Optimal Wake-up Scheduling for PSM Delay Minimization in Mobile Wireless Networks

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Abstract—In this work, we propose a Lloyd-Max based Sleep interval Determination (LMSD) algorithm towards optimal wake-up schedule computation to minimize the delay between data arrival and mobile device wake-up time, termed as the ON-OFF delay, for the Power Saving Mode in wireless networks. The proposed algorithm employs the probability distribution of the request-response time to compute the optimal sleep interval lengths for average wake-up delay minimization. Analytical results are derived to demonstrate convergence of the proposed LMSD algorithm. We also derive a novel information-theoretic bound to characterize the minimum average wake-up delay for a maximum threshold on the energy consumption for any generic wake-up schedule determination scheme. The LMSD algorithm is also seen to be computationally efficient due to the convexity of the average delay optimization objective. We compare the proposed approach with the existing schemes that improve the delay performance of the standard Binary Truncated Exponent algorithm and demonstrate that it achieves a lower average wake-up delay for a given average energy consumption while asymptotically achieving the delay bound.

I. INTRODUCTION

Energy is a critical resource in wireless mobile networks. Therefore, it has to be employed judiciously to maximize the lifetime of wireless nodes while minimizing the end-to-end delay thereby improving Quality of Service (QoS) in wireless packet networks. In this context, the Power Saving Mode (PSM) adopted by the IEEE 802.11 and IEEE 802.16 [1], [2] based wireless networks is ideally suited for optimal wireless device power utilization. In PSM, an idle mobile station (MS) can enter the sleep mode by switching off the device modem thereby significantly reducing the power consumption during periods of RF inactivity. The device then wakes-up during predefined intervals of short duration to receive paging beacons intimating the presence of buffered packets at the access point (AP) or base station (BS) in the wireless network. In the presence of any such data packets, the MS continues in the active wake-up mode to receive the buffered packets, while re-entering the sleep mode during subsequent periods of inactivity. This cyclic procedure comprising of alternating sleep and wake-up intervals leads to substantial energy conservation in wireless devices arising from limiting the device RF activity, which accounts for a significant fraction of the power consumed by the device [3].

In this scenario, data packets corresponding to a mobile station (MS), which are received during the PSM, are queued at the BS to be transmitted during the next scheduled wake-up interval. Thus the PSM procedure inherently leads to an extra delay in packet delivery to the mobile nodes, which is termed as the ON-OFF delay in such networks. It can be readily seen that while long sleep intervals reduce the energy consumption of the MS, they lead to an increased average ON-OFF delay, thereby potentially leading to buffer overflow at the BS and violation of the QoS packet delay constraints. Although shorter sleep intervals reduce delay, they lead to faster draining of the limited battery resources due to the frequent wake-up periods. Thus it is key to design an efficient wake-up schedule for the wireless device to optimize the device energy consumption while minimizing the ON-OFF delay of packet delivery.

The PSM in IEEE 802.16e adopts a simplistic Binary Truncated Exponent (BTE) algorithm to determine sleep intervals. Related works such as [4], [5] have described schemes to improve the trade-off between energy consumption and delay. The Bounded Slow-Down protocol in [4] proposes a sequential sleep interval determination algorithm in which the next sleep interval is chosen dynamically to limit the maximum delay to be lower than a fraction of the total delay, while providing energy savings compared to the static PSM scheme. The Smart PSM scheme in [5] proposes a trade-off between the ON-OFF delay and energy consumption for two different classes of penalty functions. In the work therein, the two-stair delay penalty is shown to lead to a wake-up schedule such that the maximum sleep-interval is lower than a given bound, while the power-penalty delay function minimizes a joint function of energy consumed and delay. In [6] the authors propose a Probabilistic Sleep Interval Determination (PSID) scheme based wake-up scheduling for average ON-OFF delay reduction in wireless networks, based on a characterization of the request-response delay probability distribution function for the wireless device. However, the scheme therein is restrictive in nature since it is limited to delay distributions belonging to the class of hypoexponential distributions and cannot be applied for any generic distribution. Hence, motivated by this, we propose a Lloyd Max Sleep interval Determination (LMSD) scheme to derive the optimal wake-up schedule in a wireless network for an arbitrary delay distribution. Due to its computational efficiency the Lloyd-Max algorithm is attractive for real-time implementation and is used for several wireless applications such as channel feedback quantization etc. [7], [8]. Further, we develop a novel rate-distortion theory based information theoretic bound to explicitly characterize the optimal achievable delay performance for any given network delay distribution. We also demonstrate convergence of the proposed LMSD scheme. Moreover, in the case of hypoexponential distributions this results in an efficient procedure to compute the optimal wake-up schedule owing to the convexity of the
average ON-OFF delay objective function.

The remainder of this paper is organized as follows. We describe the wireless PSM network model and the associated request-response delay distribution in Section II-A. The LMSD scheme for optimal sleep interval computation and the delay bound for an arbitrary distribution function are described in Section II-B. We also prove the convergence of the proposed scheme to the optimal wake-up schedule therein. Comparison of the LMSD scheme with the existing PSID for delay minimization in PSM of wireless networks with respect to the average energy consumption, expected ON-OFF delay, and delay stability is presented in Section III. Section IV concludes the paper.

II. SLEEP INTERVAL DETERMINATION FOR PSM

A. Request-Response Delay Model

Below we describe a model for the ON-OFF delay in PSM for wireless networks. The request-response time for an MS is defined as the delay between the time instants corresponding to the data request by the MS and the arrival of the data at the BS for transmission over the MS-BS wireless link. Naturally, the request-response delay for a given MS application is erratic in nature owing to the random queue sizes and processing delays at the intermediate routers and the application server. Thus, the stochastic nature of the request-response cycle can be effectively captured by the delay distribution function of the MS. For instance in [6] a simplistic model has been presented to describe the PSM request-response delay $Y$ as,

$$Y = Y_{MS-BS} + Y_{SERVER} + Y_{BS-MS},$$

where $Y_{SERVER}$ is the time it takes for the BS to request the data from the network, and $Y_{MS-BS}$ and $Y_{BS-MS}$ are the delays corresponding to data transmission-reception over the MS-BS and BS-MS links, respectively. Thus, the total request-response delay can be modeled as,

$$Y = a + \sum_{i=1}^{M} X_i,$$

where $M$ denotes the number of hops, $a$ the fixed delay, and $X_i$ the variable delay of each router. A significant shortcoming of the work therein is that it assumes all the delays $X_i$ to be exponentially distributed with parameter $\lambda_i$. For the above model, the pdf of the request-response delay is given as,

$$p_Y(y) = \sum_{i=1}^{M} C_{i,M} \lambda_i e^{-\lambda_i(y-a)} \quad \text{for } y > a,$$

where $C_{i,M} = \Pi_{j\neq i} \lambda_j / (\lambda_j - \lambda_i)$. Next, we present the LMSD scheme for optimal wake-up schedule computation for an arbitrary request-response delay distribution.

B. Lloyd Max based Sleep interval Determination (LMSD)

Let the wake-up times be denoted by $\delta = \{\delta_0, \delta_1, \delta_2, \ldots, \delta_{N_b}\}$ with $\delta_0 = a, \delta_{N_b} = b$, where the interval $[a, b]$ denotes the support of the request-response delay distribution, similar to the existing framework for the BTE and PSID algorithms. Thus, $N_b$ denotes the number of wake-up instants in the interval $[a, b]$. Since for a response arrival instant $t \in (\delta_{i-1}, \delta_i)$, $i \in \{1, \ldots, N_b\}$, the ON-OFF delay introduced is $(\delta_i - t)$, the expected ON-OFF delay for a given wake-up schedule $\delta$ can be computed as follows,

$$D(\delta) = \sum_{i=1}^{i=N_b} \int_{\delta_{i-1}}^{\delta_i} p_Y(t)(\delta_i - t)dt,$$

where $\delta_0 = a$ and $\delta_{N_b} = b$ are fixed and $\delta_0 \leq \delta_1 \leq \cdots \leq \delta_{N_b-1} \leq \delta_{N_b}$, and $p_Y(t)$ is an arbitrary request-response delay distribution. Note that here we have neglected the wake-up interval length $T_w$, since it is of a short duration and does not significantly impact the computation of the optimal wake-up schedule $\delta$. We compute the wake-up instants $\delta$ using an iterative method. Let $\hat{\delta}^{(n)} = (\hat{\delta}_1^{(n)}, \ldots, \hat{\delta}_{N_b-1}^{(n)})$ denote the wake-up instants at iteration $n$, for the LMSD scheme. The wake-up instant vector at the $(n+1)^{th}$ iteration, $\tilde{\delta}^{(n+1)}$, is computed from $\hat{\delta}^{(n)}$ by changing its $k^{th}$ component $\hat{\delta}_k^{(n)}$ as follows,

$$\tilde{\delta}_k^{(n)+1} = \arg \min_x \delta_k^{(n)} \in [x \in [\delta_0, \delta_{N_b}]] \quad \text{subject to }$$

where $k(n) = 1 + ((k(n-1) + 1) \mod N_b)$ and $D_k(x)$ is defined as,

$$D_k(x) = D(\delta_1, \ldots, \delta_k^{(n)-1}, x, \delta_k^{(n)+1}, \ldots, \delta_{N_b}^{(n)}) - D_k^{(n)}.$$}

$$\hat{\delta}^{(0)}$$ can be expressed as $\hat{\delta}^{(0)} = (\delta_1^{(0)}, \ldots, \delta_{k(n)-1}^{(0)}, \delta_{k(n)+1}^{(0)}, \ldots, \delta_{N_b}^{(0)})$. The initial wake-up instants $\hat{\delta}^{(0)}$ are initialized to be equally spaced between $a$ and $b$, that is, $\hat{\delta}_k^{(0)} = a + \frac{(b-a)}{N_b}$. Below we demonstrate that the iterative procedure described above, for the computation of the optimal wake-up schedule $(\delta_0, \ldots, \delta_{N_b})$, converges.

**Lemma 1.** The sequence $\{D(\hat{\delta}^{(n)})\}$ corresponding to the average request-response delay at the $n^{th}$ iteration of the LMSD scheme is a convergent sequence.

**Proof:** From the definition of $\tilde{\delta}^{(n+1)}$ in (4) it can be seen that $\tilde{\delta}^{(n+1)}$ minimizes $D(\tilde{\delta}^{(n)})$ with respect to $\hat{\delta}_k^{(n)}$. Thus, it follows that,

$$D(\tilde{\delta}^{(n+1)}) \leq D(\hat{\delta}^{(n)}).$$

Each of the components in (3), $\int_{\delta_i}^{\delta_{i+1}} p_Y(t)(\delta_i - t)dt \geq 0$, $i = 1, \ldots, N_b$, we also have $D(\tilde{\delta}^{(n)}) \geq 0$, $\forall n$. Thus, $D(\tilde{\delta}^{(n)})$ is a monotonically decreasing and bounded sequence. Hence, it converges to the limit $\lim_n D(\tilde{\delta}^{(n)}) = \inf\{D(\tilde{\delta}^{(n)}): n \in \mathbb{N}\}$ [9, Ch. 3].

Interestingly, the computation of the optimal wake-up schedule $\delta$ is related to computing the optimal quantizer $\hat{Y}$ of the request-response delay random variable $Y$ described by the codebook $\mathcal{C} = \{\delta_k\}_{k=0}^{N_b}$ and the set of partition cells $\{I_k\}_{k=0}^{N_b-1}$ where $I_k = \{\delta_k < t \leq \delta_{k+1}\}$, $k = 0, \ldots, N_b - 1$, where $t$ denotes the time of packet arrival. This analogy can be readily seen by likening the expected wake-up delay $E(|Y - \hat{Y}|) = D(\delta)$ to the average quantizer distortion. The size of the reconstruction set $\hat{Y}$ is equal to the number of
wake-up intervals \( N_b \) and the reconstruction points denote the wake-up instants. The optimal quantizer thus computes the partition cell boundaries such that the delay distortion \( D(\delta) \) is minimized corresponding to the \( \log_2 N_b \) bits required to represent the \( N_b \) quantization levels.

The result below gives a lower bound on the achievable ON-OFF PSM delay for any wake-up schedule \( \delta \) employing the analogy described above.

**Lemma 2.** For any arbitrary wake-up schedule \( \delta \), the minimum delay \( D(\delta) \) can be lower-bounded as,

\[
D(\delta) \geq \exp ((h(Y) - \log_2 N_b) \ln 2 - 1),
\]

where \( h(Y) = -\int_a^b p_Y(y) \log(p_Y(y)) \, dy \) is the differential entropy corresponding to the probability density function \( p_Y(y) \) of the request-response delay \( Y \).

Proof: Let \( p_{X,Y}(y,\tilde{y}) \) denote the joint probability density function corresponding to the request-response delay \( Y \) and the wake-up intervals \( \tilde{Y} \). The mutual information between \( Y \) and \( \tilde{Y} \), \( I(Y;\tilde{Y}) \), is given from [10, Ch. 2] by the well known expression,

\[
I(Y;\tilde{Y}) = \int_{-\infty}^{\infty} p_{X,Y}(y,\tilde{y}) \log \frac{p_{X,Y}(y,\tilde{y})}{p_X(y)p_{\tilde{Y}}(\tilde{y})} \, dyd\tilde{y}.
\]

Hence, from the converse of the rate distortion theorem [10, Ch. 13.4], the rate \( \log_2 N_b \) corresponding to the number of wake-up intervals \( N_b \) with the maximum average quantizer distortion \( E(|Y-\tilde{Y}|) \) constrained such that \( E(|Y-\tilde{Y}|) \leq D(\delta) \) is given as,

\[
\log_2 N_b \geq \min_{p(\tilde{Y}|Y), \mathbb{E}|Y| \leq D(\delta)} h(Y) - h(Y|\tilde{Y})
\]

\[
= \min_{p(\tilde{Y}|Y), \mathbb{E}|Y| \leq D(\delta)} h(Y) - h(|Y-\tilde{Y}|)\tilde{Y})
\]

\[
\geq \min_{p(\tilde{Y}|Y), \mathbb{E}|Y| \leq D(\delta)} h(Y) - h(|Y-\tilde{Y}|)
\]

where the last inequality follows from the conditional entropy property \( h(Y-\tilde{Y}|\tilde{Y}) \leq h(Y-\tilde{Y}) \). Let \( Z \) denote the delay random variable \( \tilde{Y} - Y \) and \( \Phi_D(\delta) \) denote the exponential distribution with mean \( D(\delta) \), i.e., \( \Phi_D(\delta)(z) = \frac{1}{D(\delta)} \exp(-z/D(\delta)) \). Let \( g_2(z) \) denote the distribution for \( Z \). Then, we have \( \int g_2(z) \, dz = D(\delta) \). Hence, for any linear function \( (\alpha + \beta)z \), we have, \( \int (\alpha + \beta)g_2(z) \, dz = (\alpha D(\delta) + \beta) \). From the property of the Kullback-Leibler divergence \( D(g||\Phi_D(\delta)) \) [10], it follows that,

\[
0 \leq D(g||\Phi_D(\delta)) = -h(g) - \int \frac{\Phi_D(\delta) \log(\Phi_D(\delta)) \, dz}{h(\Phi_D(\delta))}
\]

where the above equality follows since \( \log(\Phi_D(\delta)) \) is a linear function of \( z \) and \( \int \Phi_D(\delta)(z) \, dz = D(\delta) \). Hence, we have \( h(g) \leq h(\Phi_D(\delta)) \). Employing this result in (8) above leads to,

\[
\log_2 N_b \geq h(Y) - h(\Phi_D(\delta)) = h(Y) - \left(1 + \ln D(\delta) \right) / \ln 2 \quad (9)
\]

Thus, from (9), the average ON-OFF delay \( D(\delta) \), for a given wake-up schedule \( \delta \) corresponding to \( N_b \) wake-up instants can be lower bounded as,

\[
D(\delta) \geq \exp ((h(Y) - \log_2 N_b) \ln 2 - 1).
\]

The above result yields a fundamental limit and provides key insights on the average ON-OFF delay in the PSM mode for wireless devices. Finally we demonstrate that for a typical request-response distribution similar to the one in (1), the LMSD iteration shown in (4) is of low computational complexity due to the convexity of the cost function \( D_k(x) \).

**Lemma 3.** The function \( D_k(x) \) defined in (5) is convex for \( x \in [\delta_{k(n-1)}, \delta_{k(n+1)}] \) for the hypo-exponentially distributed request-response delay \( Y \) given in (2).

Proof: The terms independent of \( x \) in \( D_k(x) \) (5) are denoted \( c_k' \). Thus \( D_k(x) \) can be written as,

\[
D_k(x) = c_k' - F_Y(\delta_{k-1})(x - \delta_{k-1}) - F_Y(x)(\delta_{k+1} - x), \quad (10)
\]

where \( F_Y(t) \) is the cumulative distribution function of \( Y \) given by, \( F_Y(t) = 1 - \sum_{i=1}^M C_i e^{-\lambda_i(t-a)} \) [6]. Thus, using (10) we have,

\[
\alpha D_k(x) + (1 - \alpha) D_k(y) \geq c_k' - F_Y(\delta_{k-1})(x - \delta_{k-1}) - F_Y(x)(\delta_{k+1} - x) - \left( (a-1) \delta_{k-1} - (a-1) \right) F_Y(y)(\delta_{k+1} - y) \quad (11)
\]

\[
= D_k(x) \quad (12)
\]

where inequality (a) follows from the concavity of the function \( (a - x) F_Y(x) \) in \( [0, a] \). Due to space limitations, the detailed proof of (10) and the concavity of \( (a - x) F_Y(x) \) in \( [0, a] \) (11) is given in [11].

The average energy consumed by a wireless device in PSM, \( \xi(\delta) \), for a wake-up schedule \( \delta \) can be computed as,

\[
\xi(\delta) = \sum_{n=1}^{\infty} \left( P_{\text{slp}} (a - n) + P_{\text{act}} - P_{\text{slp}} \right) T_w \times \int_{\delta_{k(n-1)}}^{\delta_{k(n+1)}} \int \, dt \right),
\]

where \( P_{\text{slp}} \) and \( P_{\text{act}} \) are the power consumed during the sleep and wake-up intervals, respectively. In the next section we compare the performance of the LMSD scheme with existing algorithms.

### III. Results and Discussion

In our simulation setup we consider a frame duration of \( T_w = 5ms \), with the request-response delay parameters \( M = 3 \) and the initial fixed delay \( a = 60ms \) with \( b = E[Y] + k \times \sigma_Y \) for \( k = 3 \). The random variables \( X_i \) are assumed to be exponential with parameters \( \lambda_i = i/20 \). The device sleep and active mode powers considered are \( P_{\text{slp}} = 0.045W \) and \( P_{\text{act}} = 1.5W \) respectively, similar to [6]. Moreover, to illustrate the applicability of the proposed framework for wake-up schedule determination for an arbitrary request-response delay distribution \( p_Y(y) \) we plot the performance results corresponding to a truncated Gaussian distribution with the mean and variance (of the un-truncated distribution) same as that for the hypo-exponential distribution.

Fig. 1(a) shows the average ON-OFF delay for the PSID and the proposed LMSD algorithms as a function of \( N_b \) for both distributions. From this figure we can observe that the LMSD algorithm achieves a lower average delay compared to the existing PSID algorithm for different values of the parameter \( N_b \). The proposed LMSD algorithm can be seen to
asymptotically achieve the lower bound in (7). We also note that the average delay decreases monotonically as a function of \( N_b \) which is expected since the frequency of the wake-up instants increases with \( N_b \). Also since the PSID algorithm constrains the probabilities in each interval to be the same, it results in a suboptimal wake-up schedule. This is reflected in the poor sensitivity of the PSID scheme to the request-response distribution. We demonstrate the stability of the LMSD scheme in Fig. 1(b) by comparing the variance in the delay performance for both distributions, hypo-exponential and truncated Gaussian. Since the variance of the LMSD scheme is lower, it is more stable than the existing PSID approach.

Fig. 2 shows the average energy consumption for the PSID and LMSD algorithms as a function of \( N_b \) for both the distributions. We observe that the energy consumption increases with \( N_b \) and the LMSD algorithm performs better than the PSID algorithm for all \( N_b \). The trade-off between the PSM energy consumption and the average delay \( D(\delta) \) is shown in Fig. 3. The results presented demonstrate that the proposed LMSD scheme is superior in both respects, average ON-OFF delay and energy consumption, to the existing PSID scheme.

**IV. CONCLUSION**

We have presented an optimal scheme to minimize the ON-OFF delay in PSM of mobile devices in wireless networks such as IEEE 802.16 and IEEE 802.11. The LMSD scheme employs an iterative procedure to compute the optimal wake-up schedule for any arbitrary request-response delay distribution. Thus, the proposed scheme has wide applicability compared to other existing approaches which are restricted to specific distributions. A novel rate-distortion framework based lower bound is developed to characterize the ON-OFF delay performance of any wireless wake-up scheduler. Simulation results demonstrate superior performance of the proposed scheme compared to the existing PSID scheme in terms of ON-OFF delay, energy consumption, and delay stability.

**REFERENCES**