Boundary value problems for Schrödinger equations with Hardy type potentials.

Abstract. We consider equations of the form $-L_V u = \tau$ where $L_V = \Delta + V$ and $\tau$ is a Radon measure in a Lipschitz bounded domain $D \subset \mathbb{R}^N$. The assumptions on $V$ include the condition $|V(x)| \leq a\delta(x)^{-2}$ where $a > 0$ and $\delta(x) = \text{dist}(x, \partial D)$. An additional condition guarantees the existence of a ground state $\Phi_V$. The model example is $V = \gamma \delta(x)^{-2}$ where $\gamma < c_H$ ($c_H$ = Hardy constant).

For positive solutions of the equation we define the $L_V$ boundary trace. If $\int_D \Phi_V \, d\tau < \infty$, the boundary trace is well defined as a positive, bounded measure $\nu$ on $\partial D$. We consider the corresponding b.v.p., namely, $-L_V u = \tau$ in $D$, $u = \nu$ on $\partial D$. We show that, for $\tau$ and $\nu$ as above, the b.v.p. has a unique solution. Further, under some conditions on the ground state - satisfied for a large family of potentials - we obtain sharp two sided estimates of positive solutions of the b.v.p. Finally we discuss some applications to semilinear problems involving the operator $L_V$. 