v}}_i$  are chosen randomly according to the distribution of the transmit beamforming vectors  $\mathbf{w}_j, 1 \leq j \leq m$ . The final codebook  $\mathcal{C}$  obtained as a result of the above iterative process is denoted by  $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_N\}$ .

### B. Codebook Based Transmit Beamforming

The codebook  $\mathcal{C}$  constructed through the scheme described above is utilized to quantize each beamforming vector  $\mathbf{w}_i$  at the receiver as  $\hat{\mathbf{w}}_i = \mathbf{c}_j = \mathcal{Q}(\mathbf{w}_i)$  where,

$$j = \mathcal{I}\{\mathcal{Q}(\mathbf{w}_i)\} = \arg \max_{1 \leq l \leq N} |\langle \mathbf{w}_i, \mathbf{c}_l \rangle|.$$

The quantity  $\mathcal{I}\{\mathcal{Q}(\mathbf{v})\}$  represents the index of the quantized vector in the codebook  $\mathcal{C}$  and is fed back to the transmitter on the reverse link. The MIMO quantization codebook  $\mathcal{C}$  is known to the transmitter since it is computed offline and depends only on the ensemble statistical information and not on the instantaneous time-varying CSI. Hence, the transmitter employs the quantized vectors  $\{\hat{\mathbf{w}}_1, \hat{\mathbf{w}}_2, \dots, \hat{\mathbf{w}}_m\}$  for beamforming, where  $\hat{\mathbf{w}}_i = \mathcal{Q}(\mathbf{w}_i)$  as described in (1). The resulting model for symbol detection corresponding to the  $i^{th}$  substream  $1 \leq i \leq m$  can be expressed as,

$$\tilde{y}_i(k) = \sigma_i \langle \mathbf{w}_i, \hat{\mathbf{w}}_i \rangle \tilde{x}_i(k) + \sigma_i \left( \sum_{\substack{n=1 \\ n \neq i}}^m \langle \mathbf{w}_i, \hat{\mathbf{w}}_n \rangle \tilde{x}_n(k) \right) + \eta(k), \quad (2)$$

where the inter-stream interference term  $\langle \mathbf{w}_i, \hat{\mathbf{w}}_n \rangle$  above arises due to the transmit beamformer quantization error corresponding to the limited feedback scheme. Hence,  $\text{SNR}_i$ , the received SNR corresponding to the  $i^{th}$  parallel transmit substream, is given as,

$$\text{SNR}_i = \frac{E \left\{ |\langle \mathbf{w}_i, \hat{\mathbf{w}}_i \rangle|^2 \right\} \sigma_i^2 P_i}{\sigma_\eta^2 + \sigma_i^2 \sum_{\substack{n=1 \\ n \neq i}}^m E \left\{ |\langle \mathbf{w}_i, \hat{\mathbf{w}}_n \rangle|^2 \right\} P_n}, 1 \leq i \leq m.$$

From the results in [8], the quantities  $E \left\{ |\langle \mathbf{w}_i, \hat{\mathbf{w}}_i \rangle|^2 \right\}$  and  $E \left\{ |\langle \mathbf{w}_i, \hat{\mathbf{w}}_n \rangle|^2 \right\}$  can be approximated as,

$$\begin{aligned} E \left\{ |\langle \mathbf{w}_i, \hat{\mathbf{w}}_i \rangle|^2 \right\} &= 1 - \delta \left( \frac{t+1}{t} \right) \approx 1 - \delta, \\ E \left\{ |\langle \mathbf{w}_i, \hat{\mathbf{w}}_n \rangle|^2 \right\} &= \left( \frac{\delta}{N-1} \right) \left( \frac{t-1}{t} \right) \approx \frac{\delta}{N-1}, \end{aligned}$$

where  $\delta \triangleq 2^{-\left(\frac{B}{t-1}\right)}$ . Substituting these approximations in the expression for  $\text{SNR}_i$  derived above, the resulting expression for the SNR of the  $k^{th}$  MIMO substream can be simplified as,

$$\text{SNR}_i = \frac{\beta_i \left( 1 - 2^{-\left(\frac{B}{t-1}\right)} \right) P_i}{\sigma_\eta^2 + \beta_i \sum_{\substack{n=1 \\ n \neq i}}^m \left( \frac{2^{-\left(\frac{B}{t-1}\right)}}{N-1} \right) P_n}, 1 \leq i \leq m \quad (3)$$

where  $\beta_i \triangleq E(\sigma_i^2)$ . This expression is employed in section IV below to formulate the framework for optimal MIMO singular

mode power allocation in the context of video transmission. In the next section we propose a singular mode diversity order based video substream allocation for distortion minimization in MIMO wireless systems.

### III. ORDER DIVERSITY BASED MIMO VIDEO TRANSMISSION

In the context of the quantized feedback MIMO system model described above, we consider a MIMO user with  $t_u$  transmit antennas,  $r_u$  receive antennas and  $m_u = \min(r_u, t_u)$  parallel substreams for video transmission. The singular value decomposition (SVD) of the MIMO channel results in a natural ordering of singular modes of the MIMO channel matrix corresponding to the singular values  $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_{m_u}$ . The key result below, which has been presented in [9], provides a handle on the diversity order and probability of bit-error for QPSK transmission for the  $i^{\text{th}}$  MIMO mode corresponding to  $\sigma_i$ .

*Theorem 1:* The bit-error rate (BER) corresponding to the  $k^{\text{th}}$  MIMO substream with channel gain  $\sigma_k$  has a diversity order  $G_d(k) = (m_u - k + 1)(n_u - k + 1)$ , where  $m_u = \min\{r_u, t_u\}$  and  $n_u = \max\{r_u, t_u\}$ . Further, the probability of bit-error  $\phi_e$  for QPSK modulated data transmission through this mode can be expressed as,

$$\phi_e(\text{SNR}_k) = (G_d(k)\text{SNR}_k)^{-G_d(k)} \quad (4)$$

where the array gain  $G_a(k)$  is given by the expression,

$$G_a(k) = \frac{3}{n_u} \left( \frac{(\sqrt{2}-1)a_k 2^{d_k} \Gamma(d_k + 3/2)}{\sqrt{\pi}(d_k + 1)} \right)^{-\left(\frac{1}{d_k+1}\right)}$$

The quantities  $d_k$ ,  $a_k$  and  $K_{n_u, m_u}$  are given as,

$$\begin{aligned} d_k &= G_d(k) - 1 \\ a_k &= K_{n_u, m_u}^{-1} |A(k)| |B(k)| \\ K_{n_u, m_u} &= \prod_{i=1}^{m_u} (m_u - i)!(n_u - i)! \end{aligned}$$

Let the quantity  $b(i)$  be defined as  $b(i) \triangleq n_u - m_u + i$ . The matrix  $A(k)$  can be defined as follows. For  $k = 1$ ,  $A(1) = 1$ . For other values of  $k$ , i.e.  $k \geq 2$  and  $i, j = 1, 2, \dots, (k-1)$ ,  $A(k)_{ij}$ , the  $(i, j)^{\text{th}}$  entry of  $A(k)$  is given as,

$$A(k)_{ij} = (b(i+j) + 2(m_u - k))!$$

The matrix  $B(k)$  is defined as follows. The matrix  $B(m_u) = 1$ , corresponding to  $k = m_u$ . For other values of  $k$ , the  $(i, j)^{\text{th}}$  entry of  $B(k)$  with  $1 \leq i, j \leq m_u - k$  can be expressed as,

$$B(k)_{ij} = \frac{2}{(b(i+j)^2 - 1)b(i+j)}$$

The above result plays a key role in the context of video transmission through MIMO systems as follows. Consider a MIMO system with  $r = t = m_u = n_u$ . Hence, the strongest mode corresponding to the singular value  $\sigma_1$  has a diversity order  $m_u n_u$ , while the weakest mode, corresponding to singular value  $\sigma_{m_u}$  has diversity order 1. Thus, it can be

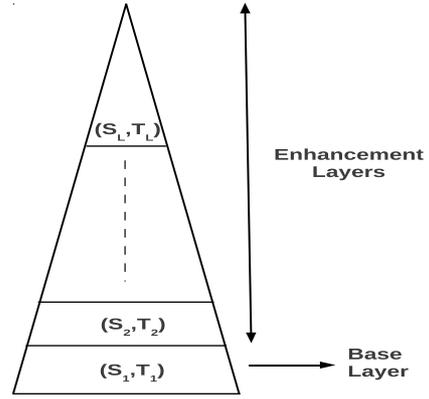


Fig. 2. Spatio-Temporal hierarchical layering of video sequence

readily seen that the BER corresponding to the strongest mode decreases as  $O(\text{SNR}^{-m_u n_u})$ , while that corresponding to the weakest mode decreases as  $O(\text{SNR}^{-1})$ . Hence, the strongest mode has a significantly lower BER and higher reliability owing to its higher diversity order. The diversity order is progressively lower for the weaker modes, eventually becoming 1 for the weakest mode. This salient property relating to MIMO substream diversity order can be advantageously employed for minimization of video distortion corresponding to video transmission over the MIMO wireless system as described below.

Any given video sequence can be readily decomposed into  $L$  hierarchical spatio-temporal layers, comprising of a base spatial layer  $S_1$ , base temporal layer  $T_1$  and  $L - 1$  spatial, temporal enhancement layers  $S_2, \dots, S_L, T_2, \dots, T_L$ , as shown schematically in Fig.2. Such a scheme has been elaborated in [6] where the spatial discrete wavelet transform (DWT) is employed for spatial layering based intra coding (I frames), coupled with the hierarchical block motion estimation algorithm (HBMA) for DPCM based temporal layering (P,B frames). In this context, a natural scheme for MIMO video transmission is diversity based hierarchical video transmission (DHVT), wherein the  $i^{\text{th}}$  spatio-temporal layers  $S_i, T_i$  are transmitted over the  $i^{\text{th}}$  MIMO singular mode corresponding to gain  $\sigma_i$  and transmit beamforming vector  $\mathbf{w}_i$ . Thus, by transmitting the base layers  $S_1, T_1$  through the strongest MIMO mode and choosing progressively weaker modes for the transmission of the enhancement layers, DHVT significantly reduces video distortion. In the next section, we describe the optimal power allocation based video transmission (OPVT) scheme for MIMO singular mode power distribution to further reduce the video distortion of DHVT.

### IV. OPTIMAL POWER ALLOCATION BASED VIDEO TRANSMISSION (OPVT)

Consider a MIMO wireless system with total transmit power  $P_T$ . The video distortion of the DHVT scheme described above can be further reduced through optimal power allocation

amongst the  $m_u$  MIMO substreams. Let  $D_l(\mathcal{V})$  denote the distortion coefficients of the video sequence  $\mathcal{V}(x, y, t)$  corresponding to the spatio-temporal layer  $l$ , where  $1 \leq l \leq m_u$ . These can be computed in a relatively straight forward manner for each spatial-temporal layer as described in [6]. The mean overall distortion  $D(\mathcal{V})$  for MIMO transmission of the video sequence  $\mathcal{V}(x, y, t)$  can be expressed as  $D(\mathcal{V}) = \sum_{l=1}^{m_u} D_l(\mathcal{V}) \phi_e(P_l)$ . Employing the DHVT based hierarchical video layer singular mode allocation, the expression for the interference constrained  $l^{th}$  mode SNR $_l$  given in (3) and the relation for  $l^{th}$  MIMO mode probability of bit-error from (4), the constrained cost function for overall video distortion  $D(\mathcal{V})$  minimization through optimal power allocation can be described as,

$$\begin{aligned} \min. & \sum_{l=1}^{m_u} D_l(\mathcal{V}) \left( \frac{\beta_l G_a(l) \left(1 - 2^{-\frac{B}{\tau-1}}\right) P_l}{\sigma_\eta^2 + \beta_l \sum_{\substack{j=1 \\ j \neq l}}^{m_u} \left(\frac{2^{-\frac{B}{\tau-1}}}{N-1}\right) P_j} \right)^{-G_d(l)} \\ \text{s.t.} & \sum_{l=1}^{m_u} P_l = P_T \\ & P_l \geq 0, 1 \leq l \leq m_u \end{aligned} \quad (5)$$

The above optimization can be solved through the following iterative procedure. Consider the optimal power allocation vector after the  $i^{th}$  iterative step given as  $\tilde{P}^{(i)} = [\tilde{P}_1^{(i)}, \tilde{P}_2^{(i)}, \dots, \tilde{P}_{m_u}^{(i)}]$ . The optimal power vector  $\tilde{P}^{(i+1)}$  corresponding to the  $(i+1)^{th}$  iteration can be obtained by solving the simplified convex optimization problem,

$$\begin{aligned} \min. & \sum_{l=1}^{m_u} B_l^{(i)}(\mathcal{V}) \left(\frac{1}{P_l}\right)^{G_d(l)} \\ \text{s.t.} & \sum_{l=1}^{m_u} P_l = P_T \\ & P_l \geq 0, 1 \leq l \leq m_u, \end{aligned} \quad (6)$$

where the coefficients  $B_l^{(i)}$ ,  $1 \leq l \leq m_u$  are defined as,

$$B_l^{(i)}(\mathcal{V}) \triangleq \frac{D_l(\mathcal{V}) \left(\beta_l G_a(l) \left(1 - 2^{-\left(\frac{B}{\tau-1}\right)}\right)\right)^{-G_d(l)}}{\left(\sigma_\eta^2 + \beta_l \sum_{\substack{j=1 \\ j \neq l}}^{m_u} \left(\frac{2^{-\left(\frac{B}{\tau-1}\right)}}{N-1}\right) \tilde{P}_j^{(i)}\right)^{-G_d(l)}}$$

It can be observed that the above constrained objective minimization paradigm is a standard form convex optimization problem. Hence, the optimal power allocation vector  $\tilde{P}^{(i+1)}$  can be readily computed from the above optimization problem using the primal-dual interior point method [10]. This iterative procedure can be initialized with the uniform power vector,

$P^{(0)} \triangleq \left(\frac{P_T}{m_u}\right) [1, 1, \dots, 1]^T$  and has been observed to converge very rapidly to the optimal power allocation solution.

Below, we describe a fast Lagrangian procedure to compute the allocation vector  $\tilde{P}^{(i+1)}$ . The Lagrangian cost function  $L(\mathcal{V}, \lambda, \tilde{\mu})$  can be formulated for the convex optimization problem described in (6) above as,

$$\sum_{l=1}^{m_u} B_l^{(i)}(\mathcal{V}) \left(\frac{1}{P_l}\right)^{G_d(l)} + \lambda \left(\sum_{j=1}^{m_u} P_j - P_T\right) - \sum_{k=1}^{m_u} \mu_k P_k.$$

From the KKT conditions for the above optimization problem, it can be shown that  $\tilde{P}_l^{(i+1)}$ , the optimal power allocation for the  $l^{th}$  mode can be expressed as a function of the dual variable  $\tilde{\lambda}^{(i+1)} > 0$  as,

$$P_l^{(i+1)} = \left(\frac{G_d(l) B_l^{(i)}(\mathcal{V})}{\tilde{\lambda}^{(i+1)}}\right)^{\frac{1}{G_d(l)+1}}. \quad (7)$$

Finally, the optimal dual variable  $\tilde{\lambda}^{(i+1)}$  can be obtained as the root of the polynomial equation,

$$\sum_{l=1}^{m_u} \left(\frac{G_d(l) B_l^{(i)}(\mathcal{V})}{\tilde{\lambda}}\right)^{\frac{1}{G_d(l)+1}} = P_T. \quad (8)$$

Hence, the optimal substream power  $\tilde{P}_l^{(i+1)}$  corresponding to the  $i^{th}$  iterative step convex optimization problem in (6) can be obtained by substituting this value of  $\tilde{\lambda}^{(i)}$  in (7). Thus, the computation of the optimal power vector  $\tilde{P}$  for each iterative step reduces essentially to finding the solution of the polynomial root finding problem in (8). This can be computed very efficiently through a Newton-step iterative procedure. It has been observed that the optimal power allocation obtained above for the hierarchical layered video transmission results in a significant reduction in video distortion compared to the sub-optimal single layer video transmission (SLVT) with equal power allocation across the singular modes corresponding to the power allocation vector  $\hat{P} = \left(\frac{P_T}{m_u}\right) [1, 1, \dots, 1]^T$  without MIMO mode diversity consideration.

## V. SIMULATION RESULTS

In our simulation setup, we consider a  $t_u = 2$  transmit antenna and  $r_u = 2$  receive antenna i.e. a  $2 \times 2$  MIMO wireless system. Each user transmits the video over the MIMO wireless system comprising of  $m_u = 2$  parallel substreams. The physical layer information symbols are QPSK modulated for wireless transmission. The reverse link channel feedback is limited to  $B = 8$  bits per MIMO mode i.e. for each of the right singular vectors  $\mathbf{w}_1, \mathbf{w}_2$ . The MIMO transmit beamforming codebook for this system therefore contains  $N = 2^B = 256$  quantized vectors and is constructed employing the MSIP minimization procedure described in section II. After each channel estimation epoch, the receiver feeds back the indices of the quantized vectors in the codebook, closest in MSIP to each of the instantaneous transmit beamforming vectors  $\mathbf{w}_1, \mathbf{w}_2$ . The transmitter beamforms the data along the corresponding quantized vectors  $\hat{\mathbf{w}}_1, \hat{\mathbf{w}}_2$  as described by

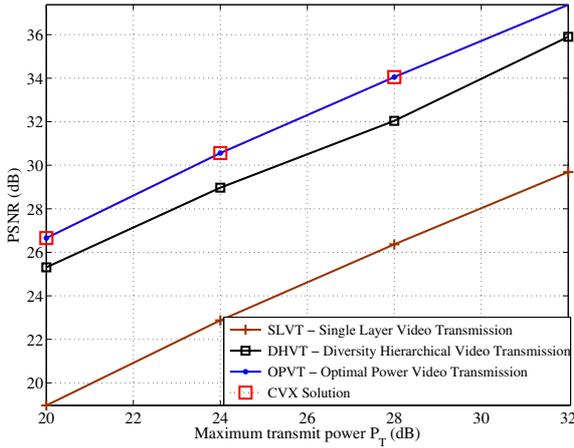


Fig. 3. PSNR plots for decoded Coastguard video sequence

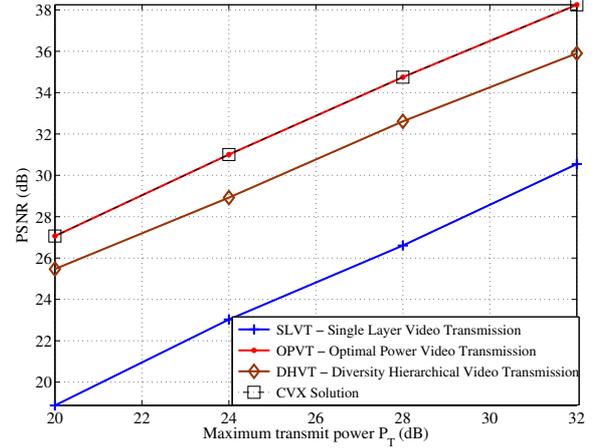


Fig. 4. PSNR plots for decoded Foreman video sequence

the system model in (2). We consider the standard Foreman, Hall, Coastguard and Mobile video sequences for MIMO based video transmission. For the hierarchical decomposition of the video sequences, we employ the level-1 Haar DWT based spatial decomposition of the intra-coded video frame into 2 hierarchical layers comprising the base spatial layer  $S_0$  and enhancement layer  $S_1$ . Similarly, 2-level HBMA [11] is employed for DPCM based hierarchical motion estimation and decomposition into temporal layers  $T_0, T_1$ . In the DHVT scheme discussed in section III, the base layers are transmitted across the strongest MIMO mode corresponding to gain  $\sigma_1$ , while the enhancement layers are transmitted over progressively weaker modes, thus significantly reducing distortion. The transmit power is distributed equally over both the spatial MIMO modes. The video distortion can be further minimized through the OPVT scheme elaborated in (5) for distribution of the limited transmit power  $P_T$  across the modes. Distortion coefficients  $D_l(\mathcal{V})$  for each video sequence are computed as described in [6]. The procedure for the optimal power vector computation is described in IV. It has been observed that the iterative process converges very rapidly to the optimal solution, requiring usually not more than 5 iterative cycles.

We compare the performance of the optimal video transmission schemes OPVT, DHVT with the sub-optimal SLVT scheme. Fig.3 and Fig.4 demonstrate the PSNR of the decoded Coastguard and Foreman video streams corresponding to the different video transmission schemes described above. It can be observed that the diversity based hierarchical transmission scheme DHVT, which employs hierarchical decomposition followed by singular mode diversity order based video layer allocation for MIMO transmission (while allocating equal power to all the substreams) results in a significant performance enhancement of around 6–7 dB in PSNR over the sub-optimal SLVT scheme. This is due to the fact that SLVT is diversity agnostic and does not employ video decomposition, which is key to achieving performance gains in the context of MIMO

video transmission. This performance of DHVT is further improved by about 1 – 2 dB in PSNR through the optimal power allocation based video transmission scheme OPVT as described in section IV. Also, the optimal power vector obtained from the constrained convex optimization problem solver CVX agrees well with the theoretical results presented in (7), thus lending support to the derivation of the optimal power allocation vector in section IV. Further, in Fig.5 we plot the decoded PSNR for several values of  $B$ , the number of feedback bits per singular mode. It can be observed therein that the PSNR achieved for the OPVT scheme for  $B = 4$ , i.e. a total feedback of  $mB = 8$  bits on the reverse link, is close to that of perfect CSI feedback (i.e.  $B = \infty$ ). Hence, the codebook based quantized beamforming vector feedback scheme is ideally suited for practical wireless systems since it achieves a performance close to the ideal system with perfect CSI without incurring large bit overheads on the reverse link.

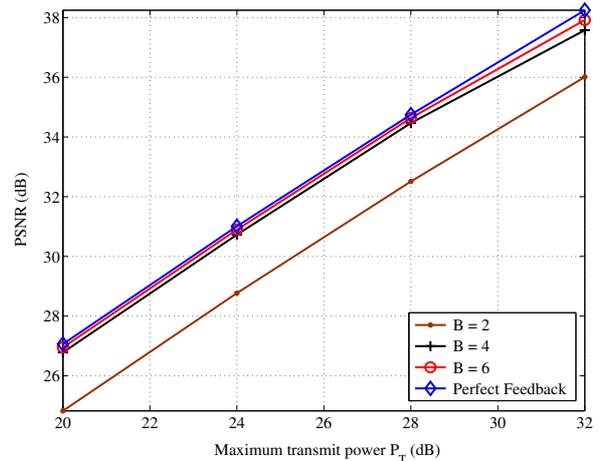


Fig. 5. PSNR values for decoded video sequence for different feedback bits ( $B = 2, 4, 6$ ) per MIMO channel model compared with perfect feedback



Fig. 6. Frame quality comparison of decoded Foreman video sequence from the transmission employing OPVT (left) and SLVT (right)

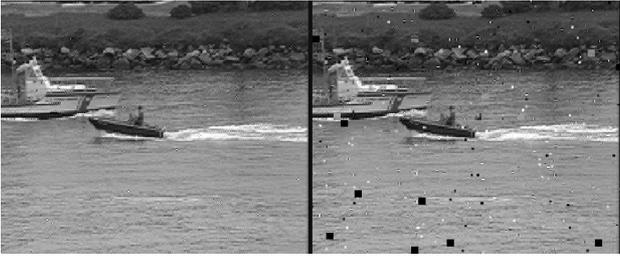


Fig. 7. Frame quality comparison of decoded coastguard video sequence from the transmission employing OPVT (left) and SLVT (right)

The superior performance of OPVT over SLVT is visually illustrated by a comparison of the decoded received frames for the Foreman and Coastguard video sequences in Fig.6 and Fig.7 respectively. The OPVT frames have significantly lower artifacts introduced by the bit-errors over the fading MIMO wireless channel. Thus, the DHVT and OPVT schemes are ideally suited for video transmission over MIMO wireless channels as they minimize the video distortion through intelligent use of the diversity properties of the MIMO transmission modes. Further, it only involves codebook based feedback of quantized transmit beamforming vectors, thereby greatly reducing the overheads on the MIMO reverse wireless link.

## VI. CONCLUSION

In this work we proposed MIMO SVD based video transmission schemes viz. DHVT, OPVT for minimization of video distortion in 4G MIMO wireless systems. Further, the proposed schemes employ codebook based quantized feedback,

thus resulting in significantly lower overheads on the reverse link, while simultaneously not compromising on the beamforming gain achievable in the MIMO system. We described the paradigm of DHVT for MIMO mode diversity order based hierarchical video allocation, which results in a significant reduction in video distortion. Subsequently, we formulated the optimization problem for OPVT taking into account the distortion caused by quantized beamforming due to limited feedback. The Lagrangian based closed form solution for iterative computation of the MIMO optimal power vector has been derived as the solution of a simplistic polynomial root computation. Simulation results show that the proposed OPVT and DHVT schemes are significantly superior to the diversity order agnostic equal power allocation based SLVT scheme for video transmission in MIMO wireless systems.

## REFERENCES

- [1] David Tse and P. Viswanath, *Fundamentals of wireless communication*, Cambridge University Press, 2005.
- [2] M.F. Sabir, A.C. Bovik, and R.W. Heath, "Unequal power allocation for JPEG transmission over MIMO systems," *Image Processing, IEEE Transactions on*, vol. 19, no. 2, pp. 410–421, Feb. 2010.
- [3] M. Tesanovic, D.R. Bull, and A. Doufexi, "Enhancing video quality for multiple-description MIMO transmission through unequal power allocation between eigen-modes," in *Image Proc., 2008. ICIP 2008. 15th IEEE International Conference on*, Oct. 2008, pp. 2024–2027.
- [4] S. Zhao, M. You, and L. Gui, "Scalable video delivery over MIMO OFDM wireless systems using joint power allocation and antenna selection," in *Multimedia Sig. Proc. 2009. MMSP '09. IEEE International Workshop on*, Oct. 2009, pp. 1–6.
- [5] June Chul Roh and B.D. Rao, "Channel feedback quantization methods for MISO and MIMO systems," in *Personal, Indoor and Mobile Radio Communications, 2004. PIMRC 2004. 15th IEEE International Symposium on*, Sept. 2004, vol. 2, pp. 805–809 Vol.2.
- [6] Sohil Mahajan and Aditya K Jagannatham, "Hierarchical DWT based optimal diversity power allocation for video transmission in 4G OFDMA wireless systems," in *Imaging Systems and Techniques, 2011. IST 2011, IEEE International Conference on*, May. 2011, pp. 279–283.
- [7] A. Scaglione, P. Stoica, S. Barbarossa, G. B. Giannakis, and H. Sampath, "Optimal designs for space-time linear precoders and decoders," in *IEEE Trans. Signal Processing*, May 2002, vol. 50, pp. 1051–1064.
- [8] J.C. Roh and B.D. Rao, "Transmit beamforming in multiple-antenna systems with finite rate feedback: a VQ-based approach," *Information Theory, IEEE Transactions on*, vol. 52, no. 3, pp. 1101–1112, march 2006.
- [9] L. Garcia-Ordenez, D.P. Palomar, A. Pages-Zamora, and J.R. Fonollosa, "Analytical BER performance in spatial multiplexing MIMO systems," in *Signal Processing Advances in Wireless Communications, 2005 IEEE 6th Workshop on*, June 2005, pp. 460–464.
- [10] Stephen Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.
- [11] Y. Wang, Y. Zhang, and J. Ostermann, *Video Processing and Communications*, Prentice Hall PTR, 2001.