1. Introduction

1.1 Historical background

Human beings are extremely interested in the observation of nature, as this was and still is of utmost importance for their survival. Human senses are especially well adapted to recognize moving objects as in many cases they mean eventual danger. One can easily imagine how the observation of moving objects has stimulated first simple experiments with set-ups and tools easily available in nature. Today the same primitive behavior becomes obvious, when small children throw little pieces of wood down from a bridge in a river and observe them floating downstream. Even this simple experimental arrangement allows them to make a rough estimate of the velocity of the running water and to detect structures in the flow such as swirls, wakes behind obstacles in the river, water shoots, etc.

However, with such experimental tools the description of the properties of the flow is restricted to qualitative statements. Nevertheless, being at the same time an artist with excellent skills and an educated observer of nature, LEONARDO DA VINCI, was able to prepare very detailed drawings of the structures within a water flow by mere observation.

Fig. 1.1. LUDWIG PRANDTL in front of his water tunnel for flow visualization in 1904
1. Introduction

A great step forward in the investigation of flows was made after it was possible to replace such passive observations of nature by experiments carefully planned to extract information about the flow utilizing visualization techniques. A well-known promoter of such a procedure was Ludwig Prandtl, one of the most prominent representatives of fluid mechanics, who designed and utilized flow visualization techniques in a water tunnel to study aspects of unsteady separated flows behind wings and other objects.

Figure 1.1 shows Ludwig Prandtl in 1904 in front of his tunnel, driving the flow manually by rotating a blade wheel [178]. The tunnel comprises an upper and lower section separated by a horizontal wall. The water recirculates from the upper open channel, where the flow may be observed, back through the lower closed duct. Two-dimensional models like cylinders, prisms, and wings can be easily mounted vertically in the upper channel, thereby extending above the level of the surface of the water.

The flow is visualized by distributing a suspension of mica particles on the surface of the water. Ludwig Prandtl studied the structures of the flow in steady as well as in unsteady flow (at the onset of flow) with this arrangement [165].

Being able to change a number of parameters of the experiment (model, angle of incidence, flow velocity, steady-unsteady flow) Prandtl gained insight into many basic features of unsteady flow phenomena. However, at that time only a qualitative description of the flow field was possible. No quantitative data about flow velocity, etc., could be achieved.

Today, 90 years after Ludwig Prandtl’s experiments, it is easily possible to also extract quantitative information about the instantaneous flow velocity field exactly from the same kind of images as were available to Prandtl. A proof for this is given in figure 1.2. A replica of Ludwig Prandtl’s water tunnel together with a flash lamp for illumination and a video camera have been employed to obtain a visualization of the flow by means of aluminum particles distributed on the water surface.

Fig. 1.2. Separated flow behind wing, visualized with modern equipment in a replica of Ludwig Prandtl’s tunnel

Evaluation of this recording by methods which will be described later resulted in a vector map of the instantaneous velocity field shown in figure 1.3. This means that the basic principles underlying the quantitative visualization technique which is the subject of this book have already been known for a long time.

However, the scientific and technical progress achieved in the last 15 years in optics, lasers, electronics, video and computer techniques was necessary to further develop a technique for qualitative flow visualization to such a stage that it can be employed for quantitative measurement of complex instantaneous velocity fields.

1.2 Principle of particle image velocimetry (PIV)

In the following the basic features of this measurement technique, most widely named “particle image velocimetry” or “PIV”, will be described briefly.

The experimental set-up of a PIV system typically consists of several subsystems. In most applications tracer particles have to be added to the flow. These particles have to be illuminated in a plane of the flow at least twice within a short time interval. The light scattered by the particles has to be

\[1\] In earlier years other names such as speckle velocimetry, particle image displacement velocimetry etc. have been used as well.
velocity vector) is calculated taking into account the time delay between the two illuminations and the magnification at imaging.

The process of interrogation is repeated for all interrogation areas of the PIV recording. With modern video cameras (1000 × 1000 sensor elements) it is possible to capture more than 100 PIV recordings per minute. The evaluation of one video PIV recording with 3600 instantaneous velocity vectors (depending on the size of the recording and of the interrogation area) is of the order of a few seconds with standard computers. If even faster availability of the data is required for on line monitoring of the flow, dedicated hardware processors are commercially available which perform evaluations of similar quality within fractions of a second.

Before going into the details of the PIV technique, some general aspects have to be discussed in order to facilitate the understanding of certain technical solutions later on.

**Nonintrusive velocity measurement.** In contrast to techniques for the measurement of flow velocities employing probes as pressure tubes or hot wires, the PIV technique being an optical technique works nonintrusively. This allows the application of PIV even in high speed flows with shocks or in boundary layers close to the wall, where the flow may be disturbed by the presence of probes.

**Indirect velocity measurement.** In the same way as with laser Doppler velocimetry the PIV technique measures the velocity of a fluid element indirectly by means of the measurement of the velocity of tracer particles within the flow, which – in most applications – have been added to the flow before the experiment started. In two phase flows particles are already present in the flow. In such a case it will be possible to measure the velocity of the particles themselves as well as the velocity of the fluid (to be additionally seeded with small tracer particles).

**Whole field technique.** PIV is a technique which allows one to record images of large parts of flow fields in a variety of applications in gaseous and liquid media and to extract the velocity information out of these images. This feature is unique to the PIV technique. Except Doppler global velocimetry (DGV) [176, 177], which is a new technique particularly appropriate for high speed air flows, all other techniques for velocity measurements only allow the measurement of the velocity of the flow at a single point, however in most cases with a high temporal resolution. With PIV the spatial resolution is large, whereas the temporal resolution (frame rate of recording PIV images) is limited due to technical restrictions. These features must be observed if comparing results obtained by PIV with those obtained with traditional techniques. Instantaneous image capture and high spatial resolution at PIV allow the detection of spatial structures even in unsteady flow fields.

**Velocity lag.** The need to employ tracer particles for the measurement of the flow velocity requires us to check carefully for each experiment whether the particles will faithfully follow the motion of the fluid elements, at least to

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Figure 1.4 briefly explains a typical set-up for PIV recording in a wind tunnel. Small tracer particles are added to the flow. A plane (light sheet) within the flow is illuminated twice by means of a laser (the time delay between pulses depending on the mean flow velocity and the magnification at imaging). It is assumed that the tracer particles move with local flow velocity between the two illuminations. The light scattered by the tracer particles is recorded via a high quality lens either on a single photographic negative or on two separate frames on a special cross correlation CCD sensor. After development the photographic PIV recording is digitized by means of a scanner. The output of the CCD sensor is stored in real time in the memory of a computer directly.

For evaluation the digital PIV recording is divided in small subareas called "interrogation areas". The local displacement vector for the images of the tracer particles of the first and second illumination is determined for each interrogation area by means of statistical methods (auto- and cross- correlation). If assumed that all particles within one interrogation area have moved homogeneously between the two illuminations. The projection of the vector of the local flow velocity into the plane of the light sheet (2-component
larger particles have to be used for water flow experiments, which can mostly be accepted since the density matching of particles and fluid is usually better. In the following three figures the normalized scattered intensity of different diameter glass particles in water according to the Mie theory are shown at $\lambda = 532$ nm.

![Image 2.5. Light scattering by a 1 μm glass particle in water](image)

![Image 2.6. Light scattering by a 10 μm glass particle in water](image)

![Image 2.7. Light scattering by a 30 μm glass particle in water](image)

As can be seen from all Mie scattering diagrams, the light intensity is not blocked by the particles but spread in all directions. Therefore, for a large number of particles inside the light sheet massive multiscattering appears. Then the light which is focused by the recording lens is not only due to direct illumination but also due to fractions of light, which have been scattered by more than one particle. In the case of heavily seeded flows this considerably increases the intensity of individual particle images, because the intensity of directly – at 90° to the incident illumination – recorded light is orders of magnitude smaller than that scattered in the forward scatter range.

One interesting implication is that not only can larger particles be used to increase the scattering efficiency but also the number density of the particles. However, two problems limit this effect from being intensively used. First, the background noise and therefore the noise on the recordings will increase significantly. Second, if – as is usually the case – polydisperse particles (i.e., particles of different sizes) are used, it is finally not sure whether the number of visible particles has been increased by simply increasing the number of very large particles. Since images of larger particles clearly dominate PIV evaluation, it would be difficult to give sure estimates on the effective particle size and the corresponding velocity lag.

### 2.1.3 Particle generation and supply

Descriptions of seeding particles and their characteristics have been given in many scientific publications. In contrast to that, little information can be found in the literature on how to practically supply the particles into the flow under investigation. Sometimes seeding can be done very easily or does not even have to be done. The use of natural seeding is sometimes acceptable, if enough visible particles are naturally present to act as markers for PIV. In almost all other work it is desirable to add tracers in order to achieve sufficient image contrast and to control particle size. For most liquid flows this can be easily done by suspending solid particles into the fluid and mixing them in order to get a homogeneous distribution.

<table>
<thead>
<tr>
<th>Type</th>
<th>Material</th>
<th>Mean diameter in μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid</td>
<td>Polystyrene</td>
<td>10 – 100</td>
</tr>
<tr>
<td></td>
<td>Aluminum</td>
<td>2 – 7</td>
</tr>
<tr>
<td></td>
<td>Glass spheres</td>
<td>10 – 100</td>
</tr>
<tr>
<td></td>
<td>Granules for synthetic coatings</td>
<td>10 – 500</td>
</tr>
<tr>
<td>Liquid</td>
<td>Different oils</td>
<td>50 – 500</td>
</tr>
<tr>
<td>Gaseous</td>
<td>Oxygen bubbles</td>
<td>50 – 1000</td>
</tr>
</tbody>
</table>

A number of different particles which can be used for flow visualization, LDV, and PIV are listed in table 2.1 for liquid and in 2.2 for gas flows. For our experiments in oil and water flows we used coated glass spheres of approximately 10 μm diameter as is shown in figure 2.8 for two different magnifications. They offer good scattering capability and a sufficiently small velocity lag.

In gas flows the supply of tracers is very often more critical for the quality and feasibility of the PIV measurement and the health of the experimentalists
if they have to breathe seeded air for example in wind tunnels in an open test section. The particles which are often used are not easy to handle because many liquid droplets tend to evaporate rather quickly and solid particles are difficult to disperse and very often agglomerate. The particles can therefore not simply be supplied a long time before the measurement, but must be injected into the flow shortly before the gaseous medium enters the test section. The injection has to be done without significantly disturbing the flow, but in a way and at a location that ensures homogeneous distribution of the tracers. Since the existing turbulence in many test set-ups is not strong enough to mix the fluid and particles sufficiently, the particles have to be supplied from a large number of openings. Distributors, like rakes consisting of many small pipes with a large number of tiny holes, are often used. Therefore, particles which can be transported inside small pipes are required.

<table>
<thead>
<tr>
<th>Table 2.2. Seeding materials for gas flows</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
</tr>
<tr>
<td>Solid</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
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<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td>Smoke</td>
</tr>
<tr>
<td>Liquid</td>
</tr>
</tbody>
</table>

A number of techniques are used to generate and supply particles for seeding gas flows [62, 75, 77]: dry powders can be dispersed in fluidized beds or by air jets. Liquids can be evaporated and afterward condensed in so-called condensation generators, or liquid droplets can directly be generated in atomizers. Atomizers can also be used to disperse solid particles suspended in evaporating liquids, or to generate tiny droplets of high vapor pressure liquids (e.g. oil) that have been mixed with low vapor pressure liquids (e.g. alco-

hol) which evaporate before the test section. For seeding wind tunnel flows condensation generators, smoke generators and monodisperse polystyrene or latex particles injected with water-ethanol are most often used for flow visualization and LDV. For most of the PIV measurements in air flows Laskin nozzle generators and oil have been used. These particles offer the advantage of not being toxic, they stay in air at rest for hours, and do not change in size significantly under various conditions. In recirculating wind tunnels they can be used for a global seeding of the complete tunnel volume or for a local seeding of a stream tube by a seeding rake with a few hundred tiny holes. A technical description of such an atomizer is given below.

![Fig. 2.9. Oil seeding generator](image)

The aerosol generator consists of a closed cylindrical container with two air inlets and one aerosol outlet. Four air supply pipes—mounted at the top—dip into vegetable oil inside the container. They are connected to one air inlet by a tube and each has a valve. The pipes are closed at their lower ends (see figure 2.10). Four Laskin nozzles, 1 mm in diameter, are equally spaced in each pipe [45].

![Fig. 2.10. Sketch of a Laskin nozzle](image)

A horizontal circular impactor plate is placed inside the container, so that a small gap of about 2 mm is formed by the plate and the inner wall of the container. The second air inlet and the aerosol outlet are connected directly to the top. Two gauges measure the pressure on the inlet of the nozzles and
inside the container, respectively. Compressed air with 0.5 to 1.5 bar pressure difference with respect to the outlet pressure is applied to the Laskin nozzles and creates air bubbles within the liquid. Due to the shear stress induced by the tiny sonic jets small droplets are generated and carried inside the bubbles towards the oil surface. Big particles are retained by the impactor plate; small particles escape through the gap and reach the aerosol outlet. The number of particles can be controlled by the four valves at the nozzle inlets. The particle concentration can be decreased by an additional air supply via the second air inlet. The mean size of the particles generally depends on the type of liquids being atomized, but is only slightly dependent on the operating pressure of the nozzles. Vegetable oil is the most commonly used liquid since oil droplets are believed to be less unhealthy than many other particles. However, any kind of seeding particles which cannot be dissolved in water should not be inhaled. Most vegetable oils (except cholesterol-free oils) lead to polydisperse distributions with mean diameters of approximately 1 μm [91].

### 2.2 Light sources

#### 2.2.1 Lasers

Lasers are widely used in PIV, because of their ability to emit monochromatic light with high energy density, which can easily be bundled into thin light sheets for illuminating and recording the tracer particles without chromatic aberrations. In figure 2.11 a typical configuration of a laser is shown. Generally speaking, as shown in figure 2.11, every laser consists of three main components.

![Schematic diagram of a laser](image)

**Fig. 2.11.** Schematic diagram of a laser

The laser material consist of an atomic or molecular gas, semiconductor or solid material.

The pump source excites the laser material by the introduction of electromagnetical or chemical energy.

The mirror arrangement allows an oscillation within the laser material.

In the following we will describe the principle of gas lasers and give an overview of the lasers used in PIV.

It is well known from quantum mechanics that each atom can be brought into various energy states by three elementary kinds of interaction with electromagnetic radiation. This can be illustrated in an energy level diagram, as shown in figure 2.12 for a hypothetical atom with only two possible energy states. An excited atom at level \(E_2\) usually drops back to the state \(E_1\) after a very short, but not exactly defined period of time and emits the energy \(E_2 - E_1 = hν\) in the form of a randomly directed photon. This process is called spontaneous emission.

However, if, on the other hand, a photon with "appropriate" frequency \(ν\) impinges on an atom, then two effects are possible: either in the case of absorption – an atom in the state \(E_1\) can receive the energy \(hν\), i.e., it becomes raised to \(E_2\) and the photon is absorbed; or the incident photon can stimulate an atom in the excited \(E_2\) state into a specific, nonspontaneous, transition to \(E_1\). Then, in addition to the incident photon, a second photon in phase with the first occurs, i.e., the impinging wave was coherently amplified (stimulated emission).

![Elementary kinds of interactions between atoms and electromagnetic radiation](image)

**Fig. 2.12.** Elementary kinds of interactions between atoms and electromagnetic radiation

When there are large numbers of atoms, one of the two processes – absorption or stimulated emission – predominates: if there are more atoms in the \(E_2\) state than in the \(E_1\) state (i.e. the population density \(N_2 > N_1\) [atoms/m\(^3\)]), then stimulated emission predominates, and, in the case of \(N_1 > N_2\), absorption.

Since the laser can only operate if a population inversion is forced to take place \((N_2 > N_1)\), external energy has transferred to the laser material because atoms usually exist in their ground state. This is achieved by a different pump mechanism depending on the kind of laser material. Solid laser materials are generally pumped by electromagnetic radiation, semiconductor lasers by electronic current, and gas lasers by collision of the atoms or molecules with electrons and ions.

It should be noted that in a system which consists of only two energy states, as described so far, no population inversion can be achieved, because
that extent required by the objectives of the investigations. Small particles will follow the flow better.

Illumination. For applications in gas flows a high power light source for illumination is required in order that the light scattered by the tiny tracer particles will expose the photographic film or the video sensor. However, the need to utilize larger particles because of their better light scattering efficiency is in contradiction to the demand to have as small particles as possible in order that they follow the flow faithfully. In most applications a compromise has to be found. In liquid flows larger particles can usually be accepted which scatter much more light. Thus, light sources of considerably lower peak power can be used here.

Duration of illumination pulse. The duration of the illumination light pulse must be short enough that the motion of the particles is "frozen" during the pulse exposure in order to avoid blurring of the image ("no streaks").

Time delay between illumination pulses. The time delay between the illumination pulses must be long enough to be able to determine the displacement between the images of the tracer particles with sufficient resolution and short enough to avoid particles with an out-of-plane velocity component leaving the light sheet between subsequent illuminations.

Distribution of tracer particles in the flow. At qualitative flow visualization certain areas of the flow are made visible by marking a stream tube in the flow with tracer particles (smoke, dye). According to the location of the seeding device the tracers will be entrained in specific areas of the flow (boundary layers, wakes behind models, etc.). The structure and the temporal evolution of these structures can be studied by means of qualitative flow visualization. For PIV the situation is different: a homogeneous distribution of medium density is desired for high quality PIV recordings in order to obtain optimal evaluation. No structures of the flow field can be detected on a PIV recording of high quality.

Density of images of tracer particles on the PIV recording. Qualitatively three different types of image density can be distinguished [28], which is illustrated in figure 1.5. In the case of low image density (figure 1.5 a), the images of individual particles can be detected and images corresponding to the same particle originating from different illuminations can be identified. Low image density requires tracking methods for evaluation. Therefore, this situation is referred to as "particle tracking velocimetry", abbreviated "PTV". In the case of medium image density (figure 1.5 b) the images of individual particles can be detected as well. However, it is no longer possible to identify image pairs by visual inspection of the recording. Medium image density is required to apply the standard statistical PIV evaluation techniques. In the case of high image density (figure 1.5 c) it is not even possible to detect individual images as they overlap in most cases and form speckles. This situation is called "laser speckle velocimetry" (LSV), a term which has been used at the beginning of the eighties for the medium image density case as well, as the (optical) evaluation techniques were quite similar for both situations.

Fig. 1.5. The three modes of particle image density: (a) low (PTV), (b) medium (PIV), and (c) high image density (LSV).

Number of illuminations per recording. For both photographic and video techniques, we have to distinguish whether it is possible to store images of the tracer particles on different frames for each illumination or whether all particle images due to the different illuminations are stored on a single frame.

Number of components of the velocity vector. Due to the planar illumination of the flow field only two (in plane) components of the velocity vector can be determined in standard PIV (2D-PIV). Methods are already available to extract the third component of the velocity vector as well (stereo techniques, dual-plane PIV, holographic recording [31]). This would be labeled 3C-PIV. Both methods work in planar domains of the flow field (2D-PIV).

Extension of observation volume. In the most general way an extension of the observation volume is possible by means of holographic techniques (3D-PIV) [115]. Other methods such as establishing several parallel light sheets in a volume [31] or scanning a volume in a temporal sequence [102, 103] would be referred to as 2+1D-PIV.

Extension in time. By means of repetitively working cameras it is already possible to record temporal sequences of PIV recordings. However, as the repetition rate of pulse lasers and cameras is limited, it is not possible to record fast enough as would be required due to the temporal scales of most flows.

Size of interrogation area. The size of the interrogation area at evaluation must be small enough that velocity gradients have no significant influence on the results. Furthermore, it determines the number of independent velocity vectors and therefore the maximum spatial resolution of the velocity map which can be obtained at a given spatial resolution of the sensor employed for recording.
1.3 Development of PIV during the last two decades

The development of particle image velocimetry during the past 15 years is characterized by the fact that analog recording and evaluation techniques have been replaced to a large extent by digital techniques. Though these analog methods have widely contributed to the fast initial success of the PIV technique, the discussion of these techniques will not be one of the main objectives of this handbook on PIV. We will rather concentrate on the description of the present state of the art of PIV.

A number of sources describing the basic principles of PIV in the context of its historical development are readily available. Thus, for further information the reader is referred to the SPIE Milestone Series 99, edited by I. Grant in 1994 [4]. This volume comprises more than 70 original papers, first published between 1932 and 1993. The majority of them originates from the eighties, including contributions about the roots of modern PIV (i.e. speckle interferometry), the early work of R. Meynart [76], the development of low and high image density PIV, optical correlation techniques, etc. Review articles by W. Lauterborn and A. Vogel (1984) [34] and by R. Adrian (1991) [28], which are also reprinted in the MS 99 on PIV, demonstrate the vast development and compilation of know-how about PIV within a decade.

The state of the art of PIV seen from the side of optics is described in the chapter “Particle Image Velocimetry” written by K. Hinsch in 1993 [30], included in a book on “Speckle metrology”. This contribution is especially useful for the understanding of the optical aspects of PIV. It includes 104 references to other literature on PIV.

At that time strong competition with respect to the better performance of optical and digital methods in the evaluation of PIV recordings took place. Details of the theoretical fundamentals of digital particle velocimetry can be found in the book Digital particle image velocimetry – Theory and practice published also in 1993 by J. Westerweel [7]. This book includes more than 100 references.

A review paper “Particle image velocimetry: a review” by I. Grant appeared in 1997 [29]. It gives a summary of different modifications of PIV illumination, recording and evaluation techniques, many of them not covered in this book. The paper includes 188 references.

As indicated all four publications mentioned above include a detailed bibliography of the literature on PIV, which the reader should use if more details are required on special aspects of PIV than it was possible to describe in this book. A further bibliography on PIV with nearly 1200 references was compiled by R. Adrian [1] and is available commercially.

The large number of references listed in the review articles demonstrate that particle image velocimetry is nowadays a well accepted tool for the investigation of velocity fields in many different areas. This also means that a number of special implementations of the PIV technique had to be developed for such different applications as, for example, in biology or in turbomachinery.

At present the widest use of PIV is made in fluid mechanics in the investigation of air and water flows. The progress made in the last two years has brought PIV to such a state that it is close to being routinely applied in aerodynamic research. This is not yet the case for more complex flows (turbomachinery, two-phase flows, flames, etc.). This means that today a nearly complete and nearly stable picture of the technical aspects of PIV can best be given if looking at the demands of applications in aerodynamics or in water flows, where the technical problems are similar but usually much less severe than in air flows. Most of the technical problems in the application of PIV encountered in this special field appear in other PIV applications as well. Many of the basic considerations can easily be transferred to other applications.

1.3.1 PIV in aerodynamics

The use of the PIV technique is very attractive in modern aerodynamics, because it helps to understand unsteady flow phenomena as, for example, in separated flows above models at high angle of attack. PIV enables spatially resolved measurements of the instantaneous flow velocity field within a very short time and allows the detection of large and small scale spatial structures in the flow velocity field. Another need of modern aerodynamics is that the increasing number and increasing quality of numerical calculations of flow fields require adequate experimental data for validation of the numer-
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In order to decide whether the physics of the problem has been modeled correctly. For this purpose carefully designed experiments have to be performed in close cooperation with those scientists doing the numerical calculations. The experimental data of the flow field must possess high resolution in time and space in order to be able to compare them with high density numerical data fields. The PIV technique is an appropriate experimental tool for this task, especially if information about the instantaneous velocity field is required.

A PIV system for the investigation of air flows in wind tunnels must be operated as well in low speed flows (e.g. flow velocities of less than 1 m/s in boundary layers) as in high speed flows (flow velocities up to 600 m/s in supersonic flows with shocks). Flow fields above solid, moving, or deforming models have to be investigated. The application of the PIV technique in large, industrial wind tunnels poses a number of special problems: large observation area, long distances between the observation area and the light source and the recording camera, restricted time for the measurement, and high operational costs of the wind tunnel.

The description of the problems as given above leads to the definition of requirements which should be fulfilled when PIV is applied in aerodynamics. First of all, a high spatial resolution of the data field is necessary in order to resolve large scale as well as small scale structures in the flow. This condition directly influences the choice of the recording medium (video or photographic recording). A second important condition is that a high density of experimental data is required for a meaningful comparison with the results of numerical calculations. Thus, the image density (i.e. number of particle images per interrogation area) must be high. A powerful seeding generator (high concentration of tracer particles in the measuring volume in the flow even at high flow velocities) is needed for this purpose. As the flow velocity is measured indirectly by means of the measurement of the velocity of tracer particles added to the flow, the tracer particles must follow the flow faithfully. This requires the use of very small tracer particles. However, small particles scatter little light. This fact results in a third important condition for the application of PTV in aerodynamics: a powerful pulse laser is required for the illumination of the flow field.

1.3 Development of PIV during the last two decades

1.3.2 Major technical milestones of PIV

Earlier in this section some references to papers describing the general historical development of PIV have been given. In a handbook more devoted to the technical aspects of PIV it might be of even greater interest to outline the development of PIV towards its applicability in complex flows in terms of the achievement of major technical milestones.

The understanding of some of the technical restrictions in the application of PIV in the past and their conquest may be useful for new users of the PIV technique, in order to assess the discussion in some older publications some-times dealing with – nowadays – “strange” looking efforts to solve technical problems which no longer exist today.

The selection of these milestones was done according to the technical progress in the past as experienced by the authors in their own work. Thus, the choice is a subjective one.

Feasibility of modern PIV. The feasibility of employing the particle image velocimetry technique for the measurement of flow velocity fields in water and even in air was demonstrated in the early eighties at the VON KARMAN institute in Brussels, mainly by R. MEYNART [76]. At that time the evaluation methods were based on the work done in the field of speckle interferometry (see references in [4]).

Reliable high power light sources for application in air. The use of double oscillator Nd:YAG lasers (two resonators; frequency doubled, to achieve a wavelength of $\lambda = 532\,\text{nm}$ in visible light) allowed for the first time the illumination of a plane in the flow with laser pulses of the same, constant energy at any time delay between the two pulses as required by the experiment at frame rates of the order of $10\,\text{Hz}$ [66]. Alignment of the light sheet optics and image acquisition was thus facilitated considerably.

Ambiguity removal. Especially with photographic recordings it was not possible in most cases to store the images of the tracer particles due to first and second illumination on two different recordings. Thus, the temporal sequence of the images of the tracer particles could not be distinguished. Methods to remove the ambiguity of the sign of the velocity vector had to be developed (see references in [4]). The most widely used technique was image shifting, which could be successfully applied later on even in high speed flows. By enabling the investigation of complex, unsteady 3D flow fields, this development contributed considerably to the increasing interest in PIV from the side of wind tunnel users and industry.

Generation and distribution of tracer particles in the flow. The development of powerful aerosol generators and the know how to distribute the tracer particles within the flow homogeneously improved the image density and thus the quality of the PIV recordings considerably.

Computer hardware. The improvement of computer hardware with respect to processor speed and larger memory still continues. Memory size of 16 MB and 32 bit processors, which is today’s standard, allow the handling of complete digital PIV recordings (even of temporal sequences of recordings) by a personal computer, which was not possible in the eighties with 8 bit processors and the 640 KB restrictions on memory.

Improved peak finders. The spread of digital particle image velocimetry was affected by the limited size and resolution of the video sensors and hence of digital PIV recordings as compared to that of photographic recordings. The development of Gaussian peak finders allowed the determination of the location of the displacement peak with further improved accuracy. Thus,
smaller interrogation windows could be utilized, leading to an increase of spatial resolution (number of vectors) in digital particle image velocimetry.

Cross correlation video camera. Today progressive scan video cameras allow users to store the images of the tracer particles on separate frames for each illumination [149]. This feature immediately solves the problem of ambiguity removal. A sensor size of 1000 × 1000 pixels together with the application of cross correlation methods with superior signal-to-noise ratio at evaluation yield velocity vector fields of nearly the same quality as was possible only with 35 mm photographic film in the past.

Theoretical understanding of PIV. At the beginning of the development of particle image velocimetry the understanding of the technique was a more intuitive one. Progress was often made just by trial and error. In the past few years the theoretical understanding of the basic principles of the PIV technique has been improved considerably. Such theoretical considerations as well as simulations of the recording and evaluation process give useful information on many parameters important for the planning of an experiment utilizing PIV.

In this chapter a brief introduction to the basic principles of PIV and to some of its problems and technical constraints to be kept in mind has been given.

Next, the different topics will be described in more detail. We will start with providing the background of the most important physical principles. In the following the mathematical background of PIV evaluation will be discussed. With this knowledge the path has been prepared for the understanding of the recording, evaluation, and post processing methods applied in PIV. Furthermore the present state of the technical development of stereo and dual-plane PIV will be described – methods allowing access to the third component of the velocity vector in planar domains. In the final chapter examples of the application of PIV will be presented, thereby explaining the specific problems experienced at each measurement due to the different properties of the flow under investigation.

2. Physical and technical background

2.1 Tracer particles

It is clear from the principle of PIV as described that PIV – in contrast to hotwire or pressure probe techniques – is based on the direct determination of the two fundamental dimensions of the velocity: length and time. On the other hand, the technique measures indirectly, because it is the particle velocity which is determined instead of fluid velocity. Therefore, fluid mechanical properties of the particles have to be checked in order to avoid significant discrepancies between fluid and particle motion.

2.1.1 Fluid mechanical properties

A primary source of error is the influence of gravitational forces if the densities of the fluid $\rho$ and the tracer particles $\rho_p$ do not match. Even if it can be neglected in many practical situations, we will derive the gravitationally induced velocity $U_g$ from Stokes drag law in order to introduce the particle’s behavior under acceleration. Therefore, we assume spherical particles in a viscous fluid at a very low Reynolds number. This yields:

$$ U_g = d_p^2 \frac{(\rho_p - \rho)}{18\mu} g \quad (2.1) $$

where $g$ is the acceleration due to gravity, $\mu$ the dynamic viscosity of the fluid, and $d_p$ is the diameter of the particle.

In analogy to equation (2.1), we can derive an estimate for the velocity lag of a particle in a continuously accelerating fluid:

$$ U_s = U_p - U = d_p^2 \frac{(\rho_p - \rho)}{18\mu} a \quad (2.2) $$

where $U_p$ is the particle velocity. The step response of $U_p$ typically follows an exponential law if the density of the particle is much greater than the fluid density:

$$ U_p(t) = U \left[ 1 - \exp \left( \frac{-t}{\tau} \right) \right] \quad (2.3) $$
when the number of atoms $N_2$ in level $E_2$ equals the number $N_1$ in level $E_1$, absorption and stimulated emission are equally likely and the material will become transparent at the frequency $\nu = (E_2 - E_1)/h$. In other words: the number of transitions from the upper level $E_2$ to the lower level $E_1$ and vice versa are on average the same. Hence, at least three energy levels of the laser medium are essential to achieve population inversion. But a three level system is not very efficient because a fraction of more than 50% of the atoms of the system have to be excited in order to amplify an impinging photon. This means that the energy needed for the excitation of this fraction is lost for the amplification. In the case of a four level laser the lower laser level $E_2$ does not coincide with the basic level $E_1$ and therefore remains unoccupied at room temperature. In this way it is easier to achieve the population inversion and a four level laser requires substantially less pumping power. This is illustrated in figure 2.13. If for instance state $E_4$ is achieved by optical pumping at frequency $\nu$ according to $\hbar \nu = E_4 - E_1$ then a rapid nonradiative transition to the upper laser level $E_2$ occurs. The atoms remain in this so-called metastable state $E_2$ for a relatively long interim period before it drops down to the unoccupied lower laser level $E_1$.

![Level diagrams of three (left) and four (right) level lasers](image)

**Fig. 2.13.** Level diagrams of three (left) and four (right) level lasers

As a consequence of population inversion through energy transfer by the pump mechanism spontaneous emission occurs in all directions which causes excitation of further neighboring atoms. This initiates a rapid increase of stimulated emission and therefore of radiation in a chain reaction.

In the case of a cylindrical shape of laser material the rapid increase of radiation occurs in a defined direction, because the amplification increases with increasing length of the laser medium. With an optical resonator (mirror arrangement) the laser material can extended to form an oscillator. The simplest way to achieve this is to place the material between to exactly aligned mirrors. In this case, a photon which impinges randomly on one of the mirror surfaces is reflected and amplified in the laser material again. This process will be repeated and generates an avalanche of light which increases exponentially with the number of reflections, finally resulting in a stationary process. In other words, standing waves are produced on the resonator length with the condition

$$L = \frac{m\lambda}{2n}$$

(2.4)

where $n$ is the refractive index, $m$ an integer number, and $L$ the resonator length. Since the frequency $\nu$ according to the transition $\nu h = E_2 - E_1$ does not correspond to exactly one wavelength, but rather to a spectrum of a certain bandwidth $\Delta \nu$ depending on the transition time $\tau$ of the process these conditions can be fulfilled by different wavelengths $\lambda$ or frequencies $\nu$ and the resonator can oscillate in many axial modes with distinct frequencies.

Consecutive modes are separated by a constant difference $\Delta \nu = c/(2L n)$, wherein $c$ is the speed of light. Moreover, the cross-section of the laser beam can be divided into several ranges oscillating in antiphase with intermediate node lines, i.e. different transverse modes can be sustained as well (see figure 2.14). Their occurrence depends on resonator design and alignment. The lowest order transverse mode TEM00 (TEM = transverse electric mode; index = node in X- and Y-direction) is most commonly used, because it produces a beam with uniform phase and a Gaussian intensity distribution versus the beam cross-section.

![Examples of different transverse modes](image)

**Fig. 2.14.** Examples of different transverse modes

There are various types of resonators with different mirror curvatures. The confocal resonator, shown in figure 2.11, is particularly stable and easy to adjust. Hemispherical resonators use one planar and one concave mirror, and critical resonators use two planar mirrors. Critical resonators offer the advantage of having no beam waist inside the laser rod and therefore using its whole volume. However, they are sensitive for thermal lens effects and misalignment.
2. Physical and technical background

In gas lasers, which are usually used for continuous operations (CW = continuous wave), free electrons are accelerated by an electrical field resulting in an excitation of the gaseous medium. The plasma tube in which the excitation takes place is closed by Brewster windows (plates tilted at the polarization angle). Therefore, these lasers emit linearly polarized light. In the case of optically pumped solid-state lasers, the light of the rod-shaped flash lamp is concentrated on the laser rod by a cylindrical mirror with an elliptical cross-section. In lasers, which basically consist of luminescence diodes, two polished, parallel surfaces work as resonator mirrors. The laser light perpendicular to the p-n junction is more divergent due to the lower aperture width.

Helium-neon lasers (He-Ne lasers $\lambda = 633$ nm) are the most common as well as the most efficient lasers in the visible range. He-Ne lasers are rather used for the optical evaluation of photographic images in PIV than for illuminating the flow. The power of commercial models ranges from less than 1 mW to more than 10 mW. The laser transitions take place in the neon atom. During the electrical discharge within a gaseous mixture of helium and neon, the helium atoms are excited by collisions with electrons. In the second step the upper laser levels of the Ne atoms are populated through collisions with metastable helium atoms. This is the main pump mechanism. The major properties of the He-Ne laser that are important for the evaluation of PIV images are the coherence of the laser beam and the Gaussian distribution of the laser beam intensity ($TE_{00}$). If necessary, beam quality can be further improved by means of spatial filters (see section 5.3.2).

Copper-vapor lasers (Cu lasers $\lambda = 510$ nm, 578 nm) are the most important neutral metal vapor lasers for PIV. The wavelengths of Cu lasers are within the yellow and green spectrum (table 2.3).

<table>
<thead>
<tr>
<th>Wavelength:</th>
<th>510.6 nm and 578.2 nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average power:</td>
<td>50 W</td>
</tr>
<tr>
<td>Pulse energy:</td>
<td>10 mJ</td>
</tr>
<tr>
<td>Pulse duration:</td>
<td>15 ns - 60 ns</td>
</tr>
<tr>
<td>Peak power:</td>
<td>&lt; 300 kW</td>
</tr>
<tr>
<td>Pulse frequency:</td>
<td>5 kHz - 15 kHz</td>
</tr>
<tr>
<td>Beam diameter:</td>
<td>40 mm</td>
</tr>
<tr>
<td>Beam divergence:</td>
<td>0.6 $\cdot$ 10$^{-3}$ rad</td>
</tr>
</tbody>
</table>

Table 2.3. Properties of a commercial Cu laser

These lasers are characterized by their high average power (typically 1–30 W) and an efficiency of up to one percent. Continuous wave operations are not possible due to the long life-span of the lower laser level. During pulse operations, repetition rates within the kHz range can be achieved.

This type of laser has intensively been developed during the past decade because the copper laser is an important pump source for dye lasers. Whereas most lasers are cooled, metal vapor lasers need thermal insulation so that the operating temperature for vaporizing the metal can typically reach 1500°C. Two electrodes are located at the ends of a thermally insulated ceramic tube with a pulsed charge burning in between them. For improving discharge quality, neon is added as the buffer gas at a pressure of around 3000 Pa. Table 2.3 lists some of the key properties of a typical Cu laser as an example.

Argon-ion lasers ($Ar^+$ lasers $\lambda = 514$ nm, 488 nm) are gas lasers, similar to the He-Ne lasers described above. In argon lasers, very high currents have to be achieved for ionization and excitation. This is technologically much more complicated compared to He-Ne lasers. Typically the efficiency of these lasers is on the order of a tenth of a percent. These lasers can supply over 100 W in the blue-green range and 60 W in the near ultraviolet range. Emission is produced at several wavelengths through the use of broadband laser mirrors. Individual wavelengths can be selected by means of Brewster prisms in the laser resonator. The individual wavelengths can be adjusted by turning the prism. The most important wavelengths are 514.5 and 488.0 nm. Nearly all conventional inert gas ion lasers supply TEM$_{00}$. Despite the extreme load on the tubes resulting from the high currents, product lives of several thousand operating hours can be achieved. Since argon lasers are frequently used for LDV measurements, they are often found in fluid mechanics laboratories. In PIV they can easily be used for low speed water investigations.

Semiconductor lasers offer the advantage to be very compact. The laser material is typically 1 cm long and has a diameter of 0.5 mm. The total efficiency of a commercial diode laser pumped Nd:YAG system is around 7%. Since heating is considerably reduced, these types of pumped lasers supply a very good beam quality of over 100 mW in the TEM$_{00}$ mode during continuous operations. The diode laser is interesting for PIV because it can be used as a seed laser to improve the coherence length of flash lamp pumped Nd:YAG lasers for use in holographic PTV. A particularly interesting variant is the combination of a diode-pumped laser oscillator and a flash lamp pumped amplifier. Together with other optical components, like vacuum-pinholes and phase-conjugated mirrors, this concept offers very good beam properties, but at the same time, its initial purchase costs are also high.

Ruby lasers ($Cr^{2+}$ lasers $\lambda = 694$ nm), historically the very first lasers, use ruby crystal rods containing $Cr^{2+}$ ions as the active medium. The are pumped optically by means of flash lamps. As already mentioned above, the ruby laser is a three level system which has the disadvantage that approximately 50% of the atoms must be excited before population inversion takes place. The high pumping energy needed can usually only be achieved during pulse mode operations. The wavelength of the ruby laser is 694.3 nm. Like other solid-state lasers, the ruby laser can also be operated normally or in Q-switched mode. (For details about Q-switched mode see the next paragraph.)
The ruby laser is particularly interesting for PIV because it delivers very high pulse energies and its beam is well suited for holographic imaging because of its good coherence. Its disadvantage is that the low repetitive rates hamper the optical alignment and its light is emitted at the edge of the visible spectrum. Photographic films are usually not sensitive for red light and also modern CCD cameras are usually optimized for smaller wavelengths.

**Neodym-YAG laser** (Nd:YAG lasers $\lambda = 532$ nm) are the most important solid-state laser for PIV in which the beam is generated by Nd$^{3+}$ ions. The Nd$^{3+}$ ion can be incorporated into various host materials. For laser applications, YAG crystals (yttrium-aluminum-garnet) are commonly used. Nd:YAG lasers have a high amplification and good mechanical and thermal properties. Excitation is achieved by optical pumping in broad energy bands and nonradiative transitions into the upper laser level.

The fact that solid-state lasers can be pumped with white light results from the arrangement of the atoms which form a lattice. The periodic arrangement leads to energy bands formed by the upper energy levels of the single atoms. Therefore, the upper energy levels of the system are not discrete as in the case of single atoms, but are continuous.

As already mentioned the Nd:YAG laser is a four-level system which has the advantage of a comparatively low laser threshold. At conventional operating temperatures, the Nd:YAG laser only emits the strongest wavelengths, 1064 nm. In the relaxation mode the population inversion takes place as soon as the threshold is reached, with this threshold value depending on the design of the laser cavity. In this way, many successive laser pulses can be obtained during the pump pulse of the flash lamp. By including a quality switch (Q-switch) inside the cavity the laser can be operated in a triggered mode. The Q-switch has the effect of altering the resonance characteristics of the optical cavity. If the Q-switch is operated, allowing the cavity to resonate at the most energetic point during the flashlamp cycle, a very powerful laser pulse, the so-called giant pulse, can be achieved. Q-switches normally consist of a polarizer and a Pockels cell, which change the quality of the optical resonator depending on the Pockels cell voltage. The Q-switched mode is in general more interesting and is usually used in PIV. Even if the Q-switches can be used to generate more than one giant pulse out of one resonator, PIV lasers are more often designed as double-oscillator systems. This enables the user to adjust the separation time between the two illuminations of the tracer particles independently of the pulse strength. The beam of Q-switch lasers is linearly polarised. For PIV, and many other applications, the fundamental wavelength of 1064 nm is frequency-doubled using special crystals.

(For details about these so called KDP crystals see the next section.) After separation of the frequency-doubled portion, approximately one-third of the original light energy is available at 532 nm. Nd:YAG lasers are usually driven in a repetitive mode. Since the optical properties of the laser cavity change with changing temperature, good and constant beam properties will only be obtained at a nominal repetition rate and flashlamp voltage. Due to thermal lensing the beam quality, which is very often poor compared to other laser types, decreases significantly when, for example, single pulses are used. This is not that critical for telescopic resonator arrangements but very important for modern critical resonator systems. The coherence length of pulsed Nd:YAG lasers is normally on the order of a few centimeters. For holographic recording, lasers with a narrow spectral bandwidth have to be used. This is usually done by injection from a smaller semiconductor laser into the cavity by a partially reflecting mirror. Then, the laser pulse builds up from this small seeding pulse of narrow bandwidth, resulting in coherence lengths of 1 or 2 meters. However, very precise laser timing and temperature control for the primary cooling circuit are required for this purpose.

### 2.2.2 Features and components of Nd:YAG lasers for PIV

Commercially available Nd:YAG laser rods are up to 150 mm long and have diameters of up to 10 mm. Typically pulse energies of 400 mJ or more can be achieved out of one oscillator. In this case, more than one flashlamp and critical resonators with plane mirror surfaces have to be used. The price paid for the high output power that can be achieved with these resonators is that the beam profile tends to be very poor: hot spots and different ring modes can often be found. In order to improve the beam profile, output mirrors with a reflectivity that varies with the radius are frequently used. However, even with these mirrors the beam profile is sometimes very poor, even if it is specified to be 80% Gaussian in the near and 98% in the far field. Two laser systems of the same manufacturer often have different beam properties depending on the laser rod properties, and the alignment of the laser. Since a good beam profile is absolutely essential for PIV (see chapter 3) it must be specified not only in the near and in the far field – as most manufacturers do – but also in the mid-field in a distance of 2–10 m from the laser. The description of the beam intensity distribution should not only be based on a good fit to a Gaussian distribution, but also on the minimum and maximum intensity in order to ensure a hole-free intensity distribution without hot spots.

In figure 2.16 the intensity profiles versus the sheet thickness measured at different distances from the laser are shown. The light sheet optics used for this experiment are shown in figure 2.20. The peak value of the distribution has been adjusted close to full scale for each position (1.8 m, 3.3 m, 4.3 m, 5.8 m). It can be seen that the thickness of the light sheet increases slowly with distance from the laser. A small side peak is visible at every position but seems to vanish at 5.8 meters. The fluctuations in the distribution are minimized at position three (4.3 m). When assessing these light sheet profiles it has to be taken into account that the loss of correlation during the evaluation of PIV recordings is mainly influenced by the light sheet intensity distribution at recording (see chapter 3). The light energy contained in the side peak will be lost in most situations, because a very small
light of identical polarization one Type II crystal is generally used. Therefore, the infrared laser light must have two polarization components. The second harmonic will then have one polarization direction parallel to one of both original components depending on the orientation of the crystal. In order to provide two components of the incident laser light it's linear polarization is turned by an angle of 45° using a polarization rotator.

A polarization rotator is a crystal which continuously rotates the polarization angle of linearly polarized light when it propagates through it. The rate of rotation is dependent on the material, its thickness, and the wavelength. A 45° rotator will be used when a Type II doubling crystal is used. A 90° rotator might be used in front of the beam combination optics, if two oscillators of identical orientation are used (see figure 2.17).

A prism harmonic separator can be used to separate the second harmonic wave by deflecting it into an energy dump. Two energy dumps are provided, one for the fundamental and one for the third harmonic wave. These separators are most efficient when used with only one polarization direction, as the reflection losses at the prism surfaces are lower for one polarization (see figure 2.18).

A dichroic mirror has maximum reflectivity for one given wavelength. The fundamental and any unwanted harmonic waves pass through such a mirror and can therefore be steered into an external energy dump.

### 2.2.3 White light sources

Even if most PIV investigations are performed using laser light sheets, white light sources might also be used. Due to the finite extension of these sources and since white light cannot be collimated as well as monochromatic light, they clearly have some disadvantages. On the other hand, the spectral output of sources, like Xeon lamps is well suited for use with CCD cameras because of their spectral sensitivity. Systems are commercially available which can easily be triggered and offer a repetition rate that matches the video rate. Two flashlamps can be linked by optical fiber bundles in order to achieve short pulse separation times. If the outputs of the fibers are arranged in line, the generation of a light sheet is considerably simplified. The main advantage of these white light sources is - besides costs - that applications are not hampered by laser safety rules.

### 2.3 Light sheet optics

This section treats the optics for the illumination of the particles by a thin light sheet. Therefore we describe three different lens configurations, which have been used during various experiments. Rules for the calculation of the light sheet intensity distribution are not given herein. The reason for that is that geometric optic rules are already sufficient for a general layout of the chosen lens configuration. They do not require a special description and can easily be found in every book on optics [13]. On the other hand, more sophisticated calculations based on Gaussian optics usually require some assumptions, which are valid only for exceptional cases. Computer programs can be used in order to predict further parameters such as the light sheet thickness at the beam waist where the theoretical (geometrical) thickness is zero, but their description is beyond the scope of this book.

Optical fibers for beam delivery can be used to improve the handling of the system or for experimental situations where mirror systems would not be feasible. For CW lasers a variety of systems are available and for their use we refer to the manufacturers. Since pulsed lasers and white light sources have only limited repetition rates fiber bundles can be used for the combination of two sources for shorter pulse separation times. New developments have already deliver more than 10 mJ per pulse and further improvements can be expected [39]. However, the use of optical fibers will always be associated with a certain loss in intensity.

The essential element for the generation of a light sheet is a cylindrical lens. When using lasers with a sufficiently small beam diameter and divergence, like e.g. Argon-ion lasers, one cylindrical lens can be sufficient to generate a light sheet of appropriate shape. For other light sources, like e.g. Nd:YAG lasers, a combination of different lenses is usually required in order to generate thin light sheets of high intensity. At least one additional lens has then to be used for focusing the light to an appropriate thickness. Such a configuration is shown in figure 2.18, where also a third cylindrical lens has been added in order to generate a light sheet of constant height.

![Fig. 2.18. Light sheet optics using three cylindrical lenses (one of them with negative focal length)](image)

The reason why a diverging lens has been used first is that focal lines should be avoided. In high power pulse lasers focal points have to be avoided, as otherwise the air close to the focal point will be ionized. Focal lines usually do
not ionize the air but dust particles might be burned if the area in the vicinity of the line is not covered or evacuated. In both cases acoustic radiation will occur and the beam properties will change significantly. For the light sheet shown in figure 2.18, the position of minimum thickness is given by the beam divergence of the light source and the focal length of the cylindrical lens on the right hand side, e.g. at a distance of 500 mm from the last lens for the conditions illustrated in figure 2.18.

The combination of a cylindrical lens together with two telescope lenses makes the system more versatile. This is shown in figure 2.19 where spherical lenses have been used, because they are in general easier to manufacture especially if short focal length lenses are required. The height of the light sheet shown in figure 2.19 is mainly given by the focal length of the cylindrical lens in the middle. A diverging lens – negative focal length – could also be used, however, since the focal line has a relatively large extension this configuration can be used also for pulsed lasers. The adaptation of the light sheet height has to be done by changing the cylindrical lens. The adjustment of the thickness can be easily done by shifting the spherical lenses with respect to each other.

![Fig. 2.19. Light sheet optics using two spherical lenses (one of them with negative focal length) and one cylindrical lens](image)

The use of spherical lenses in general does not allow light sheet height and thickness to be changed independently. This can be done by the configuration shown in figure 2.20. Additionally this set-up allows the generation of light sheets which are thinner than the beam diameter at every location. It therefore enables the generation of light sheets which are already thin shortly after the last lens. With this arrangement the thickness can be held constantly small. However, the energy per unit area of these configurations is high. When using pulse lasers the critical region close to the focal line has to be covered in order to avoid dust or seeding particles disturbing the generation of a defined light sheet. Using a diverging cylindrical lens first would solve those problems, but the combination shown in figure 2.20 has the advantage of imaging of the beam profile from a certain position in front of the lens to the observation area while keeping its properties constant.

Simple geometric considerations can be used for these lens combinations to determine from which position the laser beam has been imaged and, if the development of the beam profile of the laser is known, this information can be used to optimize the light sheet intensity distribution. For lasers with a critical beam profile this can improve the valid data yield, because the light sheet intensity distribution especially in the out-of-plane direction is essential for the quality of the measurement (see section 3). In figure 2.15 on page 30 the evolution of a light sheet profile generated by a lens configuration similar to that shown in figure 2.20 have been shown as a function of the distance from the laser.

![Fig. 2.20. Light sheet optics using three cylindrical lenses](image)

A few general rules should be given here also. Uncoated lens surfaces in air exhibit a slight reflectivity of \(\left(\frac{n-1}{n+1}\right)^2\). Since this value is on the order of 4% for common lenses the losses due to the reflection can usually be accepted. However, these reflections can cause damage if they are focused close to other optical components. In most cases this can easily be avoided by the right orientation of the lenses as demonstrated in figure 2.21.

![Fig. 2.21. General considerations on the orientation of lenses inside the light sheet optic](image)
Furthermore case c and d in figure 2.21 should be used in order to minimize aberrations. For other situations it might be possible to tilt the lens slightly in order to avoid reflections on to other lenses or towards the laser or even into the resonator.

2.4 Imaging of small particles

2.4.1 Diffraction limited imaging

This section provides a description of diffraction limited imaging, which is an effect of practical significance in optical instrumentation, and of particular interest for PIV recording. In the following we will restrict our description of imaging by considering only one-dimensional functions.

If plane light waves impinge on an opaque screen containing a circular aperture they generate a far-field diffraction pattern on a distant observing screen. By using a lens – e.g. an objective in a camera – the far field pattern can be imaged on an image sensor close to the aperture without changes. However, the image of a distant point source (e.g. a small scattering particle inside the light sheet), does not appear as a point in the image plane but forms a Fraunhofer diffraction pattern even if it is imaged by a perfectly aberration-free lens [13]. A circular pattern, which is known as the Airy disk, will be obtained for a low exposure. Surrounding Airy rings can be observed for a very high exposure.

Using an approximation (the so-called Fraunhofer approximation) for the far field it can be shown that the intensity of the Airy pattern represents the Fourier transform of the aperture’s transmissivity distribution [12, 18]. Taking the scaling theorem of the Fourier transform into account, it becomes clear that large aperture diameters correspond to small Airy disks and small apertures to large disks as can be seen in figure 2.22.

The Airy function can mathematically be represented by the square of the first order Bessel function. Therefore, the first dark ring, which defines the extension of the Airy disk, corresponds to the first zero of the first order Bessel function shown in figure 2.23. The Airy function represents the impulse response – the so-called point spread function – of an aberration-free lens. We will now determine the diameter of the Airy disk $d_{\text{Airy}}$ because it represents the smallest particle image that can be obtained for a given imaging configuration.

![Figure 2.23. Normalized intensity distribution of the Airy pattern and its approximation by a Gaussian bell curve](image)

In figure 2.23 the value of the radius of the ring and therefore of the Airy disk can be found for a given aperture diameter $D_a$ and wavelength $\lambda$:

$$\frac{I(x)}{I_{\text{max}}} = 0 \quad \Rightarrow \quad \frac{d_{\text{Airy}}}{2x_0} = 1.22$$

with

$$x_0 = \frac{\lambda}{D_a}.$$ 

If we consider imaging of objects in air – the same media on both sides of the imaging lens – the focus criterion is given by (see figure 2.24):

$$\frac{1}{z_0} + \frac{1}{Z_0} = \frac{1}{f} \tag{2.5}$$

where $z_0$ is the distance between the image plane and lens and $Z_0$ the distance between the lens and the object plane. Together with the definition of the magnification factor

$$M = \frac{z_0}{Z_0},$$

the following formula for the diffraction limited minimum image diameter can be obtained:
3. Mathematical background of statistical PIV evaluation

A detailed mathematical description of statistical PIV evaluation has been given by Adrian [36]. This early work from 1988 concentrated on auto-correlation methods and was later expanded to cross-correlation analysis [65]. Most of the characteristics and limitations of the statistical PIV evaluation have been described therein. The most complete and careful mathematical description of digital PIV has been given by Westerweel [7]. In this chapter a simplified mathematical model of the recording and subsequent statistical evaluation of PIV images will be presented. For this purpose the two-dimensional spatial estimator for the correlation will be referred to as the correlation. First, we analyze the cross-correlation of two frames of singly exposed recordings, then we expand the theory for the evaluation of doubly exposed recordings. The motivation for why auto- and cross-correlation methods are employed in PIV evaluation will be given in chapter 5.

3.1 Particle image locations

Typically, PIV recordings are subdivided into interrogation areas during evaluation. These areas are called interrogation spots – in the case of optical interrogation – or interrogation windows when digital recordings are considered. Due to reasons stated in the following, for cross-correlation analysis those interrogation areas need not necessarily be located at the same position of the PIV recording. Their geometrical backprojection into the light sheet will be referred to as interrogation volumes in the following (see figure 3.1). Two interrogation volumes used for statistical evaluation together define the measurement volume. Now, a single exposure recording is considered. It consists of a random distribution of particle images, which correspond to the following pattern of \( N \) tracer particles inside the flow:

\[
\mathbf{X} = \begin{pmatrix}
X_1 \\
X_2 \\
\vdots \\
X_N
\end{pmatrix}
\]

with

\[
\mathbf{X}_i = \begin{pmatrix}
X_i \\
Y_i \\
Z_i
\end{pmatrix}
\]
being the position of a tracer particle in a $3N$-dimensional space. $I'$ describes the state of the ensemble at a given time $t$. $\mathbf{x}_i$ is the position vector of the particle $i$ at time $t$. For more details about the mathematical description of the tracer ensemble, see [7]. Lower case letters refer to the coordinates in the image plane (figure 3.1) such that

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$$

is the image position vector in this plane.

In the remainder of this section we will assume that the particle position and the image position are related by a constant magnification factor $M$ for simplicity, such that:

$$X_i = x_i/M \quad \text{and} \quad Y_i = y_i/M.$$  

As already described in section 2.4.3, a more complex model of imaging geometry has to be used to take the effect of perspective projection into account.

![Fig. 3.1. Schematic representation of geometric imaging](image)

### 3.2 Image intensity field

In this section a mathematical representation of the intensity distribution in the image plane is given. It is assumed that the image can best be described by a convolution of the geometric image and the impulse response of the imaging system, the so-called point spread function. For infinite small particles and perfectly aberration-free well focused lenses the amplitude of the point spread function can mathematically be described by the square of the first order Bessel function the so-called Airy function (see section 2.4.4).

A more complex model of imaging has to include imperfections of lenses and photographic films or sensors. For lenses and photographic films an estimation of the main effects besides diffraction can be obtained by analyzing their modulation transfer functions (MTF's) (see section 2.4.2). For CCD sensors a careful analysis requires more complex models, which have not yet been described in the PIV literature sufficiently. The description of digital imaging of very small objects is especially important, because the systematic arrangement of sensor elements can cause significant bias errors in statistical particle image displacement estimation (so-called peak locking; see section 5.4).

In the following we assume the point spread function of the imaging lens $\tau(x)$ to be Gaussian versus $x$ and $y$ (see appendix B.2), which is a common practice in literature and a good approximation for the point spread function of real lens systems [36, 7]. The convolution product of $\tau(x)$ with the geometric image of the tracer particle at the position $\mathbf{x}$ therefore describes the image of a single particle located at position $X_i$. In the following we will assume infinitely small geometric particle images which would be the case for small particles imaged at small magnifications. Therefore, we use the Dirac delta-function shifted to position $\mathbf{x}_i$ to describe the geometric part of the particle image. Thus, the image intensity field of the first exposure may be expressed by:

$$I = I(\mathbf{x}, f') = \tau(x) \ast N_{i=1}^N \delta(x - x_i)$$

where $V_i(\mathbf{X}_i)$ is the transfer function giving the light energy of the image of an individual particle $i$ inside the interrogation volume $V_i$ and its conversion into an electronic signal or optical transmissivity\footnote{Strictly speaking equation (3.1) is valid only for incoherent light. For coherent light a term considering the interference of overlapping particle images has to be included [7]. In most practical situations the particle images do not overlap. Therefore, we use equation (3.1) also for coherent illumination.}. $\tau(x)$ is considered to be identical for every particle position. The visibility of a particle depends on many parameters as for example the scattering properties of the particle, the light intensity at the particle position, the sensitivity of the recording optics and the sensor or film at the corresponding image position. In the following we assume that the particles at every position have the same scattering properties and the recording optics and media have a constant sensitivity over the image plane.

In many situations different weight is put on different locations inside the interrogation area. This can be done by a multiplication of the recorded image intensity with weight kernels in the case of digital evaluation or implicitly due to the spatial intensity distribution of the interrogating laser beam in the case of optical evaluation. In the following we assume that $Z$ is the viewing
direction and that the light intensity inside the interrogation volume is only
a function of \( Z \) and that the image intensity finally analyzed depends on \( X \)
and \( Y \) only due to the weight function. Therefore, \( V_0(X) \) just describes
the shape, extension, and location of the actual interrogation volume:

\[
V_0(X) = W_0(X, Y) I_0(Z)
\]

(3.2)

where \( I_0(Z) \) is the intensity profile of the laser light sheet in the \( Z \) direction
and \( W_0(X, Y) \) is the interrogation window function geometrically back
projected into the light sheet. This is mathematically not correct, because it does
not consider the convolution with the point spread function. For rectangular
interrogation windows this means that in our mathematical description we
neglect the effects of partially cropped images at the edges of the interrogation
area. However, we will use this simple model of the interrogation volumes
in the flow, because it also simplifies the description of PIV evaluation:

\[
I_0(Z) = I_Z \exp \left( -8 \frac{(Z - Z_0)^2}{\Delta Z_0^2} \right)
\]

might be used to describe the Gaussian intensity profile of the laser light
sheet, where \( \Delta Z_0 \) is the thickness of the light sheet measured at the \( e^{-2} \)
points and \( I_Z \) is the maximum intensity of the light sheet. \( W_0(X, Y) \) can be
described in a similar way if a Gaussian window function with a maximum
weighting \( W_{XY} \) at position \( X_0, Y_0 \) has to be considered:

\[
W_0(X, Y) = W_{XY} \exp \left( -8 \frac{(X - X_0)^2 + (Y - Y_0)^2}{\Delta X_0^2 + \Delta Y_0^2} \right)
\]

Since many pulsed lasers used for PIV have an intensity distribution which is
closer to a top-hat function than to a Gaussian function and since digitized
recordings are commonly interrogated with rectangular windows \( V_0(X) \) can be
also defined as a rectangular box:

\[
I_0(Z) = \begin{cases} 
I_Z & \text{if } |Z - Z_0| \leq \Delta Z_0/2 \\
0 & \text{elsewhere}
\end{cases}
\]

(3.3)

\[
W_0(X, Y) = \begin{cases} 
W_{XY} & \text{if } |X - X_0| \leq \Delta X_0/2 \text{ and } |Y - Y_0| \leq \Delta Y_0/2 \\
0 & \text{elsewhere}
\end{cases}
\]

(3.4)

The factor \( I_0(Z_i) \) represents the amount of light received by the particle
inside the flow, and located at distance \( |Z_i - Z_0| \) from the center plane
of the laser light sheet. \( \Delta Z_0 \) is the light sheet thickness and therefore
the extension of the interrogation volume in the \( Z \) direction. \( \Delta X_0 = \Delta Z_0/M \)
and \( \Delta Y_0 = \Delta Y_0/M \) is the extension of the interrogation volume in the \( X \) direction
and \( Y \) direction respectively. With \( \tau(x - x_i) = \tau(x) \delta(x - x_i) \) (see appendix
B.1) and the assumption that the particle images under consideration do not
overlap equation (3.1) can alternatively be written as:

\[
I(x, \Gamma) = \sum_{i=1}^{N} V_0(X_i) \tau(x - x_i) \quad \text{(see appendix B)}
\]

(3.5)

This expression for the image intensity field will be extensively used in the fol-
lowing sections. In the following we will illustrate different representations of
the intensity field and their correlation by giving an example for the recording
of three arbitrarily located particles.

![Fig. 3.2. Example of an intensity field \( I \) (single exposure)](image)

\[\text{Fig. 3.2. Example of an intensity field } I \text{ (single exposure)}\]

### 3.3 Mean value, auto-correlation, and variance of a single exposure recording

In this section we will determine spatial estimators for the mean value
and the variance of the image intensity field, because these quantities will be used
for the normalization of the cross-correlation. Furthermore, auto-correlation
and auto-covariance of a single exposure intensity field will be introduced.
The main equations used in the following are taken from PAPOUS [21, 22]. The
spatial average is defined as:

\[
\langle I(x, \Gamma) \rangle = \frac{1}{a_1} \int_{a_1} I(x, \Gamma) \, dx
\]

where \( a_1 \) is the interrogation area. Employing equation (3.5) yields:

\[
\langle I(x, \Gamma) \rangle = \frac{1}{a_1} \int_{a_1} \sum_{i=1}^{N} V_0(X_i) \tau(x - x_i) \, dx
\]

The mean value of the intensity field can be approximated by:

\[
\mu_1 = \langle I(x, \Gamma) \rangle = \frac{1}{a_1} \sum_{i=1}^{N} V_0(X_i) \int_{a_1} \tau(x - x_i) \, dx
\]

We can now derive the auto-correlation of the single exposure intensity field
in a similar way:
We will now concentrate on this central peak in order to evaluate its features. For a Gaussian particle image intensity distribution

\[ \tau(x) = K \exp \left( -\frac{8|z|^2}{d_z^2} \right) \]

it can be shown that the auto-correlation \( R_c(s) \) is again a Gaussian function with a width that is \( \sqrt{2} d_z \) (see appendix B.3). Consequently, \( R_P(s, \Gamma) \) may be rewritten as following:

\[ R_P(s, \Gamma) = \sum_{i=1}^{N} V_0^2(x_i) \exp \left( -\frac{8|s|^2}{(\sqrt{2} d_z)^2} \right) \frac{1}{a_1} \int_{a_1} \tau^2 (x - x_i + \frac{s}{2}) \, dx \, . \]

In the remainder of this book we will always use \( R_c(s) \) instead of:

\[ \exp \left( -\frac{8|s|^2}{(\sqrt{2} d_z)^2} \right) \frac{1}{a_1} \int_{a_1} \tau^2 (x - x_i + \frac{s}{2}) \, dx \]

taking into account that its features are mainly the same also for non-Gaussian \( \tau(x) \); the maximum of \( R_c(s) \) is located at \( |s| = 0 \) and the characteristics of its shape is given by the particle images shape. Therefore, we will write \( R_P \) as\(^2\):

\[ R_P(s, \Gamma) = R_c(s) \sum_{i=1}^{N} V_0^2(x_i) \, . \]

\(^2\)The dependency of \( R_c(s) \) has been disregarded.
in that location. For intensity fields with zero mean value the auto-correlation equals the auto-covariance. For nonzero mean values of the intensity field the auto-covariance \( C_1(s) \) can be obtained by [21]:

\[
C_1(s) = R_1(s) - \mu_1^2.
\]

An estimator of the variance of the intensity field can be obtained by:

\[
\sigma_1^2 = C_1(0, \Gamma) = R_1(0, \Gamma) - \mu_1^2 = R_F(0, \Gamma) - \mu_1^2.
\]

### 3.4 Cross-correlation of a pair of two singly exposed recordings

As already mentioned before, PIV recordings are most often evaluated by locally cross-correlating two frames of single exposures of the tracer ensemble. The mathematical background of this technique will now be described.

In the following, a constant displacement \( D \) of all particles inside the interrogation volume is assumed, so that the particle locations during the second exposure at time \( t' = t + \Delta t \) are given by:

\[
X'_i = X_i + D = \begin{pmatrix}
X_i + DX \\
Y_i + DY \\
Z_i + DS
\end{pmatrix}.
\]

We furthermore assume that the particle image displacements are given by:

\[
d = \begin{pmatrix}
MD_X \\
MD_Y
\end{pmatrix}
\]

which is a simplification of the perspective projection that is only valid for particles located in the vicinity of the optical axis (see section 2.4.3).

We come to the following representation of the image intensity field for the time of the second exposure (see equation 3.5):

\[
I'(x, \Gamma) = \sum_{j=1}^{N} V_0'(X_j + D) r(x - x_j - d)
\]

where \( V_0'(X) \) defines the interrogation volume during the second exposure. If we first consider identical light sheet and windowing characteristics, the cross-correlation function of two interrogation areas can be written as:

\[
R_{II}(s, \Gamma, D) = \frac{1}{a_l} \sum_{i,j} V_0(X_i) V_0(X_j + D) \int a_l \tau(x - x_i) \tau(x - x_j + s - d) \, dx
\]

where \( s \) is the separation vector in the correlation plane. Analogous to the procedure used in the previous section we come to:

\[
R_{II}(s, \Gamma, D) = \sum_{i,j} V_0(X_i) V_0(X_j + D) R_s(x_i - x_j + s - d).
\]

By distinguishing the terms \( i \neq j \) which represent the correlation of different randomly distributed particles and therefore mainly noise in the correlation plane and the \( i = j \) terms, which contain the displacement information desired, we obtain:

\[
R_{II}(s, \Gamma, D) = \sum_{i \neq j} V_0(X_i) V_0(X_j + D) R_s(x_i - x_j + s - d) + R_s(s - d) \sum_{i=1}^{N} V_0(X_i) V_0(X_i + D).
\]

Again, we can decompose the correlation into three parts:

\[
R_{II}(s, \Gamma, D) = R_C(s, \Gamma, D) + R_F(s, \Gamma, D) + R_D(s, \Gamma, D)
\]

where \( R_D(s, \Gamma, D) \) represents the component of the cross-correlation function that corresponds to the correlation of images of particles obtained from the
first exposure with images of identical particles obtained from the second exposure \((i = j\) terms):

\[
R_D(s, \Gamma, D) = R_0(s - d) \sum_{i=1}^{N} V_0(X_i) V_0(X_i + D). \tag{3.6}
\]

Hence, for a given distribution of particles inside the flow, the displacement correlation peak reaches a maximum for \(s = d\). Therefore, as already anticipated, the location of this maximum yields the average in-plane displacement, and thus the \(U\) and \(V\) components of the velocity inside the flow.

\[
R_{II^+}
\]

**Fig. 3.7.** Schematic representation of the cross-correlation of the intensity fields \(I\) and \(I'\) given in figure 3.5

In figure 3.7 the schematic of the cross-correlation of the example intensity fields \(I\) and \(I'\) is given. Nearly the same correlation peaks occur as in the auto-correlation shown in figure 3.4 but at locations which are displaced by \(d\). Correlations of \(x'_2\) do not appear here, because this image is located outside the interrogation window (see figure 3.5).

It can be seen from equation (3.6) that the displacement correlation is a function of the random variables \((X_i)_{i=1\ldots N}\). Consequently it is a random variable itself and for different realizations at the same overall conditions we will obtain different qualities of the displacement estimation depending on the state of the tracer ensemble. In order to derive rules for a general optimization of the displacement estimation we will determine the expected value of the displacement correlation in section 3.6.

### 3.5 Correlation of a doubly exposed recording

The correlation function of a doubly (or multiply) exposed recording can be derived by analogy to the correlation for single exposed recordings. Instead of cross-correlating \(I\) with \(I'\) we will consider the correlation of the intensity field \(I^+ = I + I'\) with itself. Assuming identical light sheets and windowing characteristics the intensity field of both exposures \(I^+\) can be written as:

\[
I^+(x, \Gamma) = I(x, \Gamma) + I'(x, \Gamma)
\]

\[
= \sum_{i=1}^{N} \left( V_0(X_i) \tau(x - x_i) + V_0(X_i + D) \tau(x - x_i - d) \right).
\]

**Fig. 3.8.** The sum of the intensity fields \(I\) and \(I'\) (see figure 3.5) as obtained by a recording of the tracer ensemble at \(t\) and \(t'\) on the same frame

It can be shown that the auto-correlation of \(I^+\) consists of four terms:

\[
R_{I^+}(s, \Gamma, D) = R_I(s, \Gamma) + R_I'(s, \Gamma) + R_{II}(s, \Gamma, D) + R_{II}(-s, \Gamma, D).
\]

It is therefore appropriate to decompose the estimator into the following terms:

\[
R_{I^+}(s, \Gamma, D) = R_{C^0}(s, \Gamma, D) + R_{D^0}(s, \Gamma, D) + R_{D^+}(s, \Gamma, D) + R_{D^-}(s, \Gamma, D).
\] \tag{3.7}

**Fig. 3.9.** Components of the auto-correlation function

where \(R_{C^0}(s, \Gamma, D)\) is the convolution of the mean intensity of \(I^+\) and \(R_{D^0}(s, \Gamma, D)\) is the fluctuating noise component. \(R_{D^+}(s, \Gamma)\) is the self-correlation
peak located in the center of the correlation plane. It results from the components that correspond to the correlation of each particle image with itself. $R_{D+}(s, \Gamma, D)$ and $R_{D-}(s, \Gamma, D)$ represent the components of the correlation function which correspond to the correlation of images of particles obtained from the first exposure with that of images of identical particles obtained from the second exposure and vice versa.

When comparing the correlation of a doubly exposed recording with the correlation of a pair of two singly exposed recordings, the following statements can be made: $R_{D+}$ is symmetric with respect to its central peak $R_p$. Two identical displacement peaks $R_{D+}$ and $R_{D-}$ appear and as a consequence the sign of the displacement cannot be determined. Therefore, the correlation of a doubly exposed recording is not conclusive if the displacement field of the whole recording is not unidirectional. Another problem appears as the field contains displacements close to zero, which would lead to an overlap between the displacement peaks with the central peak. However, these problems have to be solved during recording. Precautions have to be made, that the images of identical particles due to the different exposures do not overlap and that the sign of their displacement is determined. If the flow field under investigation contains areas of reverse flow or of relative slow velocities image shifting has to be used (see section 4.3). It can be seen from figure 3.10 that the correlation of doubly exposed recordings contains more than twice the number of randomly distributed noise peaks.

![Fig. 3.10. Schematic representation of the auto-correlation of the intensity field $I + I'$ given in figure 3.8](image)

The example given in figure 3.10 shows that in situations for which the cross-correlation of single exposure yield good results, the correlation of doubly exposed recordings contains noise peaks of the strength of the displacement peak. Hence, the evaluation of multiply exposed recordings has to be performed with more particle image pairs in order to get the same performance as that of single exposure evaluation. This can be done by different methods: the seeding density, the number of exposures, or the light sheet thickness can be increased. Besides other problems related to these methods their appli-

cation is restricted due to the limited number of particle images that can be stored on the sensor without a significant overlap. Therefore, in most cases the size of the interrogation areas has to be increased compared to the evaluation of single exposures resulting in a lower spatial resolution of the measurement at the same sensor size.

### 3.6 Expected value of displacement correlation

In order to derive rules for a general optimization of the displacement estimation we will determine the expected value of the displacement correlation $E(R_D)$ for all realizations of $\Gamma$. More concretely: we want to calculate the mean correlation function of all possible "patterns" that can be realized with $N$ particles. From equation (3.8), it follows that

$$E(R_D) = E\left\{ R_+ (s - d) \sum_{i=1}^{N} V_0(X_i) V_0(X_i + D) \right\}$$

$$= R_+ (s - d) E\left\{ \sum_{i=1}^{N} V_0(X_i) V_0(X_i + D) \right\}$$

Defining $f_i(X) = V_0(X) V_0(X + D)$ yields:

$$E(R_D) = R_+ (s - d) E\left\{ \sum_{i=1}^{N} f_i(X_i) \right\}$$

(3.8)

We prove in appendix B.4 that:

$$E\left\{ \sum_{i=1}^{N} f_i(X_i) \right\} = \frac{N}{V_F} \int_{V_F} f_i(X) dX$$

where $\int_{V_F} f_i(X) dX$ is the volume integral

$$\int \int \int f_i(X, Y, Z) dX dY dZ$$

Thus:

$$E(R_D) = \frac{N}{V_F} R_+ (s - d) \int_{V_F} f_i(X) dX$$

(3.9)

Since we defined $N$ to be the number of all particles of the ensemble, $V_F$ has to be interpreted as the whole volume of fluid that has been seeded with particles. According to the above definition of $f_i(X)$ we can say in a
more practical sense that the integration has to be performed over the volume which contained all particles that were inside the interrogation volumes during the first or second exposure. We can rewrite the integral over \( f_1(X) \) as:

\[
\int_{V_p} f_1(X) \, dX = \int I_0(Z) I_0(Z + D_Z) \, dZ \\
\times \int \int W_0(X,Y) W_0(X + D_X, Y + D_Y) \, dX \, dY \\
= \int_{V_p} V_0^2(X) \, dX \cdot F_0(D_Z) F_1(D_X, D_Y)
\]

with

\[
F_1(D_X, D_Y) = \frac{\int \int W_0(X,Y) W_0(X + D_X, Y + D_Y) \, dX \, dY}{\int \int W_0^2(X,Y) \, dX \, dY}
\]

and

\[
F_0(D_Z) = \frac{\int I_0(Z) I_0(Z + D_Z) \, dZ}{\int I_0^2(Z) \, dZ}
\]

KEANE & ADRIAN [63, 64, 65] have defined \( F_1 \) as a factor expressing the in-plane loss-of-pairs, and \( F_0 \) as a factor expressing the out-of-plane loss-of-pairs. When no in-plane or out-of-plane loss-of-pairs are present the latter two are unity, respectively. Finally equation (3.9) yields:

\[
E[R_D(s, D)] = C_R \cdot R_r(s - d) \cdot F_0(D_Z) F_1(D_X, D_Y)
\]

where the constant \( C_R \) is defined as:

\[
C_R = \frac{N}{V_p} \int_{V_p} V_0^2(X) \, dX
\]

3.7 Optimization of correlation

The first parameter that has to be optimized during a PIV measurement is the pulse separation time between the successive light pulses. Besides technical limitations some general effects have to be considered. According to the principle of PIV the measured velocity is determined by the ratio of two components of the measured particle displacement between successive light pulses \( D_X \) and \( D_Y \) respectively, and the pulse separation time \( \Delta t \). Since the particle displacement \( \Delta t \) is a function of \( \Delta t \) in the following – is determined by the particle image displacement with \( D_X(\Delta t) = d_x(\Delta t)/M \) and \( D_Y(\Delta t) = d_y(\Delta t)/M \) respectively, and the measured image displacements contain certain residual errors \( \varepsilon_{\text{resid}} \) we can define the following equation for the magnitude of the locally measured velocity:

\[
|U| = \frac{|d(\Delta t)| + \varepsilon_{\text{resid}}}{M \Delta t}
\]

Since the particle image displacement for a given recording configuration reduces linearly with the pulse separation time, the first summand of the above equation stays constant for vanishing pulse separations:

\[
\lim \frac{|d(\Delta t)|}{M \Delta t} = |U|
\]

In contrast to that, the residual error contained in the measured image displacement will not be reduced below a certain limit by a reduction of the pulse separation, because the uncertainty in determining the particle image positions will be unaffected. Therefore, the second summand of equation (3.13) – which states that the measurement error is weighted with \( 1/\Delta t \) – increases rapidly with decreasing pulse separation:

\[
\lim \frac{\varepsilon_{\text{resid}}}{M \Delta t} = \infty
\]

From these considerations it can be seen that the accuracy of PIV measurements can be increased by increasing the separation time between the exposures at least within certain limits. However, for high values of \( \Delta t \) the measurement noise increases. This becomes clear when looking at the expectation of the displacement correlation given in equation (3.12). It can be seen that the average signal strength is weighted with the loss of pairs due to the particle displacement \( D(\Delta t) \). For a very large separation time the particle displacement, which increases linearly with \( \Delta t \), will exceed the extent of the interrogation volume. Then, no particle will be illuminated twice and no image correlation would be obtained. What can be done to improve the situation? First of all the pulse separation time can be reduced. This directly reduces the particle displacement and the loss of pairs.

In figure 3.11 we have tried to illustrate the two aspects of the choice of \( \Delta t \) on the quality of the PIV data: the dotted line, curve \( g \), represents the effect of the weighting of the residual error with \( \Delta t \), the solid line, curve \( f \), represents the influence of the loss of pairs. The optimum \( \Delta t \) could therefore be found by determining the maximum of a quality function \( Q_{\text{PIV}} \), for example the product of curves \( f \) and \( g \) which is represented by the dashed line.

However, the shape of curve \( f \) has been chosen arbitrarily, since a general value for the quality of a measurement is difficult to define. When using digital equipment, which allows immediate feedback during the measurement,
the optimum can be found interactively by slowly increasing the pulse separation until the number of obvious outliers within the vector map increases. However, the number of valid data yield is only one parameter of the obtained quality, but not an exact measure of it.

Another parameter, which can be used for optimization, if it is made available from the evaluation software, is the normalized strength of the displacement correlation, the so-called cross-correlation coefficient given by:

$$c_H = \frac{C_H}{\sigma_H \sigma_V} = \frac{R_H - \mu_H \mu_V}{\sigma_H \sigma_V}.$$  

When using photographic recording the choice of all recording parameters merely depends on the experiences of the experimentalist, because the valid data yield, the cross-correlation coefficient, or, in the case of optical evaluation, the visibility of the Young’s fringes, can be assessed only after several hours.

Another way to reduce the loss of pairs is to change the size of the interrogation volumes and/or to displace them slightly with respect to each other in order to compensate for the mean particle displacement. The extension of the interrogation volume in the out-of-plane direction is given by the light sheet thickness. This parameter can be increased only if enough laser power is available. If one of the two possible out-of-plane directions is predominant, the light sheet can be displaced between the successive illuminations towards the mean flow. When using double oscillator systems this can be done by a slight “misalignment” of the beam combining optics. In the case of CW lasers a displacement requires additional equipment (see e.g. section 8.4). The extension and location of the interrogation volumes in the in-plane directions is given by the size of the interrogation areas during evaluation and the magnification during recording. In the case of cross-correlation analysis the location of the interrogation windows with respect to each other can be changed. This is one main advantage of cross-correlation and the reason why it is frequently applied also for the evaluation of single frame recordings instead of auto-correlation.

The effects of the interrogation volume locations during the first and second exposure $X_0 = (X_0, Y_0, Z_0)$ and $X'_0 = (X'_0, Y'_0, Z'_0)$ respectively, can best be described by presenting equation (3.12) in a more generalized form:

$$E \{ R_D(s, D, X'_0 - X_0) \} = C_R R_r [s - d - (x'_0 - x_0)] \times F_0 [D_Z - (Z'_0 - Z_0)] \times F_1 [D_X - (X'_0 - X_0), D_Y - (Y'_0 - Y_0)].$$

From this equation the effect of an interrogation window offset $x'_0 - x_0$ can clearly be seen: the peak location has changed by the amount of the offset and the influence of the in-plane loss of correlation on to the peak strength has changed. The significance of the loss of correlation also depends on the absolute extension of the interrogation volume. This is implied in the equations for $F_0$ and $F_1$ as given in the previous section (equation (3.10) and equation (3.11)), but shall be illustrated by the following equation which has been derived for a top-hat light sheet profile (equation (3.3)) and rectangular interrogation windows (equation (3.4)):

$$E \{ R_D(s, D, X'_0 - X_0) \} = C_R R_r [s - d - (x'_0 - x_0)] \left( 1 - \frac{D_X - (X'_0 - X_0)}{\Delta X_0} \right) \left( 1 - \frac{D_Y - (Y'_0 - Y_0)}{\Delta Y_0} \right) \left( 1 - \frac{D_Z - (Z'_0 - Z_0)}{\Delta Z_0} \right).$$

Generally speaking, a stronger peak results in a better peak detection probability and in reduced influence of noise components on the determination of the peak location. In many cases prior knowledge of the main displacement due to mean flow or image shifting can be used in order to improve the result of the evaluation. In other cases more sophisticated algorithms are required in order to take advantage of this effect. The software can either work in a multiple path scheme or with different sizes of the interrogation windows to be correlated. In both cases the resolution and accuracy of the measurement can be considerably increased at the cost of more computing time. The different aspects of choosing the right interrogation window sizes and locations for the evaluation by cross-correlation are described in section 5.4 in detail. According to our experience it is advisable to apply a quick-look evaluation during recording for the optimization of the experimental param-
4. PIV recording techniques

In this chapter different approaches to PIV recording are introduced. It is important to realize that the various recording methods are not necessarily defined by the recording medium. The same approach may for instance be applied using either photography or digital recording. The PIV recording modes can be separated into two main branches: (1) methods which capture the illuminated flow on to a single frame and (2) methods which provide a single illuminated image for each illumination pulse. These branches are referred to as single frame/multi-exposure PIV and multi-frame/single exposure PIV, respectively [28].

![Diagram of PIV recording techniques]

**Fig. 4.1. Single frame techniques**