Nonlinear boundary value problems in connexion with harmonic functions

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Abstract: We study the problem of finding a function $u$ verifying $-\Delta u = 0$ in $\Omega$ under the boundary condition $\frac{\partial u}{\partial n} + g(u) = \mu$ on $\partial \Omega$ where $\Omega \subset \mathbb{R}^N$ is a smooth domain, $n$ is the normal unit outward vector to $\Omega$, $\mu$ is a measure on $\partial \Omega$ and $g$ a continuous nondecreasing function. We give sufficient conditions on $g$ for this problem to be solvable for any measure. When $g(r) = |r|^{p-1}r$, $p > 1$, we give conditions in order an isolated singularity on $\partial \Omega$ to be removable. We also give capacitary conditions on a measure $\mu$ in order the problem with $g(r) = |r|^{p-1}r$ to be solvable for some $\mu$. We also study the isolated singularities of functions satisfying $-\Delta u = 0$ in $\Omega$ and $\frac{\partial u}{\partial n} + g(u) = 0$ on $\partial \Omega \setminus \{0\}$. 