Technical Report: Robust Cooperative Spectrum Sensing for MIMO Cognitive Radio Networks under CSI Uncertainty

Adarsh Patel, *Student Member, IEEE,* Hukma Ram, *Student Member, IEEE,*
Aditya K. Jagannatham, *Member, IEEE*
and Pramod K. Varshney, *Fellow, IEEE*

I. Cooperative MIMO Spectrum Sensing with Perfect CSI

The LRT $T(\mathbf{y})$ for spectrum sensing with perfect CSI scenario based on the Neyman-Pearson (NP) criterion can be expressed as,

$$T(\mathbf{y}) = \ln\left[\prod_{i=1}^{N}\frac{p\left(\mathbf{y}_i;\mathcal{H}_1\right)}{p\left(\mathbf{y}_i;\mathcal{H}_0\right)}\right] \tag{1}$$

$$= \sum_{i=1}^{N}\ln\left[\frac{p\left(\mathbf{y}_i;\mathbf{U}_i=\mathbf{P}_1\right)\Pr\left(\mathbf{U}_i=\mathbf{P}_1|\mathcal{H}_1\right)+p\left(\mathbf{y}_i;\mathbf{U}_i=\mathbf{P}_0\right)\Pr\left(\mathbf{U}_i=\mathbf{P}_0|\mathcal{H}_1\right)}{p\left(\mathbf{y}_i;\mathbf{U}_i=\mathbf{P}_1\right)\Pr\left(\mathbf{U}_i=\mathbf{P}_1|\mathcal{H}_0\right)+p\left(\mathbf{y}_i;\mathbf{U}_i=\mathbf{P}_0\right)\Pr\left(\mathbf{U}_i=\mathbf{P}_0|\mathcal{H}_0\right)}\right] \tag{2}$$

$$= \sum_{i=1}^{N}\ln\left[\frac{P_{D,i}\exp\left(-\sum_{j=1}^{N_f}\frac{\|\mathbf{y}_{i,j}-\mathbf{P}_1\mathbf{h}_{i,j}\|^2}{\sigma^2}\right)+(1-P_{D,i})\exp\left(-\sum_{j=1}^{N_f}\frac{\|\mathbf{y}_{i,j}-\mathbf{P}_0\mathbf{h}_{i,j}\|^2}{\sigma^2}\right)}{P_{F,i}\exp\left(-\sum_{j=1}^{N_f}\frac{\|\mathbf{y}_{i,j}-\mathbf{P}_1\mathbf{h}_{i,j}\|^2}{\sigma^2}\right)+(1-P_{F,i})\exp\left(-\sum_{j=1}^{N_f}\frac{\|\mathbf{y}_{i,j}-\mathbf{P}_0\mathbf{h}_{i,j}\|^2}{\sigma^2}\right)}\right], \tag{3}$$

where the PDFs $p(\mathbf{y}_i;\mathbf{U}_i=\mathbf{P}_1)$ and $p(\mathbf{y}_i;\mathbf{U}_i=\mathbf{P}_0)$ can be written as,

$$p\left(\mathbf{y}_i;\mathbf{U}_i=\mathbf{P}_1\right)=\prod_{j=1}^{N_f}p\left(\mathbf{y}_{i,j};\mathbf{U}_i=\mathbf{P}_1\right)=\prod_{j=1}^{N_f}\frac{1}{\pi^L\sigma^{2L}}\exp\left[\frac{-1}{\sigma^2}(\mathbf{y}_{i,j}-\mathbf{P}_1\mathbf{h}_{i,j})^H\mathbf{I}_L^{-1}(\mathbf{y}_{i,j}-\mathbf{P}_1\mathbf{h}_{i,j})\right], \tag{4}$$

$$p\left(\mathbf{y}_i;\mathbf{U}_i=\mathbf{P}_0\right)=\prod_{j=1}^{N_f}p\left(\mathbf{y}_{i,j};\mathbf{U}_i=\mathbf{P}_0\right)=\prod_{j=1}^{N_f}\frac{1}{\pi^L\sigma^{2L}}\exp\left[\frac{-1}{\sigma^2}(\mathbf{y}_{i,j}-\mathbf{P}_0\mathbf{h}_{i,j})^H\mathbf{I}_L^{-1}(\mathbf{y}_{i,j}-\mathbf{P}_0\mathbf{h}_{i,j})\right]. \tag{5}$$

A. Patel, H. Ram and A. K. Jagannatham are with the Department of Electrical Engineering, Indian Institute of Technology Kanpur, Kanpur, India 208016. Email: {adarsh, hukma, adityaj}@iitk.ac.in

P. K. Varshney is with the Department of Electrical Engineering and Computer Science, Syracuse University, Syracuse, NY, 13244, USA, Email: varshney@syr.edu

Substituting $\mathbf{P}_0 = -\mathbf{P}$ and $\mathbf{P}_1 = \mathbf{P}$ in the above equation in (3), the test statistics for the antipodal signaling scenario can be simplified as,

$$T_A(\mathbf{y})$$

$$= \sum_{i=1}^{N} \ln \left[\frac{P_{D,i} \exp\left(-\sum_{j=1}^{N_f} \frac{\|\mathbf{y}_{i,j}-\mathbf{P}\mathbf{h}_{i,j}\|^2}{\sigma^2}\right) + (1-P_{D,i}) \exp\left(-\sum_{j=1}^{N_f} \frac{\|\mathbf{y}_{i,j}+\mathbf{P}\mathbf{h}_{i,j}\|^2}{\sigma^2}\right)}{P_{F,i} \exp\left(-\sum_{j=1}^{N_f} \frac{\|\mathbf{y}_{i,j}-\mathbf{P}\mathbf{h}_{i,j}\|^2}{\sigma^2}\right) + (1-P_{F,i}) \exp\left(-\sum_{j=1}^{N_f} \frac{\|\mathbf{y}_{i,j}+\mathbf{P}\mathbf{h}_{i,j}\|^2}{\sigma^2}\right)} \right] \tag{6}$$

$$= \sum_{i=1}^{N} \ln \left[\frac{\exp\left(-\sum_{j=1}^{N_f} \frac{\|\mathbf{y}_{i,j}-\mathbf{P}\mathbf{h}_{i,j}\|^2}{\sigma^2}\right) \left[P_{D,i} + (1-P_{D,i}) \exp\left(-\sum_{j=1}^{N_f} \frac{\|\mathbf{y}_{i,j}+\mathbf{P}\mathbf{h}_{i,j}\|^2 - \|\mathbf{y}_{i,j}-\mathbf{P}\mathbf{h}_{i,j}\|^2}{\sigma^2}\right)\right]}{\exp\left(-\sum_{j=1}^{N_f} \frac{\|\mathbf{y}_{i,j}-\mathbf{P}\mathbf{h}_{i,j}\|^2}{\sigma^2}\right) \left[P_{F,i} + (1-P_{F,i}) \exp\left(-\sum_{j=1}^{N_f} \frac{\|\mathbf{y}_{i,j}+\mathbf{P}\mathbf{h}_{i,j}\|^2 - \|\mathbf{y}_{i,j}-\mathbf{P}\mathbf{h}_{i,j}\|^2}{\sigma^2}\right)\right]} \right] \tag{7}$$

$$= \sum_{i=1}^{N} \ln \left[\frac{P_{D,i} + (1-P_{D,i}) \exp\left(-\sum_{j=1}^{N_f} \frac{\|\mathbf{y}_{i,j}+\mathbf{P}\mathbf{h}_{i,j}\|^2 - \|\mathbf{y}_{i,j}-\mathbf{P}\mathbf{h}_{i,j}\|^2}{\sigma^2}\right)}{P_{F,i} + (1-P_{F,i}) \exp\left(-\sum_{j=1}^{N_f} \frac{\|\mathbf{y}_{i,j}+\mathbf{P}\mathbf{h}_{i,j}\|^2 - \|\mathbf{y}_{i,j}-\mathbf{P}\mathbf{h}_{i,j}\|^2}{\sigma^2}\right)} \right] \tag{8}$$

$$= \sum_{i=1}^{N} \ln \left[\frac{P_{D,i} + (1-P_{D,i}) \exp\left(-\sum_{j=1}^{N_f} \frac{(\mathbf{y}_{i,j}+\mathbf{P}\mathbf{h}_{i,j})^H(\mathbf{y}_{i,j}+\mathbf{P}\mathbf{h}_{i,j}) - (\mathbf{y}_{i,j}-\mathbf{P}\mathbf{h}_{i,j})^H(\mathbf{y}_{i,j}-\mathbf{P}\mathbf{h}_{i,j})}{\sigma^2}\right)}{P_{F,i} + (1-P_{F,i}) \exp\left(-\sum_{j=1}^{N_f} \frac{(\mathbf{y}_{i,j}+\mathbf{P}\mathbf{h}_{i,j})^H(\mathbf{y}_{i,j}+\mathbf{P}\mathbf{h}_{i,j}) - (\mathbf{y}_{i,j}-\mathbf{P}\mathbf{h}_{i,j})^H(\mathbf{y}_{i,j}-\mathbf{P}\mathbf{h}_{i,j})}{\sigma^2}\right)} \right] \tag{9}$$

$$= \sum_{i=1}^{N} \ln \left[\frac{P_{D,i} + (1-P_{D,i}) \exp\left(-\sum_{j=1}^{N_f} \frac{\mathbf{y}_{i,j}^H\mathbf{P}\mathbf{h}_{i,j} + (\mathbf{y}_{i,j}^H\mathbf{P}\mathbf{h}_{i,j})^H - (-\mathbf{y}_{i,j}^H\mathbf{P}\mathbf{h}_{i,j} - (\mathbf{y}_{i,j}^H\mathbf{P}\mathbf{h}_{i,j})^H)}{\sigma^2}\right)}{P_{F,i} + (1-P_{F,i}) \exp\left(-\sum_{j=1}^{N_f} \frac{\mathbf{y}_{i,j}^H\mathbf{P}\mathbf{h}_{i,j} + (\mathbf{y}_{i,j}^H\mathbf{P}\mathbf{h}_{i,j})^H - (-\mathbf{y}_{i,j}^H\mathbf{P}\mathbf{h}_{i,j} - (\mathbf{y}_{i,j}^H\mathbf{P}\mathbf{h}_{i,j})^H)}{\sigma^2}\right)} \right] \tag{10}$$

$$= \sum_{i=1}^{N} \ln \left[\frac{P_{D,i} + (1-P_{D,i}) \exp\left(-\frac{4}{\sigma^2} \sum_{j=1}^{N_f} \Re(\mathbf{y}_{i,j}^H\mathbf{P}\mathbf{h}_{i,j})\right)}{P_{F,i} + (1-P_{F,i}) \exp\left(-\frac{4}{\sigma^2} \sum_{j=1}^{N_f} \Re(\mathbf{y}_{i,j}^H\mathbf{P}\mathbf{h}_{i,j})\right)} \right]. \tag{11}$$

Employing the approximations $e^{-t} \approx (1-t)$, $\ln(1+t) \approx t$, for small values of t, the test statistic $T_{\mathrm{A}}(\mathbf{y})$ in (11) can be further simplified at low SNR as,

$$T_{\mathrm{A}}(\mathbf{y}) = \sum_{i=1}^{N} \underbrace{\left[a_i \sum_{j=1}^{N_f} \Re(\mathbf{y}_{i,j}^H\mathbf{P}\mathbf{h}_{i,j}) \right]}_{\mathbb{T}(\mathbf{y}_i)}. \tag{12}$$

II. GLRT for Antipodal Signaling with CSI Uncertainty

The binary hypothesis testing problem for antipodal signal with CSI uncertainty can be formulated as,

$$\mathcal{H}_0 : \mathbf{y}_{i,j} = -\mathbf{P}\mathbf{h}_{i,j} + \mathbf{w}_{i,j} = -\mathbf{P}(\hat{\mathbf{h}}_{i,j} + \mathbf{e}_{i,j}) + \mathbf{w}_{i,j} \tag{13}$$

$$\mathcal{H}_1 : \mathbf{y}_{i,j} = \mathbf{P}\mathbf{h}_{i,j} + \mathbf{w}_{i,j} = \mathbf{P}(\hat{\mathbf{h}}_{i,j} + \mathbf{e}_{i,j}) + \mathbf{w}_{i,j} \tag{14}$$

Now let $\mathbf{v}_{i,j|1}, \mathbf{v}_{i,j|0}$ for the hypothesis $\mathcal{H}_1, \mathcal{H}_0$ be defined as,

$$\mathbf{v}_{i,j|1} = \mathbf{y}_{i,j} - \mathbf{P}\hat{\mathbf{h}}_{i,j} = \mathbf{P}\mathbf{e}_{i,j} + \mathbf{w}_{i,j} \tag{15}$$

$$\mathbf{v}_{i,j|0} = \mathbf{y}_{i,j} + \mathbf{P}\hat{\mathbf{h}}_{i,j} = -\mathbf{P}\mathbf{e}_{i,j} + \mathbf{w}_{i,j} \tag{16}$$

Under hypotheses $\mathcal{H}_1$ and $\mathcal{H}_0$, the maximum likelihood estimate of the uncertainty vector $\mathbf{e}_{i,j}$ is,

$$\hat{\mathbf{e}}_{i,j|1} = (\mathbf{P}^H\mathbf{P})^{-1}\mathbf{P}^H\mathbf{v}_{i,j|1} \tag{17}$$

$$\hat{\mathbf{e}}_{i,j|0} = -(\mathbf{P}^H\mathbf{P})^{-1}\mathbf{P}^H\mathbf{v}_{i,j|0}. \tag{18}$$

Following an approach similar to that of the non-antipodal scenario, the argument of the exponential term in the likelihood ratio $\frac{p(\mathbf{y}_{i,j}|\mathcal{H}_1;\hat{\mathbf{e}}_{i,j|1})}{p(\mathbf{y}_{i,j}|\mathcal{H}_0;\hat{\mathbf{e}}_{i,j|0})}$ is

$$\begin{aligned}
&(\mathbf{y}_{i,j} - \mathbf{P}\hat{\mathbf{h}}_{i,j} - \mathbf{P}\hat{\mathbf{e}}_{i,j|1})^H(\mathbf{y}_{i,j} - \mathbf{P}\hat{\mathbf{h}}_{i,j} - \mathbf{P}\hat{\mathbf{e}}_{i,j|1}) - (\mathbf{y}_{i,j} + \mathbf{P}\hat{\mathbf{h}}_{i,j} + \mathbf{P}\hat{\mathbf{e}}_{i,j|0})^H(\mathbf{y}_{i,j} + \mathbf{P}\hat{\mathbf{h}}_{i,j} + \mathbf{P}\hat{\mathbf{e}}_{i,j|0})\\
=&(\mathbf{v}_{i,j|1} - \mathbf{P}(\mathbf{P}^H\mathbf{P})^{-1}\mathbf{P}^H\mathbf{v}_{i,j|1})^H(\mathbf{v}_{i,j|1} - \mathbf{P}(\mathbf{P}^H\mathbf{P})^{-1}\mathbf{P}^H\mathbf{v}_{i,j|1})\\
&- (\mathbf{v}_{i,j|0} - \mathbf{P}(\mathbf{P}^H\mathbf{P})^{-1}\mathbf{P}^H\mathbf{v}_{i,j|0})^H(\mathbf{v}_{i,j|0} - \mathbf{P}(\mathbf{P}^H\mathbf{P})^{-1}\mathbf{P}^H\mathbf{v}_{i,j|0}) &(19)\\
=&\mathbf{v}_{i,j|1}^H(\mathbf{I} - \mathbf{P}(\mathbf{P}^H\mathbf{P})^{-1}\mathbf{P}^H)^H\underbrace{(\mathbf{I} - \mathbf{P}(\mathbf{P}^H\mathbf{P})^{-1}\mathbf{P}^H)}_{\mathcal{P}_{\mathbf{P}}^{\perp}}\mathbf{v}_{i,j|1}\\
&- \mathbf{v}_{i,j|0}^H(\mathbf{I} - \mathbf{P}(\mathbf{P}^H\mathbf{P})^{-1}\mathbf{P}^H)^H\underbrace{(\mathbf{I} - \mathbf{P}(\mathbf{P}^H\mathbf{P})^{-1}\mathbf{P}^H)}_{\mathcal{P}_{\mathbf{P}}^{\perp}}\mathbf{v}_{i,j|0} &(20)\\
=&\mathbf{v}_{i,j|1}^H\mathcal{P}_{\mathbf{P}}^{\perp}\mathbf{v}_{i,j|1} - \mathbf{v}_{i,j|0}^H\mathcal{P}_{\mathbf{P}}^{\perp}\mathbf{v}_{i,j|0}, &(21)
\end{aligned}$$

where we have used the property $(\mathcal{P}_{\mathbf{P}}^{\perp})^H = \mathcal{P}_{\mathbf{P}}^{\perp}$ and $\mathcal{P}_{\mathbf{P}}^{\perp}\mathcal{P}_{\mathbf{P}}^{\perp} = \mathcal{P}_{\mathbf{P}}^{\perp}$ in the above simplification. Further, observe that

$$\begin{aligned}
&\mathbf{v}_{i,j|1}^H\mathcal{P}_{\mathbf{P}}^{\perp}\mathbf{v}_{i,j|1} - \mathbf{v}_{i,j|0}^H\mathcal{P}_{\mathbf{P}}^{\perp}\mathbf{v}_{i,j|0}\\
=&(\mathbf{y}_{i,j} - \mathbf{P}\hat{\mathbf{h}}_{i,j})^H\mathcal{P}_{\mathbf{P}}^{\perp}(\mathbf{y}_{i,j} - \mathbf{P}\hat{\mathbf{h}}_{i,j}) - (\mathbf{y}_{i,j} + \mathbf{P}\hat{\mathbf{h}}_{i,j})^H\mathcal{P}_{\mathbf{P}}^{\perp}(\mathbf{y}_{i,j} + \mathbf{P}\hat{\mathbf{h}}_{i,j}) &(22)\\
=&(\mathbf{y}_{i,j} - \mathbf{P}\hat{\mathbf{h}}_{i,j})^H\mathcal{P}_{\mathbf{P}}^{\perp}\mathcal{P}_{\mathbf{P}}^{\perp}(\mathbf{y}_{i,j} - \mathbf{P}\hat{\mathbf{h}}_{i,j}) - (\mathbf{y}_{i,j} + \mathbf{P}\hat{\mathbf{h}}_{i,j})^H\mathcal{P}_{\mathbf{P}}^{\perp}\mathcal{P}_{\mathbf{P}}^{\perp}(\mathbf{y}_{i,j} + \mathbf{P}\hat{\mathbf{h}}_{i,j}) &(23)\\
=&(\mathcal{P}_{\mathbf{P}}^{\perp}\mathbf{y}_{i,j} - \mathcal{P}_{\mathbf{P}}^{\perp}\mathbf{P}\hat{\mathbf{h}}_{i,j})^H(\mathcal{P}_{\mathbf{P}}^{\perp}\mathbf{y}_{i,j} - \mathcal{P}_{\mathbf{P}}^{\perp}\mathbf{P}\hat{\mathbf{h}}_{i,j}) - (\mathcal{P}_{\mathbf{P}}^{\perp}\mathbf{y}_{i,j} + \mathcal{P}_{\mathbf{P}}^{\perp}\mathbf{P}\hat{\mathbf{h}}_{i,j})^H(\mathcal{P}_{\mathbf{P}}^{\perp}\mathbf{y}_{i,j} + \mathcal{P}_{\mathbf{P}}^{\perp}\mathbf{P}\hat{\mathbf{h}}_{i,j}) &(24)\\
=&(\mathcal{P}_{\mathbf{P}}^{\perp}\mathbf{y}_{i,j})^H\mathcal{P}_{\mathbf{P}}^{\perp}\mathbf{y}_{i,j} - (\mathcal{P}_{\mathbf{P}}^{\perp}\mathbf{y}_{i,j})^H\mathcal{P}_{\mathbf{P}}^{\perp}\mathbf{y}_{i,j} &(25)\\
=&0 &(26)
\end{aligned}$$

where we have used the property $\mathcal{P}_{\mathbf{P}}^{\perp}\mathcal{P} = \mathbf{0}$ in the above simplification.

The argument of the exponential in the LRT is simply zero at each receive antenna for each user. This basically arises because the standard GLRT is simply concerned with the component of $\mathbf{y}_{i,j}$ in the Null space of the beacon matrix. Since the beacon matrices are $\mathbf{P}$, $-\mathbf{P}$ for the antipodal signaling format, their nullspaces are identical, which renders the GLRT inapplicable. By considering the estimate of the unknown parameter $\mathbf{e}_{i,j}$ for $\mathcal{H}_1$ and $\mathcal{H}_0$ the GLRT simply adjusts the estimate to maximize the likelihoods corresponding to $\mathbf{P}$, $-\mathbf{P}$ which makes the net likelihoods identical. In order to overcome this problem, it is, therefore, essential to consider a framework which also takes into account the prior probability of the unknown parameter $\mathbf{e}_{i,j}$ or basically *regularize the likelihood* in order to favor smaller weighted norms of the uncertainty parameter.

III. Alternative Approach Using Regularized GLRT

To overcome this problem, one can consider a Bayesian version of the GLRT for the antipodal scenario as follows. The quantities $\mathbf{z}_{i,j|1}$, $\mathbf{z}_{i,j|0}$ are defined as,

$$\underbrace{\mathbf{y}_{i,j} - \mathbf{P}\hat{\mathbf{h}}_{i,j}}_{\mathbf{v}_{i,j|1}} = \mathbf{A}_{i|1}\mathbf{z}_{i,j|1}, \tag{27}$$

$$\underbrace{\mathbf{y}_{i,j} + \mathbf{P}\hat{\mathbf{h}}_{i,j}}_{\mathbf{v}_{i,j|0}} = \mathbf{A}_{i|0}\mathbf{z}_{i,j|0}, \tag{28}$$

and

$$\mathbf{z}_{i,j} = \begin{bmatrix} \mathbf{e}_{i,j} \\ \mathbf{w}_{i,j} \end{bmatrix}, \tag{29}$$

where $\mathbf{A}_{i|1} = \begin{bmatrix} \mathbf{P} & \mathbf{I} \end{bmatrix}$, $\mathbf{A}_{i|0} = \begin{bmatrix} -\mathbf{P} & \mathbf{I} \end{bmatrix}$. We consider the likelihood ratio $\frac{p(\mathbf{y}_{i,j},\mathbf{z}_{i,j}|\mathbf{U}_i=\mathbf{P})}{p(\mathbf{y}_{i,j},\mathbf{z}_{i,j}|\mathbf{U}_i=-\mathbf{P})}$ which can be simplified as,

$$\frac{p(\mathbf{y}_{i,j},\mathbf{z}_{i,j|1}|\mathbf{U}_i=\mathbf{P})}{p(\mathbf{y}_{i,j},\mathbf{z}_{i,j|0}|\mathbf{U}_i=-\mathbf{P})} = \frac{p(\mathbf{y}_{i,j}|\mathbf{z}_{i,j|1},\mathbf{U}_i=\mathbf{P})p(\mathbf{z}_{i,j|1}|\mathbf{U}_i=\mathbf{P})}{p(\mathbf{y}_{i,j}|\mathbf{z}_{i,j|0},\mathbf{U}_i=-\mathbf{P})p(\mathbf{z}_{i,j|0}|\mathbf{U}_i=-\mathbf{P})} \tag{30}$$

$$= \begin{cases} \dfrac{\frac{1}{\pi^{N_c+L}|\mathbf{R}_{z,i}|}\exp(-\mathbf{z}_{i,j|1}\mathbf{R}_{z,i}^{-1}\mathbf{z}_{i,j|1})}{\frac{1}{\pi^{N_c+L}|\mathbf{R}_{z,i}|}\exp(-\mathbf{z}_{i,j|0}\mathbf{R}_{z,i}^{-1}\mathbf{z}_{i,j|0})} & \text{if} \quad \begin{array}{l} \underbrace{\mathbf{y}_{i,j} - \mathbf{P}\hat{\mathbf{h}}_{i,j}}_{\mathbf{v}_{i,j|1}} - \mathbf{A}_{i|1}\mathbf{z}_{i,j|1} = 0, \\ \underbrace{\mathbf{y}_{i,j} + \mathbf{P}\hat{\mathbf{h}}_{i,j}}_{\mathbf{v}_{i,j|0}} - \mathbf{A}_{i|0}\mathbf{z}_{i,j|0} = 0 \end{array} \\ 0 & \text{Otherwise} \end{cases} \tag{31}$$

where $\mathbf{R}_{z,i} = \begin{bmatrix} \mathbf{R}_{e,i} & \mathbf{0} \\ \mathbf{0} & \sigma^2\mathbf{I} \end{bmatrix}$. The GLRT can now be obtained by replacing $\mathbf{z}_{i,j|1}$, $\mathbf{z}_{i,j|0}$ with their maximum likelihood estimates (MLEs). As rightly pointed by you, once the MLE has been obtained, the marginal likelihood of $\mathbf{y}_{i,j}$ is a Dirac mass at $\mathbf{A}_{i|1}, \hat{\mathbf{z}}_{i,j|1}, \mathbf{A}_{i|0}, \hat{\mathbf{z}}_{i,j|0}$

for hypotheses $\mathcal{H}_1$, $\mathcal{H}_0$ respectively. This is now reflected in the GLRT above. Further, the MLEs are used since the GLRT is based on the maximum likelihood estimate. The respective MLEs, which maximize the likelihood corresponding to each hypothesis now be obtained as described in the paper and shown below. For hypothesis $\mathcal{H}_1$,

$$\hat{\mathbf{z}}_{i,j|1} = \arg\min_{\mathbf{z}_{i,j|1}} \ \mathbf{z}_{i,j|1}^H \mathbf{R}_{z|i}^{-1} \mathbf{z}_{i,j|1}, \qquad \mathbf{v}_{i,j|1} = \mathbf{A}_{i|1} \mathbf{z}_{i,j|1} \tag{32}$$

The resulting MLE of $\hat{\mathbf{z}}_{i,j|1}$ is $\mathbf{R}_{z,i}\mathbf{A}_{i|1}^H(\mathbf{A}_{i|1}\mathbf{R}_{z,i}\mathbf{A}_{i|1}^H)^{-1}\mathbf{v}_{i,j|1}$. Similarly, MLE of $\hat{\mathbf{z}}_{i,j|0}$ can be determined as $\mathbf{R}_{z,i}\mathbf{A}_{i|0}^H(\mathbf{A}_{i|0}\mathbf{R}_{z,i}\mathbf{A}_{i|0}^H)^{-1}\mathbf{v}_{i,j|0}$. Finally, the argument in the exponent of the GLRT can be simplified as,

$$\hat{\mathbf{z}}_{i,j|0}^H \mathbf{R}_{z,i}^{-1} \hat{\mathbf{z}}_{i,j|0} - \hat{\mathbf{z}}_{i,j|1}^H \mathbf{R}_{z,i}^{-1} \hat{\mathbf{z}}_{i,j|1} = \mathbf{v}_{i,j|0}^H (\mathbf{A}_{i|0}\mathbf{R}_{z,i}\mathbf{A}_{i|0}^H)^{-1} \mathbf{v}_{i,j|0} - \mathbf{v}_{i,j|1}^H (\mathbf{A}_{i|1}\mathbf{R}_{z,i}\mathbf{A}_{i|1}^H)^{-1} \mathbf{v}_{i,j|1} \tag{33}$$

Observe that $\mathbf{A}_{i|1}\mathbf{R}_{z,i}\mathbf{A}_{i|1}^H = \mathbf{A}_{i|0}\mathbf{R}_{z,i}\mathbf{A}_{i|0}^H = \mathbf{P}\mathbf{R}_{e,i}\mathbf{P}^H + \sigma^2\mathbf{I}_L = \mathbf{\Gamma}_i$. Hence, the expression above can be further simplified as,

$$\begin{aligned} &\mathbf{v}_{i,j|0}^H \mathbf{\Gamma}_i^{-1} \mathbf{v}_{i,j|0} - \mathbf{v}_{i,j|1}^H \mathbf{\Gamma}_i^{-1} \mathbf{v}_{i,j|1} \\ &= (\mathbf{y}_{i,j} + \mathbf{P}\hat{\mathbf{h}}_{i,j})^H \mathbf{\Gamma}_i^{-1} (\mathbf{y}_{i,j} + \mathbf{P}\hat{\mathbf{h}}_{i,j}) - (\mathbf{y}_{i,j} - \mathbf{P}\hat{\mathbf{h}}_{i,j})^H \mathbf{\Gamma}_i^{-1} (\mathbf{y}_{i,j} - \mathbf{P}\hat{\mathbf{h}}_{i,j}) && (34) \\ &= 4\Re(\mathbf{y}_{i,j}^H \mathbf{\Gamma}_i^{-1} \mathbf{P}\hat{\mathbf{h}}_{i,j}) && (35) \end{aligned}$$

The robust GLRT for the cooperative decision rule at the fusion center is,

$$\begin{aligned} &T_{\text{A-RGLRT}}(\mathbf{y}) \\ &= \sum_{i=1}^{N} \ln \left[\frac{P_{D,i} \prod_{j=1}^{N_f} p(\mathbf{y}_{i,j}, \mathbf{z}_{i,j|1} | \mathbf{U}_i = \mathbf{P}; \mathcal{H}_1) + (1 - P_{D,i}) \prod_{j=1}^{N_f} p(\mathbf{y}_{i,j} \mathbf{z}_{i,j|0} | \mathbf{U}_i = -\mathbf{P}; \mathcal{H}_1)}{P_{F,i} \prod_{j=1}^{N_f} p(\mathbf{y}_{i,j}, \mathbf{z}_{i,j|1} | \mathbf{U}_i = \mathbf{P}; \mathcal{H}_0) + (1 - P_{F,i}) \prod_{j=1}^{N_f} p(\mathbf{y}_{i,j}, \mathbf{z}_{i,j|0} | \mathbf{U}_i = -\mathbf{P}; \mathcal{H}_0)} \right] && (36) \\ &= \sum_{i=1}^{N} \ln \left[\frac{P_{D,i} \prod_{j=1}^{N_f} p(\mathbf{y}_{i,j}, \hat{\mathbf{z}}_{i,j|1} | \mathbf{U}_i = \mathbf{P}; \mathcal{H}_1) + (1 - P_{D,i}) \prod_{j=1}^{N_f} p(\mathbf{y}_{i,j} \hat{\mathbf{z}}_{i,j|0} | \mathbf{U}_i = -\mathbf{P}; \mathcal{H}_1)}{P_{F,i} \prod_{j=1}^{N_f} p(\mathbf{y}_{i,j}, \hat{\mathbf{z}}_{i,j|1} | \mathbf{U}_i = \mathbf{P}; \mathcal{H}_0) + (1 - P_{F,i}) \prod_{j=1}^{N_f} p(\mathbf{y}_{i,j}, \hat{\mathbf{z}}_{i,j|0} | \mathbf{U}_i = -\mathbf{P}; \mathcal{H}_0)} \right] && (37) \\ &= \sum_{i=1}^{N} \ln \left[\frac{P_{D,i} + (1 - P_{D,i}) \prod_{j=1}^{N_f} \frac{1}{\pi^{N_c+L} |\mathbf{R}_{z,i}|} \exp\left\{ - \mathbf{v}_{i,j|1}^H (-\mathbf{A}_{i|1}\mathbf{R}_{z,i}\mathbf{A}_{i|1}^H)^{-1} \mathbf{v}_{i,j|1} \right\}}{P_{F,i} + (1 - P_{F,i}) \prod_{j=1}^{N_f} \frac{1}{\pi^{N_c+L} |\mathbf{R}_{z,i}|} \exp\left\{ - \mathbf{v}_{i,j|0}^H (-\mathbf{A}_{i|0}\mathbf{R}_{z,i}\mathbf{A}_{i|0}^H)^{-1} \mathbf{v}_{i,j|0} \right\}} \right]. && (38) \end{aligned}$$

At low SNR, the resulting test statistic can be simplified to obtain the final test statistic given in the paper as,

$$T_{\text{A-RGLRT}}(\mathbf{y}) = \sum_{i=1}^{N} \left[a_i \sum_{j=1}^{N_f} \Re(\mathbf{y}_{i,j}^H \mathbf{\Gamma}_i^{-1} \mathbf{P}\hat{\mathbf{h}}_{i,j}) \right]. \tag{39}$$

IV. Robust GLRT for Scenarios with Unknown Covariance

Finally, if the covariance $\mathbf{R}_{e,i}$ is unknown, one can choose $\widetilde{\mathbf{R}}_{z,i} = \begin{bmatrix} \lambda \mathbf{I}_{N_c} & \mathbf{0} \\ \mathbf{0} & \sigma^2 \mathbf{I}_L \end{bmatrix}$, where λ is a suitably chosen parameter. The resulting test can be simplified as,

$$T_{\text{A-RGLRT}}(\mathbf{y}) = \sum_{i=1}^{N} \left[a_i \sum_{j=1}^{N_f} \Re(\mathbf{y}_{i,j}^H \widetilde{\mathbf{\Gamma}}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}) \right]. \tag{40}$$

where $\widetilde{\mathbf{\Gamma}}_i^{-1} = \mathbf{A}_{i|1} \widetilde{\mathbf{R}}_{z,i} \mathbf{A}_{i|1}^H = \mathbf{A}_{i|0} \widetilde{\mathbf{R}}_{z,i} \mathbf{A}_{i|0}^H = \lambda \mathbf{P}\mathbf{P}^H + \sigma^2 \mathbf{I}$.