ON THE EQUATION \((\nabla u)^t H(u)(\nabla u) = G\)

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Abstract

Let \(n \in \mathbb{N}, n \geq 2\) and let \(\Omega \subseteq \mathbb{R}^n\) be open. Let \(H, G : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}\) be of appropriate regularity. We discuss the existence of an immersion \(u : \Omega \rightarrow \mathbb{R}^n\) of appropriate regularity, satisfying

\[(\nabla u)^t H(u)(\nabla u) = G\text{ in } \Omega.\]  

We consider Cauchy, Dirichlet and Dirichlet-Neumann problems.

Equation (1) comes up in diverse contexts. When \(H\) (and hence \(G\)) is symmetric and positive definite, Equation (1) is connected to the problem of equivalence of Riemannian metrics. The symmetric case is also important in the non-linear elasticity theory because of its connection with the Cauchy-Green deformation tensor. When \(H\) (and hence \(G\)) is skew-symmetric, Equation (1) comes up in the context of the problem of equivalence of differential two-forms.

The aim of the talk is to present a survey of recent progress and advances made in the context of Equation (1). We shall also discuss the general case when \(H, G\) are neither symmetric nor skew-symmetric.