ISI-Free Pulses with Reduced Sensitivity to Timing Errors

P. Sandeep, S. Chandan, and A. K. Chaturvedi, *Senior Member, IEEE*

***Abstract*— We consider the linear combination of two ISI free pulses, the Raised Cosine (RC) and the recently proposed Better Than Raised Cosine (BTRC). We determine their optimum combination using the distribution of timing error. The obtained pulses perform better than RC and BTRC for fixed as well as randomly distributed timing errors.**

***Index Terms*— Intersymbol interference, pulse analysis.**

I. Introduction

IN data communications there is a need for pulses that are ISI free and at the same time have low sensitivity to timing errors. One popular pulse that satisfies these requirements is Raised Cosine (RC). Recently a novel pulse, referred to as Better Than Raised Cosine (BTRC), has been proposed [1]. In terms of ISI error probabilities, BTRC outperforms RC in the presence of fixed timing errors despite the fact that the tail of BTRC decays as t^{-2} in contrast to t^{-3} of RC. Subsequently, two more pulses with still better performance have been reported [2].

The ISI error probabilities computed in [1] and [2] are for fixed timing errors only. However, timing error is usually random and can be typically characterized by a stationary random process independent of the data sequence [3]. As pointed out in [3], it can be considered to be uniformly or Gaussian distributed.

In this letter, we propose a linear combination of two pulses. We seek to optimize this combination so as to minimize the expected ISI error probability for a given distribution of timing error. We evaluate the performance of the resulting pulse to fixed as well as randomly distributed timing errors.

II. Linear Combination of Pulses

Linear combination of two pulses (that completely overlap in spectral domain) ensures that the resulting pulse has a bandwidth not greater than that of the constituent pulse with larger bandwidth. Further, if the constituent pulses are ISI free, then the resulting pulse will also be ISI free.

We consider the linear combination of pulses $p_1(t)$ and $p_2(t)$ to obtain a new pulse $p(t)$ as

$$p(t) = ap_1(t) + (1-a)p_2(t) \qquad (1)$$

Let ϵ represent the timing error. If the probability density function of timing error is $f(\epsilon)$, then the expected value of ISI error probability is

$$E[P_e] = \int P_e(\epsilon) f(\epsilon) d\epsilon \qquad (2)$$

where $P_e(\epsilon)$ is the ISI error probability for pulse $p(t)$ with timing error ϵ. Considering the case of binary antipodal signaling and Additive White Gaussian Noise (AWGN) in the channel, $P_e(\epsilon)$ can be evaluated as [4]

$$P_e(\epsilon) = \tfrac{1}{2} - \tfrac{2}{\pi} \textstyle\sum_{m=1,m=odd}^{M} \cdot \left\{ \frac{\exp\left(-m^2\omega^2/2\right)\sin(m\omega g_o)}{m} \right\} \prod_{k=N_1,k\neq 0}^{N_2} \cos(m\omega g_k) \qquad (3)$$

Here M represents the number of coefficients considered in the approximate Fourier series of noise complementary distribution function, $\omega = \frac{2\pi}{T_f}$ where T_f is the period used in the series, N_1 and N_2 represent the number of interfering symbols before and after the transmitted symbol and $g_k = p(kT + \epsilon)$ where $p(t)$ is the pulse shape used and T is the bit interval.

It is desired to determine the value of a in (1) so that $E[P_e]$ is minimized. We refer to this value of a as a_{opt} and the pulse obtained by using a_{opt} in (1) as the combination pulse. Since analytical solution for a_{opt} seems intractable, we resort to numerical evaluation.

III. Numerical Results

The linear combination technique can be applied to any pair of pulses from among RC, BTRC and the pulses in [2] to obtain new and useful pulses. In fact, the combination method is potentially applicable even to pulses that may be discovered in the future. To illustrate the proposed method we have used BTRC for $p_1(t)$ and RC for $p_2(t)$.

ISI error probabilities (for fixed timing errors) and expected ISI error probabilities (for randomly distributed timing errors) were determined for different situations. In each case we consider 2^{10} interfering symbols. As mentioned in [4], the error probabilities were calculated using $T_f = 30$ and $M = 31$. In Tables I and II the combination pulses have been optimized with respect to uniformly distributed timing error in $[-0.15T, 0.15T]$.

Table I lists the performance of combination, BTRC and RC pulses for fixed timing errors. Clearly, the combination pulses outperform BTRC and RC over the complete range of fixed timing errors and all the values of roll-off factor α considered.

Since the calculation of a_{opt} needs the knowledge of SNR, we explore the sensitivity of performance of combination pulses to mismatch in SNR. From Table II, it can be observed

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The authors are with the Department of Electrical Engineering, Indian Institute of Technology Kanpur, Kanpur, India (e-mail: akc@iitk.ac.in).

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TABLE I
ISI ERROR PROBABILITY FOR FIXED TIMING ERRORS AND SNR 15 dB (COMBINATION PULSE OPTIMIZED FOR UNIFORMLY DISTRIBUTED ϵ IN $[-0.15T, 0.15T]$, SNR 15 dB)

α	a_{opt}	$\frac{\epsilon}{T}$	Combination	BTRC	RC
0.25	1.82	±0.05	5.119e-8	5.812e-8	8.219e-8
		±0.10	1.027e-6	1.298e-6	2.818e-6
		±0.20	2.697e-4	3.568e-4	9.746e-4
		±0.25	2.264e-3	2.946e-3	6.773e-3
0.35	1.59	±0.05	3.503e-8	3.925e-8	6.000e-8
		±0.10	4.456e-7	5.402e-7	1.390e-6
		±0.20	8.283e-5	1.013e-4	3.908e-4
		±0.25	7.694e-4	9.354e-4	3.199e-3
0.50	1.41	±0.05	2.208e-8	2.413e-8	3.972e-8
		±0.10	1.612e-7	1.858e-7	5.489e-7
		±0.20	1.962e-5	2.088e-5	1.022e-4
		±0.25	1.957e-4	2.015e-4	9.469e-4
1.00	0.69	±0.05	1.229e-8	1.315e-8	1.528e-8
		±0.10	2.942e-8	3.569e-8	5.872e-8
		±0.20	1.362e-6	1.614e-6	3.654e-6
		±0.25	1.897e-5	2.227e-5	3.925e-5

TABLE II
ISI ERROR PROBABILITY FOR FIXED TIMING ERROR $\epsilon/T = \pm0.15$ (COMBINATION PULSE OPTIMIZED FOR UNIFORMLY DISTRIBUTED ϵ IN $[-0.15T, 0.15T]$, SNR 10 dB)

α	a_{opt}	SNR	Combination	BTRC	RC
0.25	2.20	5dB	5.290e-2	5.369e-2	5.562e-2
		10dB	4.220e-3	4.629e-3	5.726e-3
		15dB	2.127e-5	2.622e-5	7.153e-5
0.35	1.97	5dB	5.102e-2	5.164e-2	5.387e-2
		10dB	3.281e-3	3.587e-3	4.725e-3
		15dB	7.279e-6	8.081e-6	2.862e-5
0.50	1.65	5dB	4.936e-2	4.953e-2	5.172e-2
		10dB	2.437e-3	2.595e-3	3.620e-3
		15dB	1.852e-6	1.940e-6	8.174e-6

that the pulse optimized for SNR=10dB performs better even if the actual SNR varies from 5dB to 15dB. In general, the sensitivity of combination pulse to SNR mismatch is a function of α.

In Table III, we consider two cases. In the first case, the timing error is uniformly distributed in the range $[-0.15T, 0.15T]$ and the combination pulses have been optimized for the same. In the second case, the timing error distribution is Gaussian and the combination pulses have been optimized accordingly. A zero mean Gaussian random variable lies between $\pm3\sigma$ in 99.73% cases. By chossing σ to be $0.05T$, we model the case where the timing error is less than $\pm0.15T$, most of the time. As expected the combination pulses exhibit lower expected error probabilities for all α, in both the cases. Values for the Gaussian case are shown in brackets.

TABLE III
EXPECTED ISI ERROR PROBABILITIES FOR UNIFORMLY AND GAUSSIAN DISTRIBUTED TIMING ERRORS FOR SNR = 15 dB

α	a_{opt}	Combination	BTRC	RC
0.25	1.82	$2.279e-6$	$2.967e-6$	$7.636e-6$
	(1.82)	$(3.950e-7)$	$(5.089e-7)$	$(1.263e-6)$
0.35	1.59	$8.205e-7$	$1.004e-6$	$3.221e-6$
	(1.60)	$(1.516e-7)$	$(1.827e-7)$	$(5.508e-7)$
0.50	1.41	$2.451e-7$	$2.770e-7$	$1.017e-6$
	(1.42)	$(5.309e-8)$	$(5.887e-8)$	$(1.849e-7)$
1.00	0.69	$3.225e-8$	$3.893e-8$	$7.085e-8$
	(0.68)	$(1.454e-8)$	$(1.603e-8)$	$(2.207e-8)$

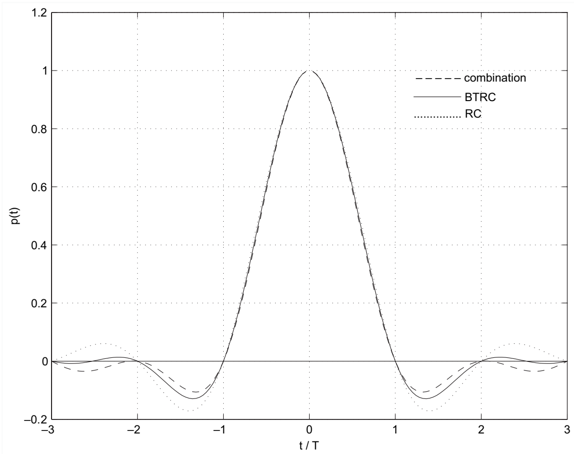

Fig. 1. Waveforms of combination ($a_{opt} = 1.59$), BTRC and RC pulses for $\alpha = 0.35$.

A. Uniqueness of a_{opt}

Let us consider the case when $|a| \gg 1$ and $|t| \neq 0$, in (1). Then

$$p(t) \simeq a(p_1(t) - p_2(t)) \tag{4}$$

As the tail of $p_2(t)$ decays much faster than $p_1(t)$, it can be inferred that the larger the value of a, the larger will be the sidelobes of $p(t)$. These large sidelobes will make $p(t)$ more sensitive to timing error, so it is unlikely that $E[P_e]$, as a function of a, has a minimum for $|a| \gg 1$. This expected behavior was corroborated during the numerical evaluation of $E[P_e]$. We observed that for all the cases considered, there was a unique minimum a_{opt} which was obtained in the range [0.69, 2.20]. The value of $E[P_e]$ was observed to be monotonically increasing over a large range of a on both sides of a_{opt}.

IV. DISCUSSION

Fig. 1 shows the waveforms of combination pulse (used in Table I), BTRC and RC pulses. The better performance of the combination pulses can possibly be attributed to the fact that the first two sidelobes are smaller than those of RC and BTRC. Even though the combination pulse is SNR dependent and does not have a unique eye diagram for a given α, we compared the eye diagrams of the combination pulse of Table I with BTRC and RC, for $\alpha = 0.35$. In terms of maximum distortion and eye opening, the combination pulse

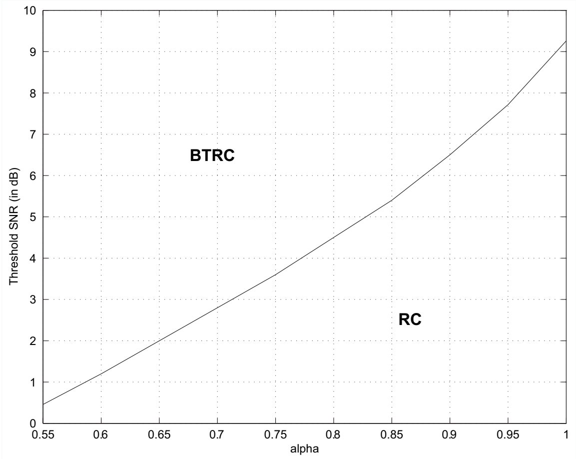

Fig. 2. Threshold value of SNR below which BTRC performs worse than RC in terms of ISI error probability for $\epsilon/T = \pm 0.15$.

was observed to be better than RC. However, it was slightly inferior to BTRC. One reason for this could be the fact that the combination pulse has not been optimized for minimizing the maximum distortion. In any case, maximum distortion is not always a good indicator of ISI error probability.

Given the fact that the combination pulse is SNR dependent, we investigated whether the relative performance of BTRC and RC is also SNR dependent. Interestingly, in terms of ISI error probabilities in the presence of fixed timing errors, BTRC performs worse than RC below a threshold value of SNR. This was not observed in [1] because it assumed the SNR to be 15dB and the threshold value is less than that. The curve depicting the threshold as a function of α is given in Fig. 2.

V. Conclusion

Two ISI free pulses, RC and BTRC, were linearly combined and optimized using the distribution of timing error. The resulting pulses perform better than BTRC and RC for fixed as well as randomly distributed timing errors while having the same bandwidth. Better results are also expected if this pulse is used in OFDM systems [5]. The proposed linear combination technique can be used to obtain several good pulses by considering combinations of the two pulses in [2] or combinations of one of them with RC/BTRC. In recent times it has been shown that for time varying channels, a fixed coding/modulation design is not optimum [6], [7]. In this paper we show that this may be true even for pulse shaping in the presence of timing errors.

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