# Power Allocation for MIMO-OFDM based CR with Spatial Constraints and CSI Uncertainty

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Abstract—This paper presents optimal power allocation strategies for a MIMO (Multiple Input Multiple Output) OFDM (Orthogonal Frequency Division Multiplexing) based cognitive radio (CR) system. The proposed power allocation schemes maximize the downlink transmission rate of the CR users under spatial interference constraints, considering both the availability and absence of the primary user (PU) Channel State Information (CSI). It is demonstrated that the isotropic interference minimization in the absence of PU CSI can be formulated as a semi-definite program (SDP) while it reduces to linear interference constraints based CR user sum-rate maximization in the presence of PU CSI. Closed form power allocation expressions are derived for the above scenarios under a sum-trace interference relaxation. Further, we also consider the MIMO-OFDM rate maximization with CSI uncertainty and formulate separate optimal power allocation schemes for the stochastic and worst case scenarios. Simulation results presented validate the performance of the proposed schemes.

#### I. INTRODUCTION

Under the Cognitive Radio (CR) paradigm, vacant primary user (PU) licensed spectral bands or spectral holes are opportunistically allocated to secondary users (SUs) to improve the efficiency of spectrum utilization. Further, MIMO-OFDM [1] based CR systems have gained significant appeal for usage in futuristic dynamic spectrum access based wireless networks. Recently, the authors in [2] presented an optimal scheme for interweave CR system rate maximization based on a novel spectral distance dependent characterization of the interference. Based on this, a similar scheme has been presented for MIMO OFDM power allocation in [3]. However, the scheme presented there in is suboptimal as they consider per antenna power allocation, while it is well known that per singular mode power allocation is optimal in MIMO systems. Further, the model is restrictive as it considers only single antenna and not MIMO wireless systems for the narrowband PUs. Also, they do not consider the PU interference in SU communication. Hence, we propose new schemes for optimal CR power allocation based on spatial interference constraints in a MIMO OFDM wireless network, considering both the presence and absence of PU channel state information (CSI). Moreover, we consider power allocation under CSI uncertainty using the separate frameworks of stochastic and worst case rate maximization. Simulation results demonstrate the efficacy of the proposed schemes compared to the conventional uniform, Aditya K. Jagannatham Department of Electrical Engineering Indian Institute of Technology Kanpur Kanpur-208016, India Email: adityaj@iitk.ac.in

proportional power allocation schemes and the scheme [3] in existing literature.

The rest of the paper is organized as follows. Section II describes the MIMO-OFDM CR system model followed by optimal power allocation schemes in section III. Section IV describes rate maximization with uncertainty in the MIMO CSI. Simulation results are presented in section V and we conclude with section VI.

## II. SYSTEM MODEL

We consider a MIMO-OFDM based CR system comprising of a base station (BS) with a total of N subchannels for the CR users. The CR base station is equipped with  $N_t$ transmit antennas while each CR receiver possesses  $N_r$  receive antennas. Similarly, the MIMO PU system consists of a PU base station with  $L_t$  transmit antennas and L PUs each with  $L_r$  receive antennas. The  $l^{th}$  PU occupies a spectral band of bandwidth  $B_l$  Hz, while the CR users occupy spectral holes of bandwidth  $\Delta f$  Hz each. The MIMO channel between the CR base station and the CR user allocated the  $n^{th}$  CR subchannel is denoted by  $\mathbf{H}_n \in \mathbb{C}^{N_r \times N_t}$  while  $\mathbf{G}_n^l \in \mathbb{C}^{L_r \times N_t}$  denotes the channel matrix for the interference channel between the CR base station and the  $l^{th}$  PU corresponding to the  $n^{th}$ CR subchannel. The received vector  $\mathbf{y}_n^c(k) \in \mathbb{C}^{N_r \times 1}$  over the  $n^{th}$  CR subchannel corresponding to the transmit vector  $\mathbf{x}_n^c(k) \in \mathbb{C}^{N_t \times 1}$  at the  $k^{th}$  time instant is given as,

$$\mathbf{y}_{n}^{c}\left(k\right) = \mathbf{H}_{n}\mathbf{x}_{n}^{c}\left(k\right) + \eta\left(k\right),$$

where  $\eta \in \mathbb{C}^{N_r \times 1}$  is the spatio-temporally additive white Gaussian noise vector with covariance  $\mathbb{E} \{\eta \eta^H\} = \sigma_0^2 \mathbf{I}_{N_r}$ . Let the SVD of the channel matrix  $\mathbf{H}_n$  be given as,

$$\mathbf{H}_{n} = \mathbf{U}_{H}^{n} \boldsymbol{\Sigma}_{H}^{n} \left( \mathbf{V}_{H}^{n} \right)^{H},$$

where  $\mathbf{U}_{H}^{n}, \mathbf{V}_{H}^{n}$  are unitary matrices and  $\Sigma_{H}^{n}$  is the diagonal matrix containing the non-negative singular values  $\sigma_{n,i}$ . Let  $N_{\min}$  denote the number of non-zero singular values, where  $N_{\min} \leq \min \{N_r, N_t\}$ . It is well known that the optimal transmit precoding matrix corresponding to the MIMO channel matrix  $\mathbf{H}_n$  is  $\mathbf{V}_{H}^{n}$ . Hence, the transmit vector  $\mathbf{x}_{n}^{c}(k)$  is given as  $\mathbf{x}_{n}^{c}(k) = \mathbf{V}_{H}^{n}\tilde{\mathbf{x}}_{n}^{c}(k)$ , where  $\tilde{\mathbf{x}}_{n}^{c}(k)$  is the vector of modulated constellation symbols. The transmit Covariance

matrix for the  $n^{th}$  subchannel  $\mathbf{R}_n(\mathbf{p}_n^c)$  is given as,

$$\mathbf{V}_{H}^{n} \mathrm{E}\left\{\tilde{\mathbf{x}}_{n}^{c}\left(k\right)\left(\tilde{\mathbf{x}}_{n}^{c}\right)^{H}\left(k\right)\right\}\left(\mathbf{V}_{H}^{n}\right)^{H} = \underbrace{\mathbf{V}_{H}^{n} \mathcal{D}\left(\mathbf{p}_{n}^{c}\right)\left(\mathbf{V}_{H}^{n}\right)^{H}}_{\mathbf{R}_{n}\left(\mathbf{p}_{n}^{c}\right)}$$

where  $\mathcal{D}(\mathbf{p}_n^c)$  is the diagonal matrix, with the principal diagonal as  $\mathbf{p}_n^c = \left[P_{n,1}^c, P_{n,2}^c, ..., P_{n,N_{\min}}^c\right]^T$  and  $P_{n,i}^c$  is the power allocated to the  $i^{th}$  MIMO mode aligned with the transmit beamforming vector  $\mathbf{v}_H^n(i)$ , the  $i^{th}$  column of the transmit precoding matrix  $\mathbf{V}_H^n$ . The interference introduced by the  $n^{th}$  CR subchannel in the  $l^{th}$  PU band is denoted by  $\mathbf{J}_{n,l}(d_{n,l}, \mathbf{p}_n^c) \in \mathbb{C}^{L_r \times L_r}$ , which can be expressed as,

$$S_{n,l}^{c} \mathbb{E}\left\{\mathbf{G}_{n}^{l} \mathbf{x}_{n}^{c}\left(k\right) \mathbf{x}_{n}^{c}^{H}\left(k\right) \left(\mathbf{G}_{n}^{l}\right)^{H}\right\} = \underbrace{S_{n,l}^{c} \mathbf{G}_{n}^{l} \mathbf{R}_{n}\left(\mathbf{p}_{n}^{c}\right) \mathbf{G}_{n}^{l}}_{\mathbf{J}_{n,l}\left(d_{n,l},\mathbf{p}_{n}^{c}\right)}$$

where  $d_{n,l}$  is the spectral distance between the  $n^{th}$  CR subchannel and the  $l^{th}$  PU band. The quantity  $S_{n,l}^c$  is the interference factor for the  $l^{th}$  PU due to the  $n^{th}$  CR subchannel, defined as  $S_{n,l}^c \triangleq T_s \int_{d_{n,l} - \frac{B_l}{2}}^{d_{n,l} + \frac{B_l}{2}} \left(\frac{\sin \pi f T_s}{\pi f T_s}\right)^2 df$ , where  $T_s$  is the OFDM symbol time. Considering a raised cosine spectrum with rolloff factor  $\beta$  for the narrowband PUs, the spectral mask P(f) of the  $l^{th}$  PU can be represented by the expression [4],

$$P(f) = \begin{cases} \frac{1}{B_l} & 0 \le |f| < f_1\\ \frac{1}{2B_l} \left(1 - \sin \frac{\pi(|f| - B_l/2)}{B_l - 2f_1}\right) & f_1 \le |f| < B_l - f_1\\ 0 & |f| \ge B_l - f_1 \end{cases}$$

where  $f_1$  is a frequency parameter that is defined as  $f_1 = \frac{1}{2}(1-\beta)B_l$  and the  $l^{th}$  narrowband is restricted to  $\left[-\frac{1}{2}B_l, \frac{1}{2}B_l\right]$ . Let the channel matrix for the interference channel between the PU base station and the  $n^{th}$  CR user subchannel induced by the  $l^{th}$  PU signal be denoted by  $\mathbf{W}_n^l \in \mathbb{C}^{N_r \times L_t}$ . Hence, the corresponding interference covariance denoted by  $\mathbf{Q}_{n,l}\left(d_{n,l}, P_l^T\right) \in \mathbb{C}^{N_r \times N_r}$  is,

$$\mathbf{Q}_{n,l}\left(d_{n,l}, P_l^T\right) = S_{n,l}^p \mathbf{W}_n^l \mathbf{R}_l \left(P_l^T\right) \left(\mathbf{W}_n^l\right)^H \qquad (1)$$

where  $S_{n,l}^p$  is the power spilling factor for the  $n^{th}$  subchannel caused due to the  $l^{th}$  PU defined as  $S_{n,l}^p \triangleq \int_{d_{n,l}-\frac{1}{2}\Delta f}^{d_{n,l}+\frac{1}{2}\Delta f} |P(f)|^2 df$ . In the absence of PU CSI, the transmit covariance matrix can be assumed to be isotropic with  $\mathbf{R}_l \left(P_l^T\right) = \frac{1}{L_t} P_l^T \mathbf{I}_{L_t}$ , where  $P_l^T$  is the total allocated power to the  $l^{th}$  user at the PU base station. Also, the interference  $I_{n,l}^j$ experienced by the  $j^{th}$  receive mode of the  $n^{th}$  CR subchannel due to the  $l^{th}$  PU can be computed as,

$$I_{n,l}^{j} = \left(\mathbf{u}_{H}^{n}\left(j\right)\right)^{H} \mathbf{Q}_{n,l}\left(d_{n,l}, P_{l}^{T}\right) \mathbf{u}_{H}^{n}\left(j\right), \qquad (2)$$

where  $1 \leq j \leq N_{\min}$  and  $\mathbf{u}_{H}^{n}(j)$  is the  $j^{th}$  column of the receive beamforming matrix  $\mathbf{U}_{H}^{n}$ , which is the receive beamformer for the  $j^{th}$  mode of the  $n^{th}$  CR subchannel.

#### III. OPTIMAL MIMO-OFDM POWER ALLOCATION

We consider an interference threshold of  $I_{th}$  for the PUs. Since PUs employ a MIMO wireless system, in principle it is essential to limit the interference caused by the CR users at each mode of each PU. However, in the absence of PU CSI, this can be formulated as limiting the worst case isotropic interference caused by the CR users. Hence, the optimal power allocation for the CR user subchannels is obtained as a solution to a convex semi-definite programming (SDP) problem [5] described as,

$$\max \cdot \sum_{n=1}^{N} \sum_{i=1}^{N_{\min}} \Delta f \log \left( 1 + \frac{P_{n,i}^{c} \sigma_{n,i}^{2}}{\sigma_{0}^{2} + \sum_{l=1}^{L} I_{n,l}^{i}} \right)$$
  
s.t. 
$$\sum_{n=1}^{N} \mathbf{J}_{n,l} \left( d_{n,l}, \mathbf{p}_{n}^{c} \right) \preceq \frac{1}{L_{r} L} I_{th} \mathbf{I}_{L_{r}}, \ 1 \leq l \leq L \quad (3)$$
$$\mathbf{p}_{n}^{c} \succeq 0, \ 1 \leq n \leq N.$$

As demonstrated above, the isotropic interference constraint at each of the L users reduces to an SDP constraint, where the generalized inequality is on the convex cone of positive semi-definite matrices. Further, this guarantees a low level of interference at each PU. A relaxed constraint problem [2], considering the sum of total interference across all PUs can be formulated by replacing (3) as,

$$\sum_{l=1}^{L} \sum_{n=1}^{N} \operatorname{tr} \left( \mathbf{J}_{n,l} \left( d_{n,l}, \mathbf{p}_{n}^{c} \right) \right) \leq I_{th}.$$
(4)

This sum-trace relaxed constraint optimization problem yields the optimal power allocation for capacity maximization as,

$$P_{n,i}^{c} = \left( \left( \lambda \ \alpha_{n,i} \right)^{-1} - \left( \gamma_{n,i} \right)^{-1} \right)^{+}$$
(5)

where  $x^+ = x$  if x > 0 and 0 otherwise. The quantity  $\gamma_{n,i} = \sigma_{n,i}^2 \left(\sigma_0^2 + \sum_{l=1}^L I_{n,l}^i\right)^{-1}$  and  $\alpha_{n,i} = \sum_{l=1}^L b_n^l(i)$ , where  $b_n^l(i)$  denotes the  $i^{th}$  diagonal element of the matrix  $\mathbf{B}_n^l = S_{n,l}^c(\mathbf{V}_H^n)^H (\mathbf{G}_n^l)^H \mathbf{G}_n^l \mathbf{V}_H^n$ . The quantity  $\lambda$  is the Lagrange dual variable, derived such that  $\sum_{n=1}^N \sum_{i=1}^{N_{\min}} \alpha_{n,i} P_{n,i}^c = I_{th}$ . Naturally, the rate achieved with this relaxed constraint is much higher than the rate with individual isotropic interference constraints. However, the above relaxed constraint can result in asymmetric interference, with high interference at PUs close to a CR with a good channel, and lower interference at others.

## A. Perfect PU CSI at CR Base Station

In the presence of CSI, let the MIMO channel between the PU base station and the  $l^{th}$  PU be denoted by the channel coefficient matrix  $\mathbf{D}_l \in \mathbb{C}^{L_r \times L_t}$ . Let the SVD of  $\mathbf{D}_l$  be given as  $\mathbf{D}_l = \mathbf{U}_D^l \mathbf{\Sigma}_D^l (\mathbf{V}_D^l)^H$ . The beamforming matrix  $\mathbf{V}_D^l$  is the optimal precoding matrix for the transmit vector  $\mathbf{x}_l^p(k)$  of the  $l^{th}$  PU. Hence, the transmit covariance matrix corresponding to the  $l^{th}$  PU can be expressed as,  $R_l(\mathbf{p}_l^p) = \mathbf{V}_D^l \mathcal{D}(\mathbf{p}_l^p) (\mathbf{V}_D^l)^H$ , where  $\mathbf{p}_l^p = \begin{bmatrix} P_{l,1}^p, P_{l,2}^p, ..., P_{l,L_{\min}}^p \end{bmatrix}^T$  is the power allocation vector of the  $l^{th}$  PU. Therefore, the interference introduced by the  $l^{th}$  PU at the  $n^{th}$  CR subchannel is denoted by  $\mathbf{Q}_{n,l}(d_{n,l}, \mathbf{p}_l^p) \in \mathbb{C}^{N_r \times N_r}$ , which can be

expressed as,

$$\mathbf{Q}_{n,l}\left(d_{n,l},\mathbf{p}_{l}^{p}\right) = S_{n,l}^{p}\mathbf{W}_{n}^{l}\mathbf{V}_{D}^{l}\mathcal{D}\left(\mathbf{p}_{l}^{p}\right)^{H}\left(\mathbf{W}_{n}^{l}\right)^{H}.$$

Further, the interference  $K_{n,l}^{j}$  by the  $l^{th}$  PU at the  $j^{th}$  receive mode of the  $n^{th}$  CR subchannel, corresponding to the  $j^{th}$  receive beamforming vector is  $K_{n,l}^{j} = (\mathbf{u}_{H}^{n}(j))^{H} \mathbf{Q}_{n,l} (d_{n,l}, \mathbf{p}_{l}^{p}) \mathbf{u}_{H}^{n}(j)$ . Hence, in the presence of PU CSI the equivalent framework for CR rate maximization can be formulated as,

$$\begin{aligned} \max &: \sum_{n=1}^{N} \sum_{i=1}^{N_{\min}} \Delta f \log \left( 1 + \frac{P_{n,i}^{c} \sigma_{n,i}^{2}}{\sigma_{0}^{2} + \sum_{l=1}^{L} K_{n,l}^{i}} \right) \\ \text{s.t.} \left( \mathbf{u}_{D}^{l}\left(j\right) \right)^{H} \left( \sum_{n=1}^{N} \mathbf{J}_{n,l}\left(d_{n,l}, \mathbf{p}_{n}^{c}\right) \right) \mathbf{u}_{D}^{l}\left(j\right) \leq \frac{I_{th}}{L_{\min}L} \\ &1 \leq l \leq L, 1 \leq j \leq L_{\min} \\ &\mathbf{p}_{n}^{c} \succeq 0, \ 1 \leq n \leq N \end{aligned}$$

where  $\mathbf{u}_D^l(j)$  is the receive beamforming vector representing the  $j^{th}$  receive mode of the  $l^{th}$  PU. The linear interference constraints limit the interference in each of the  $L_{\min}$  receive modes at each of the L users to  $\frac{1}{L_{\min}L}$  of the threshold  $I_{th}$ , where  $L_{\min} \leq \min \{L_r, L_t\}$ . The above paradigm can be readily solved by a convex solver to obtain the optimal CR power allocation. Sum-trace relaxation of the above constraint yields,

$$\sum_{l=1}^{L}\sum_{j=1}^{L_{\min}}\operatorname{tr}\left(\left(\sum_{n=1}^{N}\mathbf{J}_{n,l}\left(d_{n,l},\mathbf{p}_{n}^{c}\right)\right)\mathbf{u}_{D}^{l}\left(j\right)\left(\mathbf{u}_{D}^{l}\left(j\right)\right)^{H}\right) \leq I_{th}$$

The optimal power allocation corresponding to the above relaxation is  $P_{n,i}^c = \left( \left( \tilde{\lambda} \ \tilde{\alpha}_{n,i} \right)^{-1} - \left( \tilde{\gamma}_{n,i} \right)^{-1} \right)$ , where  $\tilde{\gamma}_{n,i} \triangleq \sigma_{n,i}^2 \left( \sigma_0^2 + \sum_{l=1}^L K_{n,l}^i \right)^{-1}$  and  $\tilde{\alpha}_{n,i} = \sum_{l=1}^L \tilde{b}_n^l(i)$  with  $\tilde{b}_n^l(i)$  denoting the diagonal elements of the matrix  $\tilde{\mathbf{B}}_n^l = S_{n,l}^c (\mathbf{V}_H^n)^H (\mathbf{G}_n^l)^H \left( \sum_{j=1}^{L_{\min}} \mathbf{u}_D^l(j) \left( \mathbf{u}_D^l(j) \right)^H \right) \mathbf{G}_n^l \mathbf{V}_H^n$ . The Lagrangian dual variable  $\tilde{\lambda}$  is obtained using a procedure similar to the one described previously.

## IV. CR POWER ALLOCATION WITH CSI UNCERTAINTY

We consider the case of imperfect CSI of the interference channel  $\mathbf{G}_n^l$  induced between the CR base station and the  $l^{th}$  PU corresponding to different CR subchannels  $1 \le n \le N$ . This uncertainty arises due to the estimation error inherent in the estimates of the CSI at the CR receiver and also due to limited feedback between the CR receiver and the CR base station.

#### A. Stochastic Uncertainty

Consider the true channel coefficient matrix  $\mathbf{G}_n^l$  for the interference channel between the  $n^{th}$  CR user subchannel and the  $l^{th}$  PU to be given as  $\mathbf{G}_n^l = \bar{\mathbf{G}}_n^l + \Psi_n^l$ , where  $\bar{\mathbf{G}}_n^l$  is the nominal interference channel matrix obtained through the channel estimation procedure and  $\Psi_n^l$  is the error matrix with circularly symmetric uncorrelated zero mean random variables



Fig. 1: Maximum transmission rate of the CR users versus the interference threshold  $(I_{th})$ 

of variance  $\sigma_G^2$ . Hence, the average interference covariance  $\mathbf{J}_{n,l}^s(d_{n,l}, \mathbf{p}_n^c)$  introduced to the  $l^{th}$  PU due to transmission on the  $n^{th}$  CR subchannel can be simplified as,

$$\underbrace{S_{n,l}^{c}\left(\bar{\mathbf{G}}_{n}^{l}\mathbf{R}_{n}\left(\mathbf{p}_{n}^{c}\right)\left(\bar{\mathbf{G}}_{n}^{l}\right)^{H}+\sigma_{G}^{2}\left(\sum_{i=1}^{N_{\min}}P_{n,i}^{c}\right)\mathbf{I}_{L_{r}}\right)}_{\mathbf{J}_{n,l}^{s}\left(d_{n,l},\mathbf{p}_{n}^{c}\right)}$$

Therefore, the stochastic CSI uncertainty considered above can be incorporated in the interference constraint in the original framework (3) by recasting the interference constraint as,

$$\sum_{n=1}^{N} \mathbf{J}_{n,l}^{s} \left( d_{n,l}, \mathbf{p}_{n}^{c} \right) \preceq \frac{1}{L_{r} L} I_{th} \mathbf{I}_{L_{r}}, \ 1 \leq l \leq L.$$

The sum-trace relaxation of the above constraint simplifies it as  $\sum_{l=1}^{L} \sum_{n=1}^{N} \operatorname{tr} \left( \mathbf{J}_{n,l}^{s} (d_{n,l}, \mathbf{p}_{n}^{c}) \right) \leq I_{th}$ . The closed form expression for optimal power allocation can be derived as  $P_{n,i}^{c} = \left( \left( \hat{\lambda} \ \hat{\alpha}_{n,i} \right)^{-1} - (\gamma_{n,i})^{-1} \right)$ , where the quantity  $\hat{\alpha}_{n,i}$ is defined as  $\hat{\alpha}_{n,i} = \sum_{l=1}^{L} b_{n}^{l}(i) + \sigma_{G}^{2} S_{n,l}^{c} L_{r}$ .

## B. Worst Case Uncertainty

Let the channel coefficient matrix  $\mathbf{G}_n^l$  between the  $n^{th}$  CR user subchannel and the  $l^{th}$  PU be given as  $\mathbf{G}_n^l = \bar{\mathbf{G}}_n^l + u_n \Delta \mathbf{G}_n^l$ , where  $\bar{\mathbf{G}}_n^l$  is the nominal channel estimate similar to above,  $\Delta \mathbf{G}_n^l$  is the variation matrix and the random variable  $u_n$  is the uncertainty parameter, characterized by the probability density function  $f_U(u)$ . We assume that the region of support of u lies in the interval [-a, a]. The expression for the interference  $\mathbf{J}_{n,l}^w(d_{n,l}, \mathbf{p}_n^c, u_n)$  introduced to the  $l^{th}$  PU due to the  $n^{th}$  CR user subchannel is given as,

$$S_{n,l}^{c} \mathbf{E} \left\{ \left( \bar{\mathbf{G}}_{n}^{l} + u_{n} \Delta \mathbf{G}_{n}^{l} \right) \mathbf{R}_{n} \left( \mathbf{p}_{n}^{c} \right) \left( \bar{\mathbf{G}}_{n}^{l} + u_{n} \Delta \mathbf{G}_{n}^{l} \right)^{H} \right\}$$

It can be seen that  $\mathbf{J}_{n,l}^w(d_{n,l}, \mathbf{p}_n^c, u_n)$  is a convex quadratic function of  $u_n$ . Therefore the maximum value or worst case interference will occur for  $u_n \in \{a, -a\}$ . Hence, the constraint in our original optimization problem (3) can be replaced by



Fig. 2: Maximum interference at any PU receive mode vs.  $I_{th}$ 

the following equivalent constraint,

$$\sum_{n=1}^{N} \mathbf{J}_{n,l}^{w} \left( d_{n,l}, \mathbf{p}_{n}^{c}, (2i_{n}-1) \, a \right) \preceq \frac{1}{L_{r} \, L} I_{th} \mathbf{I}_{L_{r}}$$

where  $0 \le i_n \le 1, 1 \le l \le L$ . The interference introduced to each PU in the worst case scenario is described by the above linear matrix inequalities (LMI). Solving this with an SDP solver yields the optimal MIMO-OFDM power allocation which limits the worst case interference.

## V. SIMULATION AND RESULTS

The CR MIMO-OFDM system was simulated in MATLAB with L = 4 PUs and 4 CR users. Both the CR and PU system employ  $4 \times 4$  MIMO wireless systems, i.e.  $L_r = L_t = N_r =$  $N_t = 4$ . Each CR user is allocated 15 OFDM subchannels where each subchannel is a group of 24 subcarriers of 10.94 KHz frequency each. The frequency bands assigned to PUs,  $B_1, B_2, B_3, B_4$  are 4.923 MHz, 8.205 MHz, 8.205 MHz, 10.174 MHz respectively. We consider a noise variance of  $\sigma_0^2 = -30$  dB. The fading channel coefficients of the matrices  $\mathbf{H}_n, \mathbf{D}_n, \mathbf{W}_n^l$  and  $\mathbf{G}_n^l$  are assumed to be Rayleigh fading with an average channel power gain of 1 dB. We assumed a total PU transmit power of 4 mW.

In Fig.1 we compare the transmission rates achieved by the different power allocation schemes presented above. The plots reflect that for a given interference threshold, optimal power allocation for the relaxed sum-trace interference constraint (5) achieves the highest transmission rate for the CR users. The performance of the optimal power allocation schemes with directional and isotropic interference constraints, corresponding to availability and absence of PU CSI respectively, achieve slightly lower rate owing to stringent per PU interference restrictions. For comparison, we also plot the performance of proportional and uniform power allocation schemes. In proportional power allocation (PPA), each CR user is assigned a power inversely proportional to the sum interference factors given as  $P_{n,i}^{\text{PPA}} = (\alpha_{n,i}NN_{\min})^{-1} I_{th}$ . Similarly, in uniform power allocation (UPA), the interference threshold  $I_{th}$  is allocated uniformly over each subchannel irrespective

of its spectral distance from the PU, to yield the allocation



Fig. 3: Total capacity of the CR users versus the interference threshold  $(I_{th})$ 

 $P_{n,i}^{\text{UPA}} = \left(\sum_{n=1}^{N} \sum_{i=1}^{N_{\min}} \alpha_{n,i}\right)^{-1} I_{th}$ . The advantage of PPA is that it has a significantly low implementation complexity compared to the optimal power allocation schemes. UPA naturally results in significant rate degradation arising from ignoring the dependence of interference on the SU channel state information. In Fig.2 we plot the maximum interference along the PU receive modes. It can be observed that the uncertainty aware schemes presented in section IV result in significantly lower PU interference compared to schemes that ignore uncertainty, thus ensuring reliable PU communication in CR scenarios.

In Fig.3 we compare the performance of the scheme (4) with a similar sum-interference constraint based MIMO-OFDM power allocation scheme employed in [3]. It can be readily seen that the performance of our proposed scheme in (4) is superior due to the fact that our scheme considers the spatial covariance based per mode allocation and thus achieves higher capacity for CR subchannels, while limiting the interference.

## VI. CONCLUSION

In this paper we have presented optimal power allocation schemes for the downlink transmission scenario of MIMO-OFDM based CR systems with spatial interference constraints. Closed form expressions for power allocation have been derived considering both the availability and non-availability of PU CSI. Results have also been presented for rate maximization under stochastic and worst case CSI uncertainty. Simulation results show that the proposed Optimal schemes have a superior performance compared to existing schemes.

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