

# **MATHEMATICAL MODELING OF GLOBAL WARMING: CAUSES, CONTROLS AND EFFECTS**

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# Introduction for Global Warming Research

- **Research is to discover some new characteristics about a system/situations which has not been explored before.**
- ***“People see things and ask why. I see things that are not and ask why not”*** ...George Bernard Shaw
- ***“The greatest force on earth is the human soul (mind) on fire”*** ...Forsyth
- 
- **Researchers therefore must have that kind of vision and mind for quality research**
- **Research is multidimensional and its goal is of paramount importance. The various variables governing the problem (goal) in a model are interdependent and amenable to simulation leading towards achieving the result.**

# **Environmental Awareness**

- **It was the first time that in 1972 United Nation Conference on Human Environment in Stockholm Sweden the awareness about environment was focussed.**
- **The Indian Prime Minister Late Indira Gandhi addressed this conference and said, I quote**
- **Poverty is the worst form of pollution**
- **22<sup>nd</sup> century ago Emperor Asoka forbade the killing of large number of animal for food and sport and also felling of trees.**
- **In Atharveda it is mentioned that if you take anything from earth you must give the same back i.e. you cut the tree you must replace by planting another tree.**

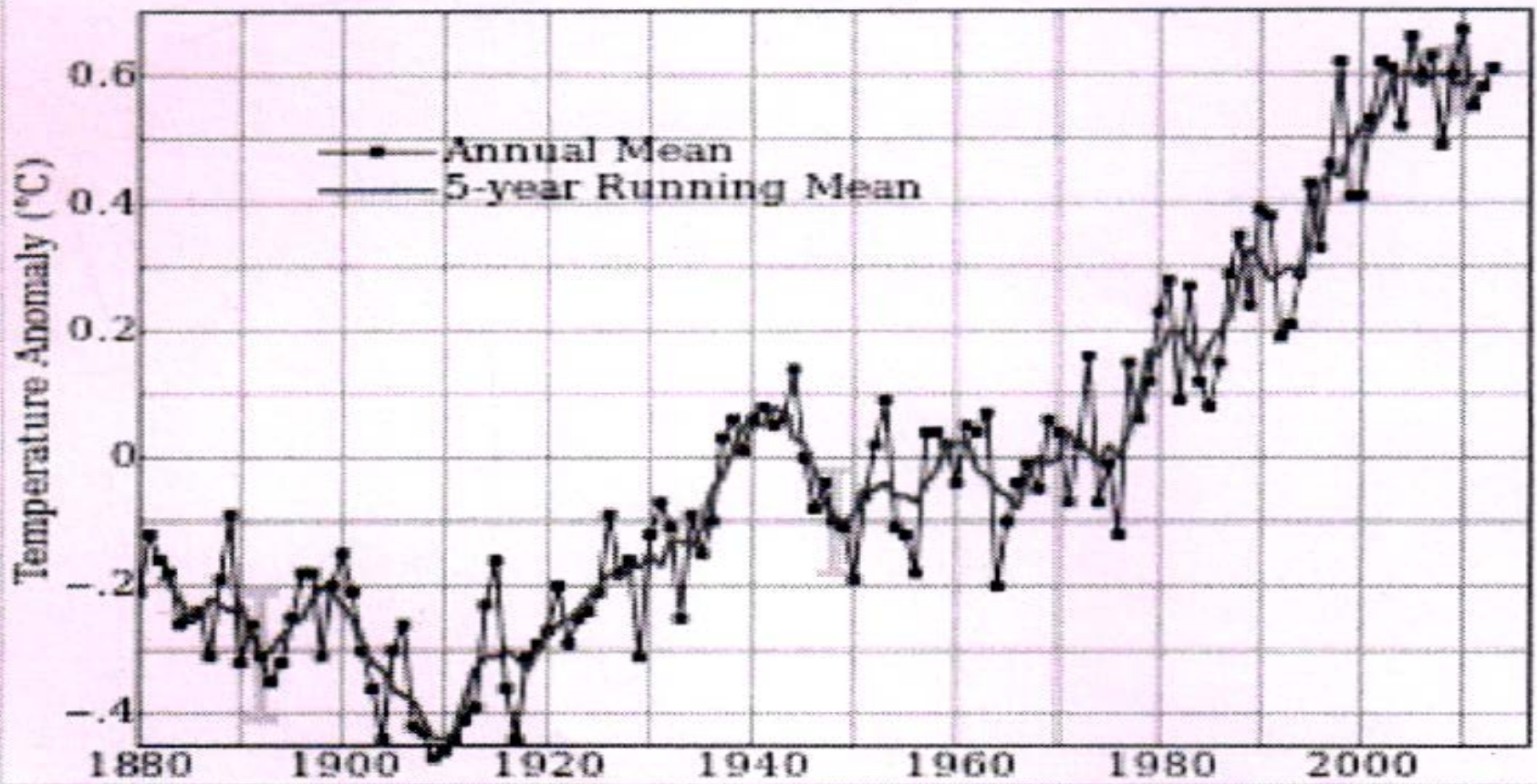
- **Poor suffer the most from environment pollution as they live nearest the polluting industries and toxic wastes.**
- **Poor contribute relatively nothing to pollution.**
- **Eliminating poverty should be one of the best form of environmental protection.**
- **Since then many conference have been held regarding environment and development.**
- **UN Conference on Environment and Development (UNCED), The Earth Summit, June 3-14, 1992. Rio de Janeiro.**
- **Copen Hagen climate change conference December 2009.**
- **United National Conference on sustainable development, June 2012 Rio de Janeiro.**
- **The focus has been on conservation of resources and economic development.**

- **Maurice Strong, the Secretary General of the Summit at Rio has pointed out that every nation should have its own model for development using local resources. The resources should be used in such a way that it remains available for future generation.**
- **Inter Government Panel (IPCC) looking after the climate change project on a global level.**
- **The Focus is on research awareness programme and advocacy Network and Climate change.**

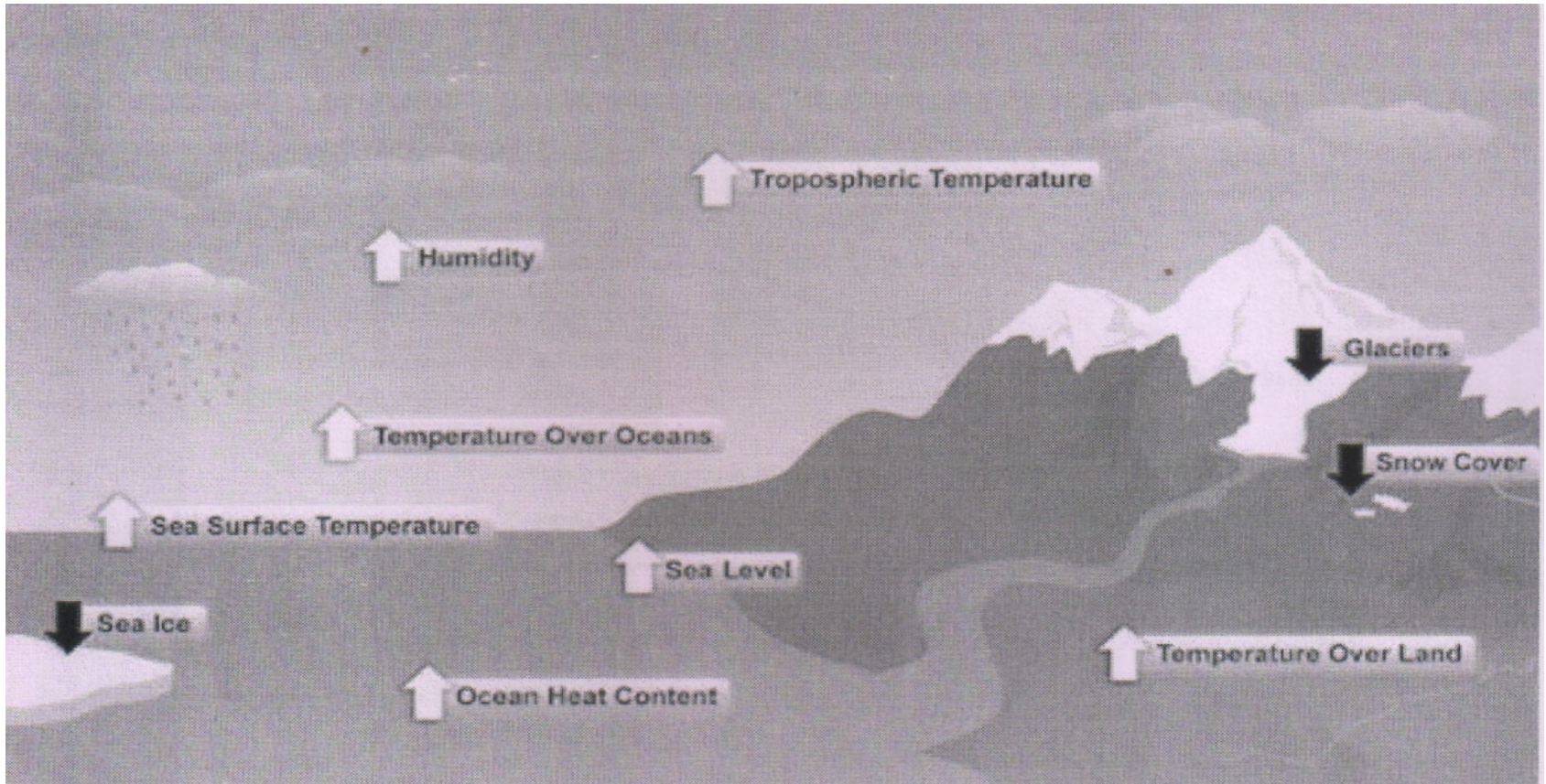
# Global Warming

- **Global warming refers to an increase in average global temperatures.**
- **Since the early 20th century, the global air and sea surface temperature has increased about 0.8°C (1.4°F), with about two-thirds of the increase occurring since 1980. Each of the last three decades has been successively warmer at the Earth's surface than any preceding decade since 1850.**

### Global Land-Ocean Temperature Index

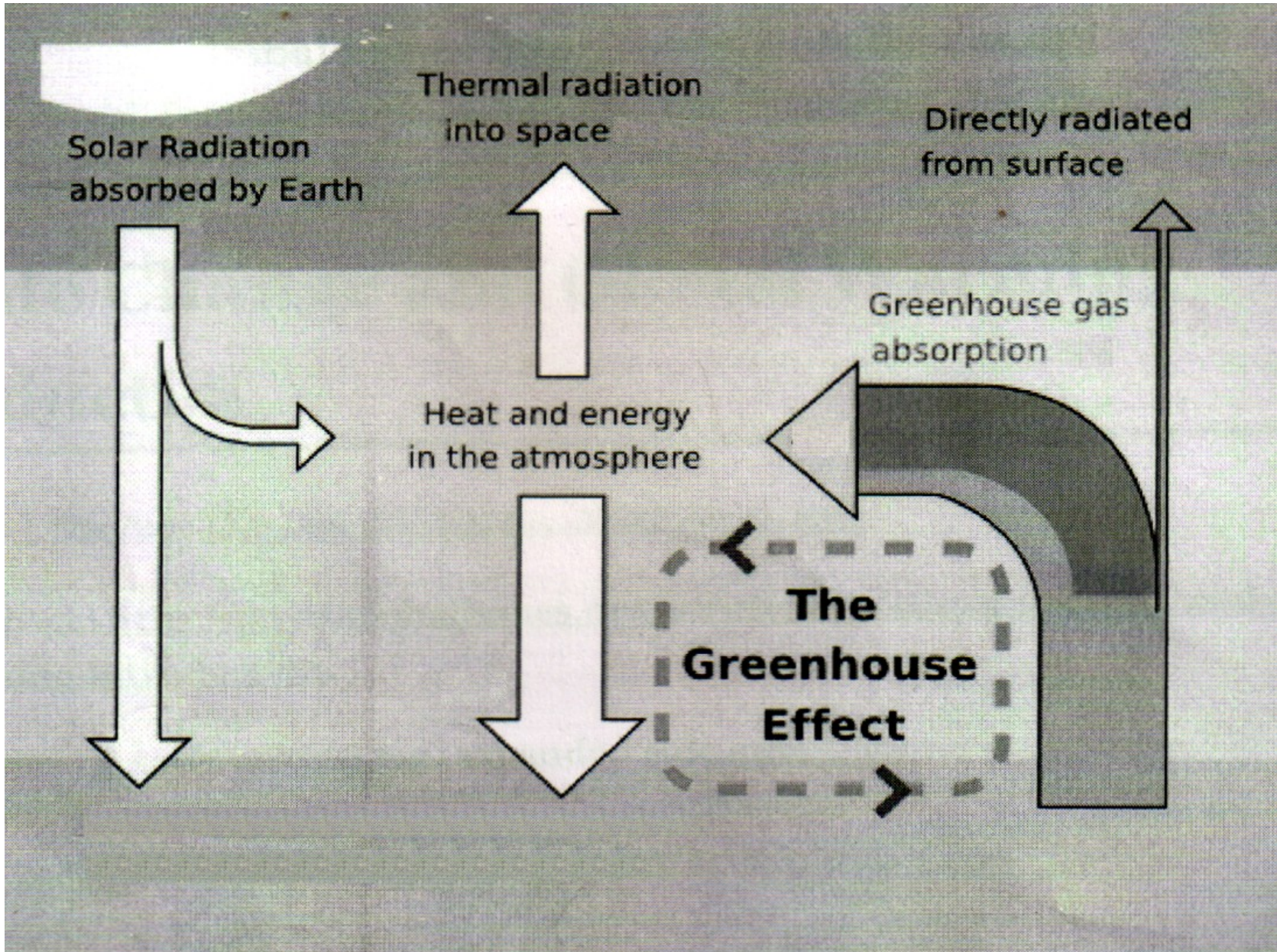


# The Greenhouse Effect?



Ten indicators for a warming world, Past Decade Warmest on Record According to Scientists in 48 Countries, NOAA, July 28, 2010





- **The term greenhouse is used in conjunction with the phenomenon known as the greenhouse effect.**
- **• Energy from the sun drives the earth's weather and climate, and heats the earth's surface;**
- **• In turn, the earth radiates energy back into space;**
- **• Some atmospheric gases (water vapor, carbon dioxide, and other gases) trap some of the outgoing energy, retaining heat somewhat like the glass panels of a greenhouse;**
- **• These gases are therefore known as greenhouse gases;**
- **• The greenhouse effect is the rise in temperature on Earth as certain gases in the atmosphere trap energy.**
- **Many of these greenhouse gases are actually life-enabling, for without them, heat would escape back into space and the Earth's average temperature would be a lot colder.**
- **However, if the greenhouse effect becomes stronger, then more heat gets trapped than needed, and the Earth might become less habitable for humans, plants and animals.**

# Causes of Global Warming

**A. Increase in concentration of CO<sub>2</sub> caused by,**

- **Increase in industrialization**
- **Increase in human population, associated activities**
- **Land use pattern, deforestation**
- **Land clearing for agriculture etc.**

## **B. Increase in concentration of Methane**

- **(i) Increase in Cattle population (Cows, Sheep, etc.)**
- **(ii) Coal mining**
- **(iii) Land fills, waste dumps**
- **(iv) Paddy fields (Rice)**
- **Methane (CH<sub>4</sub>) (which is 20 times as potent a greenhouse gas as carbon dioxide). Also nitrous oxide (N<sub>2</sub>O), plus other fluorinated industrial gases i.e. hydrofluorocarbons (HFCs), and sulphur hexafluoride (SF<sub>6</sub>) etc. Contribute to global warming. Water vapor is also considered a greenhouse gas.**

# Effects Of Global Warming

- **Biodiversity – With increase in global warming  $\frac{1}{4}$  of the earth species may be headed for extinction by the year 2050**
- **Sea level rise – The sea coast line of India is about 7000km and the sea level is rising. It can affect the city like Kolkata and Mumbai.**
- **Draught and flood**
- **Storms and Hurricanes**
- **Melting of ice and glaciers, etc.**
- **Heat related illness and diseases**





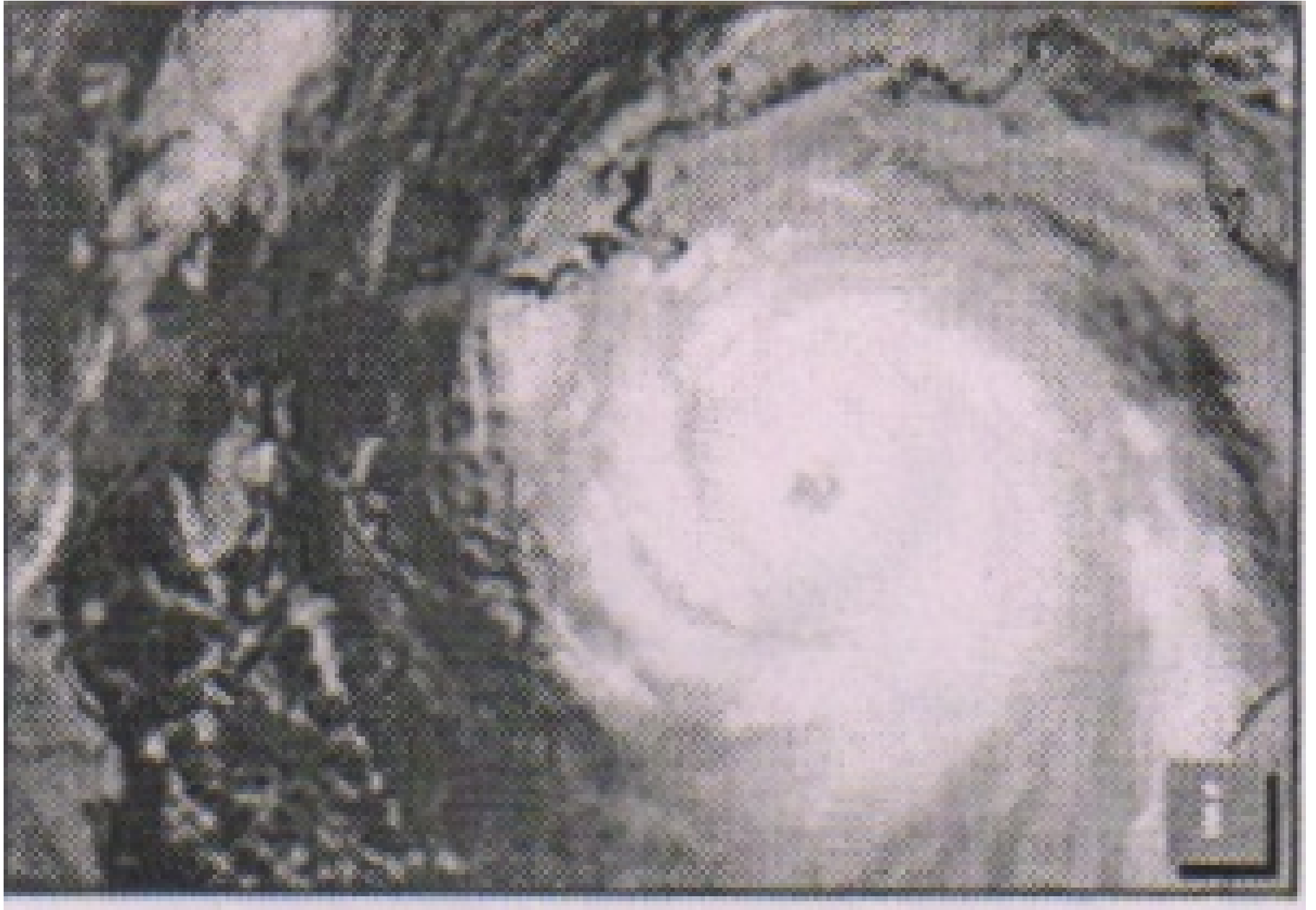
















# Mathematical Models

# CAUSES OF GLOBAL WARMING

- **We describe onwards some models for causes of global warming:**
- **1. Depletion of forests causing increase of CO<sub>2</sub> concentration.**
- **2. Increase in Population and Industrialization.**
- **3. Increase in the concentration of CH<sub>4</sub>**
- **4. Decrease in the thickness of Ozone layer causing global warming**



# **Effect of depletion of land cover (forests, grasslands, greenbelt) on decreasing CO<sub>2</sub> concentration leading to global warming**

- **Global warming is one of its complexities which is caused by higher concentrations of green house gases such as carbon dioxide (one of its most important gases) emitted by thermal power stations and other industrial and human sources. Due to the depletion of land cover (ecology, biota) by human population and industrialization, the sink of carbon dioxide decreases causing the increase of its concentration in to the atmosphere leading to higher average temperature causing climate change.**
- **The major human activities related to land use are for agriculture, industrialization leading to deforestation, desertification, soil erosion etc. Due to increase in human population, the cropped areas in recent decades have been extended to forest areas, grasslands causing loss in biota, thus decreasing the sink of carbon dioxide.**

- **To study this phenomenon using a mathematical model the following variables have to be considered in the modelling process.**
- **B, The average density of biomass of the vegetation (forests, grasslands, greenbelts, etc.) covering the land (Earth surface).**
- **N, The density of human population involved in the depletion of land cover reducing biomass density.**
- **I, The density of industrialization using the biomass, forestland, grassland, for an industrial growth.**
- **C, concentration of CO<sub>2</sub>, C<sub>0</sub> being the usual equilibrium concentration not creating global warming**
- **T, Average Temperature Rise in the Atmosphere from average Temperature T<sub>0</sub>**

- **The model is proposed as follows with appropriate initial conditions:**

$$\frac{dB}{dt} = bB - \frac{b_0 B^2}{L} - b_1 BN + kBC - k_1 BI$$

$$\frac{dN}{dt} = rN - \frac{r_0 N^2}{K} + r_1 BN + \phi NI$$

$$\frac{dI}{dt} = q + sB - s_0 I + s_1 BI + \psi NI$$

$$\frac{dC}{dt} = \alpha_0 (C - C_0) - \alpha CB + \alpha_1 N + \alpha_2 I$$

$$\frac{dT}{dt} = -\theta(T - T_0) + \theta_1 (C - C_0)$$

- where the coefficients have their usual meanings.
- The model analysis shows that as the biomass density decreases, the global warming temperature increases.

- **REFERENCES:**
- **J.B. Shukla, H.I. Freedman, V.N. Pal, O.P. Misra, M. Agarwal, and A. Shukla [1989], Degradation and Subsequent Regeneration of a Forestry Resource, Ecol. Model. 44, 219—229.**
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# **CONTROL OF GLOBAL WARMING CAUSED BY CO<sub>2</sub>**

- **We describe two models for the removal of CO<sub>2</sub> causing global warming.**
- **1. Removal of carbon-di-oxide fro the atmosphere by using liquid droplets and particulate matters.**
- **2. Removal of carbon-di-oxide fro the atmosphere by using liquid droplets, particulate matters and green belts of .**

# Removal of Carbon-di-oxide from the atmosphere

**Case I:** Removal of CO<sub>2</sub> by externally introduced liquid droplets and particulate matters in the upper atmosphere in a region.

The problem is governed by the following nonlinear ordinary differential equations,

$C$ , The concentration of CO<sub>2</sub> in the atmosphere

$C_i$ , : The concentration of externally introduced liquid species

$C_p$ , The concentration of particulate matters

$C_a, C_{pa}$ , The concentrations of the secondary species formed in the atmosphere due to reactions of CO<sub>2</sub> with externally introduced liquid species and conducive particulate matters respectively

$Q$ , The rate of introduction of CO<sub>2</sub> into the regional atmosphere

$$\frac{dC}{dt} = Q - \alpha_0 C - \alpha_1 C C_1 - \alpha_2 C C_p$$

$$\frac{dC_1}{dt} = \lambda C - \lambda_0 C_i - \lambda_1 C C_i$$

$$\frac{dC_p}{dt} = r C - r_0 C_p - \lambda_2 C C_p$$

$$\frac{dC_a}{dt} = \theta_1 C C_i - \theta_{10} C_a$$

$$\frac{dC_{pa}}{dt} = \theta_2 C C_p - \theta_{20} C_{pa}$$

with  $C(0) > 0$ ,  $C(0) > 0$ ,  $C_p(0) > 0$ ,  $C_a(0) > 0$ ,  $C_{pa} > 0$



The symbols are described below

$\alpha_1 \sim \lambda_1, \alpha_2 \sim \lambda_2$ , Rates of interaction (depletion rates) of CO<sub>2</sub> with and C<sub>i</sub>, respectively

$\lambda$ , Rate of introduction of liquid species in the regional atmosphere

$r$ , Rate of introduction of particulate matters in the regional atmosphere

$\alpha_0, \lambda_0, r_0$ , Natural depletion rate coefficients of C, C<sub>i</sub> and C<sub>p</sub> respectively

$\theta_1, \theta_2$ , Rate of formation of C<sub>a</sub> and C<sub>pa</sub> respectively

$\theta_{10}, \theta_{20}$ , Natural depletion rate coefficients of C<sub>a</sub> and C<sub>pa</sub> respectively

## **Case II: Removal of CO<sub>2</sub> from the near earth atmosphere by introducing a liquid species as well as by planting green belt near the sources of emissions.**

Let

$C$ , The concentration of CO<sub>2</sub> in the atmosphere

$C_i$ , The concentration of externally introduced liquid species

$C_p$ , The concentration of particulate matters

$C_a, C_{pa}$ , The concentrations of the secondary species formed in the atmosphere due to reactions of CO<sub>2</sub> with externally introduced liquid species and conducive particulate matters respectively

$Q$ , The rate of introduction of CO<sub>2</sub> into the regional atmosphere

The problem is governed by the following nonlinear ordinary differential equations,

$$\frac{dC}{dt} = Q - \alpha_0 C - \lambda_1 C C_i - \lambda_2 B C$$

$$\frac{dC_i}{dt} = \lambda C - \lambda_0 C_i - \lambda_1 C C_i$$

$$\frac{dB}{dt} = rB - \frac{r_0 B^2}{K} + \lambda_2 B C$$

$$\frac{dC_p}{dt} = \theta C C_i - \theta_0 C_p$$

with  $C(0) > 0$ ,  $C_i(0) > 0$ ,  $C_p(0) > 0$ ,  $C_a(0) > 0$ ,  $C_{pa} > 0$

The symbols are described below

$\lambda_1, \lambda_2$ , Rates of interaction (depletion rates) of CO<sub>2</sub> with and C<sub>i</sub>, respectively

$\lambda$ , Rate of introduction of liquid species in the regional atmosphere

$r$ , Rate of introduction of particulate matters in the regional atmosphere

$\alpha_0, \lambda_0, r_0$ , Natural depletion rate coefficients of C, C<sub>i</sub> and C<sub>p</sub> respectively

$\theta_1, \theta_2$ , Rate of formation of C<sub>a</sub> and C<sub>pa</sub> respectively

$\theta_{10}, \theta_{20}$ , Natural depletion rate coefficients of C<sub>a</sub> and C<sub>pa</sub> respectively.

# Reference

- Shukla J.B and M.S Chauhan, Shyam Sundar and Naresh R (2015), Removal of Carbon Dioxide from the atmosphere to reduce global warming: a modeling study
- Int. J. Global Warming, Vol 7, No 2 ,pp 270-292

# **Control Of Global Warming By Using As Green Belts Plantation And Seaweeds Cultivation**

- **forest biomass density**
- **seaweeds density**
- **C: Concentration of in the environment**
- **T: temperature in the environment**

$$\frac{dB_1}{dt} = r_1 B_1 \left(1 - \frac{B_1}{K_1}\right) + \pi_1 \alpha_1 B_1 C,$$

$$\frac{dB_2}{dt} = r_2 B_2 \left(1 - \frac{B_2}{K_2}\right) + \pi_2 \alpha_2 B_2 C,$$

$$\frac{dC}{dt} = Q - \alpha_0 C - \alpha_1 B_1 C - \alpha_2 B_2 C,$$

$$\frac{dT}{dt} = \theta(C - C_0) - \theta_0(T - T_0),$$

$$B_1(0) \geq 0, B_2(0) \geq 0, C(0) \geq 0, T(0) \geq 0.$$

**$C_0$ :**

- **$C_0$  is critical level of the concentration of which does not create global warming,**
- **$T_0$  is equilibrium level of the temperature.**



# Some Models For Global Warming Caused By Methane and its Control

## Model 1

- Let  $B$  the density of biomass of rice paddy
- $C_a$  the density of cattle population
- $C$  concentration of methane  $\text{CH}_4$
- $M$  mitigation effort to control  $\text{CH}_4$
- $C_1, C_2$  concentration of methane  $\text{CH}_4$  due to  $B, C_a$  respectively

$$\frac{dC_a}{dt} = r_a C_a - \frac{r_{a0} C_a^2}{K_a} + r_{a1} C_a N$$

$$\frac{dN}{dt} = rN - \frac{r_0 N^2}{K} + r_1 C_a N$$

$$\frac{dC_1}{dt} = \alpha C_a - \alpha_0 C_1 - \alpha_1 C_1 M$$

$$\frac{dC_2}{dt} = \beta N - \beta_0 C_2 - \beta_1 C_2 M$$

$$\frac{dC}{dt} = Q_0 + \lambda_1 C_1 + \lambda_2 C_2 - \lambda_0 C$$

$$\frac{dM}{dt} = \theta(C - C_0) - \theta_0 M$$

$$\text{Where } C_0 = \frac{Q_0}{\lambda_0}$$

# Effect of Methane (CH<sub>4</sub>) On Temperature

$$\frac{dT}{dt} = -\theta(T - T_0) + \theta_1(C - C_0)$$

## Model 2

- **Let B** biomass density of rice paddy
- **N** the density of human population
- **C** concentration of methane  $\text{CH}_4$
- **M** mitigation effort to control  $\text{CH}_4$
- **$C_1, C_2$**  concentration of precursor methane from N and B

$$\frac{dB}{dt} = sB - \frac{s_0 B^2}{L} - s_1 BN$$

$$\frac{dN}{dt} = rN - \frac{r_0 N^2}{K} + r_1 BN$$

$$\frac{dC_1}{dt} = \alpha B - \alpha_0 C_1 - \alpha_1 C_1 M$$

$$\frac{dC_2}{dt} = \beta N - \beta_0 C_2 - \beta_1 C_2 M$$

$$\frac{dC}{dt} = Q_0 + \lambda_1 C_1 + \lambda_2 C_2 - \lambda_0 C$$

$$\frac{dM}{dt} = \theta(C - C_0) - \theta_0 M$$

$$\text{Where } C_0 = \frac{Q_0}{\lambda_0}$$

## Model 3

- **Let  $N$  the density of human population**
- **$C_a$  the density of cattle population**
- **$C$  concentration of methane  $CH_4$**
- **$M$  mitigation effort to control  $CH_4$**
- **$C_1, C_2$  concentration of precursor methane from  $C_a$  and  $N$**

$$\frac{dB}{dt} = sB - \frac{s_0 B^2}{L} - s_1 B C_a$$

$$\frac{dC_a}{dt} = rC_a - \frac{r_0 C_a^2}{K} + r_1 B C_a$$

$$\frac{dC_1}{dt} = \alpha B - \alpha_0 C_1 - \alpha_1 C_1 M$$

$$\frac{dC_2}{dt} = \beta C_a - \beta_0 C_2 - \beta_1 C_2 M$$

$$\frac{dC}{dt} = Q_0 + \lambda_1 C_1 + \lambda_2 C_2 - \lambda_0 C$$

$$\frac{dM}{dt} = \theta(C - C_0) - \theta_0 M$$

- **Where,**  $C_0 = \frac{Q_0}{\lambda_0}$
- **And  $C_1, C_2$  concentration of precursor/bi-products**
-



# **EFFECTS OF GLOBAL WARMING**

- **We describe onwards some models for the effects of global warming.**
- **1. On carrier dependent diseases.**
- **2. On Malaria**
- **3. On sea rise.**
- **4. On glacial melting.**

# **EFFECT OF GLOBAL WARMING ON THE SPREAD OF CARRIER DEPENDENT INFECTIOUS DISEASES : A MODEL**

- Due to higher concentration of CO<sub>2</sub> and increased rain fall ecology of the region changes which provide shelters to many species including carriers and vectors which cause tropical infectious diseases. In rural and near forest areas living conditions, consumption, patterns of the people also change, leading to variety of household discharges affecting population of carriers such as flies and the population of vectors such as mosquitoes responsible for increased spread of infectious diseases (Cholera, T. B., Malaria, etc.) (see ref.).
- The modelling and analysis of the effect of climate change due to increase in temperature on the spread of carrier dependent infectious diseases is an important problem to be studied by using a mathematical model.

- Let  $X$  be the density of susceptible human population
- $Y$  is the density of infected human population
- $C_r$  is the density of carrier population
- $C$  is the concentration of  $\text{CO}_2$
- $T$  is the increase in temperature from  $T_0$

- **The model is proposed as follows with appropriate initial conditions:**

$$\frac{dX}{dt} = A - \beta XY - dX + \mu Y - kC_r X$$

$$\frac{dY}{dt} = \beta XY - dY - \mu Y - \alpha Y + kC_r X$$

$$\frac{dC_r}{dt} = \phi C_r (T - T_0) + s C_r - s_0 \frac{C_r^2}{L}$$

$$\frac{dC}{dt} = Q - \alpha_0 C + \alpha_1 (X + Y)$$

$$\frac{dT}{dt} = -\theta(T - T_0) + \theta_1 (C - C_0)$$

- **where the coefficients have their usual meanings.**
- **The model analysis shows as the global warming temperature increases, the spread of spread of carrier dependent infectious diseases increases.**

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- **Ghosh M., Chandra P., Sinha P., Shukla J.B., Modeling the spread of carrier-dependent infectious diseases with environmental effect, *Appl. Math. Comp.*, 152, 385–402, 2004.**
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- **Singh S., Chandra P., Shukla J.B., Modeling and analysis of the spread of carrier dependent infectious diseases with environmental effects, *J. Biol. Sys.*, 11(3), 325–335, 2003.**
- **Singh S., Shukla J.B., Chandra P., Modeling and analysis of the spread of malaria: environmental and ecological effects, *J. Biol. Sys.*, 13(1), 1–11, 2005.**

# **Effect of Global Warming on the spread of malaria: A model**

- **Due to climate change caused by emissions of green house gases such as CO<sub>2</sub>, CH<sub>4</sub> etc. into the environment, characteristics of each regional habitat on this planet earth related to air, water, land resources change causing undesirable consequences. These effects are locally reflected in the form of severe rain fall at some places while drought at some other places.**
- **Due to higher concentration of CO<sub>2</sub> and increased rain fall ecology of the region also changes which provide shelters to many species including carriers and vectors which cause tropical infectious diseases. In rural and near forest areas living conditions, consumption, patterns of the people also change, leading to variety of household discharges affecting population of carriers such as flies and the population of vectors such as mosquitoes responsible for increased spread of infectious diseases (Cholera, T. B., Malaria, etc.) (see ref.).**
- **The climate change also affects the nature of infrastructures needed for housing and health and if suitable actions not taken in time undesirable consequences may occur.**

- **The modelling and analysis of the effect of climate change due to increase in temperature on the spread of malaria caused by mosquito vectors is an important problem to be studied by using a mathematical model.**
- **Let  $X_1$  be the density of susceptible human population**
- **$Y_1$  is the density of infected human population**
- **$X_2$  is the density of susceptible mosquito population**
- **And  $Y_2$  is the density of infected mosquito population**
- **$C$  is the concentration of  $CO_2$  and**
- **$T$  is the increase in temperature from  $T_0$**



- **The model is proposed as follows with appropriate initial conditions:**

$$\frac{dX_1}{dt} = A - \beta_1 X_1 Y_2 - dX_1 + \mu Y_2$$

$$\frac{dY_2}{dt} = \beta_1 X_1 Y_2 - dY_2 - \mu Y_2 - \alpha Y_2$$

$$\frac{dX_1}{dt} = \phi_1 X_2 (T - T_0) + sN_2 - s_0 \frac{N_2^2}{L} - \beta_2 X_2 Y_1 - d_2 X_2$$

$$\frac{dY_2}{dt} = \phi_1 Y_2 (T - T_0) + \beta_2 X_2 Y_1 - d_2 Y_2$$

$$\frac{dC}{dt} = Q - \alpha_0 C + \alpha_1 (X_1 + Y_1)$$

$$\frac{dT}{dt} = -\theta(T - T_0) + \theta_1 (C - C_0)$$

- **where the coefficients have their usual meanings.**
- **The model analysis shows as the global warming temperature increases, the spread of malaria increases.**

# References

- Ghosh M., Shukla J.B., Chandra P., Sinha P., An Epidemiological model for carrier dependent infectious diseases with environmental effect, *Int. J. Appl. Sc. Comp.*, 7, 188–204, 2000.
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# **Modeling the role of global warming on sea rise**

- **I : density of ice on the planet**
- **density of Total population**
- **density of industrialization**
- **C: Concentration of in the environment**
- **T: temperature in the environment**

$$\frac{dI}{dt} = q - mI - nI(T - T_0)$$

$$\frac{dW}{dt} = nI(T - T_0) - k_0(W - W_0)$$

$$\frac{dL}{dt} = k_0(W - W_0) - p_0(L - L_0)$$

$$\frac{dn}{dt} = rN \left(1 - \frac{N}{K}\right) - r_0 N^2 (L - L_0)$$

$$\frac{dN_c}{dt} = sN - s_0 N_c^2 (L - L_0)$$

$$\frac{dC}{dt} = Q - \alpha_0 C + \alpha N$$

$$\frac{dT}{dt} = \theta(C - C_0) - \theta_0(T - T_0),$$



$$B(0) \geq 0, N(0) \geq 0, I(0) \geq 0, C(0) \geq 0, T(0) \geq 0, N_c(0) \geq 0.$$

- **$C_0$  is critical level of the concentration of  $CO_2$  which does not create global warming,**
- **$T_0$  is equilibrium level of the temperature.**

# Modeling the role of Global Warming On Glacial Melting

- **G** : density of ice on glaciers
- forest biomass density
- density of population
- density of industrialization
- **C**: Concentration of in the environment
- **T**: temperature in the environment

$$\frac{dB}{dt} = sB \left(1 - \frac{B}{L}\right) - s_1BN - s_2BI + \pi_3 \alpha_3 BC,$$

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K}\right) + \pi_1 s_1 BN,$$

$$\frac{dI}{dt} = q - k_0 I + \pi_2 s_2 BI,$$

$$\frac{dC}{dt} = Q - \alpha_0 C + \alpha_1 N + \alpha_2 I - \alpha_3 BC,$$

$$\frac{dT}{dt} = \theta(C - C_0) - \theta_0(T - T_0),$$

$$\frac{dG}{dt} = Qc - mG - nG(T - T_0)$$

$$B(0) \geq 0, N(0) \geq 0, I(0) \geq 0, C(0) \geq 0, T(0) \geq 0, G(0) \geq 0.$$



# COMMENTS

- **Climate change affect all living beings. It affects human health and may caused infectious disease. It may also cause migration of people from coastal areas due to sea level rise.**

**THANKS**

