Technical Report: Distributed Detection in Massive MIMO Wireless Sensor Networks under Perfect and Imperfect CSI

Apoorva Chawla, *Student Member, IEEE,* Adarsh Patel, *Member, IEEE,*
Aditya K. Jagannatham, *Member, IEEE*
and Pramod K. Varshney, *Fellow, IEEE*

I. Signaling Matrices

For the antipodal signaling scenario with $\mathbf{u}_{k,0} = -\mathbf{u}_k$ and $\mathbf{u}_{k,1} = \mathbf{u}_k$, substituting the expressions for $\mu_{T_\mathrm{A}|\mathcal{H}_0}$, $\mu_{T_\mathrm{A}|\mathcal{H}_1}$, $\sigma^2_{T_\mathrm{A}|\mathcal{H}_0}$ from Theorem 1 in (55) in [1], the deflection coefficient $d^2_\mathrm{A}(\mathbf{u})$ for $\mathbf{u} = \mathbf{u}_\mathrm{A} = \mathrm{vec}(\mathbf{U}_A^T)$, can be derived as

$$\begin{aligned} d^2_\mathrm{A}(\mathbf{u}_\mathrm{A}) &= \frac{(\mu_{T_\mathrm{A}|\mathcal{H}_1} - \mu_{T_\mathrm{A}|\mathcal{H}_0})^2}{\sigma^2_{T_\mathrm{A}|\mathcal{H}_0}} \\ &= \frac{\left(\sum_{k=1}^K \sqrt{p_u} M\beta_k a_k (b_k - c_k)\|\mathbf{u}_k\|^2\right)^2}{\sum_{k=1}^K M\beta_k a_k^2 \|\mathbf{u}_k\|^2 \left(p_u M\beta_k(1-c_k^2)\|\mathbf{u}_k\|^2 + \frac{\sigma_n^2}{2}\right)} \\ &= \frac{(\mathbf{u}_\mathrm{A}^H \boldsymbol{\Gamma}_L \mathbf{u}_\mathrm{A})^2}{(\mathbf{u}_\mathrm{A}^H \boldsymbol{\Psi}_L \mathbf{u}_\mathrm{A})^2 + \mathbf{u}_\mathrm{A}^H \boldsymbol{\Theta}_L \mathbf{u}_\mathrm{A}}. \end{aligned}$$

The numerator of the above expression can be further expressed as

$$\mathbf{u}_\mathrm{A}^H \boldsymbol{\Gamma}_L \mathbf{u}_\mathrm{A} = [\mathbf{u}_1^H, \mathbf{u}_2^H, \ldots, \mathbf{u}_K^H](\boldsymbol{\Gamma} \otimes \mathbf{I}_L) \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \\ \vdots \\ \mathbf{u}_K \end{bmatrix} = \sum_{k=1}^K \sqrt{p_u} M\beta_k a_k (b_k - c_k)\|\mathbf{u}_k\|^2,$$

A. Chawla and A. K. Jagannatham are with the Department of Electrical Engineering, Indian Institute of Technology Kanpur, Kanpur, India 208016. Email: {capoorva, adityaj}@iitk.ac.in.

A. Patel and P. K. Varshney are with the Department of Electrical Engineering and Computer Science, Syracuse University, Syracuse, NY, 13244, USA. Email: {apatel31, varshney}@syr.edu

where $\otimes$ represents the Kronecker product, the matrices $\mathbf{\Gamma}_L \in \mathbb{C}^{KL\times KL}$ and $\mathbf{\Gamma} \in \mathbb{C}^{K\times K}$ are diagonal. The matrix $\mathbf{\Gamma}_L$ can be further expanded as

$$\begin{aligned}\mathbf{\Gamma}\otimes\mathbf{I}_L &= (\sqrt{p_u}M\mathbf{D}\text{diag}(a_1,a_2,\ldots,a_K)\text{diag}((b_1-c_1),(b_2-c_2),\ldots,(b_K-c_K)))\otimes\mathbf{I}_L\\ &= \sqrt{p_u}M\text{diag}(\beta_1 a_1(b_1-c_1)\mathbf{I}_L,\beta_2 a_2(b_2-c_2)\mathbf{I}_L,\ldots,\beta_K a_K(b_K-c_K)\mathbf{I}_L).\end{aligned}$$

Hence, the numerator of (56) in [1] is obtained. From the above analysis, it can be concluded that the kth diagonal term of the matrix $\mathbf{\Gamma}$ is $\sqrt{p_u}M\beta_k a_k(b_k-c_k)$ in (57) in [1]. On similar lines, the two terms in the denominator can be obtained. Hence, the other two matrices can be defined as $\mathbf{\Psi}_L = \mathbf{\Psi}\otimes\mathbf{I}_L$, $\mathbf{\Theta}_L = \mathbf{\Theta}\otimes\mathbf{I}_L$ and the principal diagonal elements of the matrices $\mathbf{\Psi}\in\mathbb{C}^{K\times K}$, $\mathbf{\Theta}\in\mathbb{C}^{K\times K}$ can be expressed as

$$[\mathbf{\Psi}]_{k,k}=\sqrt{p_u}M\beta_k a_k\sqrt{1-c_k^2},\ [\mathbf{\Theta}]_{k,k}=\frac{\sigma_n^2}{2}M\beta_k a_k^2.$$

REFERENCES

[1] A. Chawla, A. Patel, A. K. Jagannatham, and P. K. Varshney, “Distributed Detection in Massive MIMO Wireless Sensor Networks under Perfect and Imperfect CSI,” *IEEE Trans. Signal Process.*, 2019.