Analog Multipliers

Nonlinear operations on continuous-valued analog signals are often required in instrumentation, communication, and control-system design.

These operations include

- rectification,
- modulation,
- demodulation,
- frequency translation,
- multiplication, and
- division.

In this chapter we analyze the most commonly used techniques for performing multiplication and division within a monolithic integrated circuit.
In analog-signal processing the need often arises for a circuit that takes two analog inputs and produces an output proportional to their product.

Such circuits are termed *analog multipliers*.

In the following sections we examine several analog multipliers that depend on the *exponential* transfer function of bipolar transistors.
The Emitter-Coupled Pair as a Simple Multiplier

- The emitter-coupled pair, was shown in to produce output currents that were related to the differential input voltage by:

\[
I_{c1} = \frac{I_{EE}}{1 + \exp(-V_{id}/V_T)} \quad I_{c2} = \frac{I_{EE}}{1 + \exp(V_{id}/V_T)}
\]

\[
\Delta I_c = I_{c1} - I_{c2} = I_{EE} \tanh(V_{id}/2V_T)
\]

- This relationship is plotted and shows that the emitter-coupled pair by itself can be used as a primitive multiplier.

\[
\text{or assuming } (V_{id}/2V_T) \ll 1, \Rightarrow \Delta I_c = I_{EE} (V_{id}/2V_T)
\]
The current $I_{EE}$ is actually the bias current for the emitter-coupled pair.

With the addition of more circuitry, we can make $I_{EE}$ proportional to a second input signal.

Thus we have

$$I_{EE} \cong K_o (V_{i2} - V_{BE(on)})$$

The differential output current of the emitter-coupled pair can be calculated to give

$$\Delta I_c \cong \frac{K_o V_{id} (V_{i2} - V_{BE(on)})}{2V_T}$$
Two-Quadrant restriction

- Thus we have produced a circuit that functions as a multiplier under the assumption that $V_i d$ is small, and that $V_i 2$ is greater than $V_{BE(on)}$.
- The latter restriction means that the multiplier functions in only two quadrants of the $V_i d - V_i 2$ plane, and this type of circuit is termed a two-quadrant multiplier.
- The restriction to two quadrants of operation is a severe one for many communications applications, and most practical multipliers allow four-quadrant operation.
- The Gilbert multiplier cell, shown, is a modification of the emitter-coupled cell, which allows four-quadrant multiplication.
The Gilbert multiplier cell is the basis for most integrated-circuit balanced multiplier systems.

The series connection of an emitter-coupled pair with two cross-coupled, emitter-coupled pairs produces a particularly useful transfer characteristic.

\[
I_{c3} = \frac{I_{c1}}{1 + \exp(-V_1/V_T)}
\]

\[
I_{c4} = \frac{I_{c1}}{1 + \exp(V_1/V_T)}
\]

\[
I_{c5} = \frac{I_{c2}}{1 + \exp(V_1/V_T)}
\]

\[
I_{c6} = \frac{I_{c2}}{1 + \exp(-V_1/V_T)}
\]

\[
l_{\text{out}} = l_{c3-5} - l_{c4-6}
\]
The two currents \( I_{c1} \) and \( I_{c2} \) are related to \( V_2 \):

\[
I_{c1} = \frac{I_{EE}}{1 + \exp(-V_2 / V_T)} \quad I_{c2} = \frac{I_{EE}}{1 + \exp(V_2 / V_T)}
\]

Substituting \( I_{c1} \) and \( I_{c2} \) in expressions for \( I_{c3} \), \( I_{c4} \), \( I_{c5} \) and \( I_{c6} \) get:

\[
I_{c3} = \frac{I_{EE}}{[1 + \exp(-V_1 / V_T)][1 + \exp(-V_2 / V_T)]}
\]

\[
I_{c4} = \frac{I_{EE}}{[1 + \exp(V_1 / V_T)][1 + \exp(-V_2 / V_T)]}
\]

\[
I_{c5} = \frac{I_{EE}}{[1 + \exp(V_1 / V_T)][1 + \exp(V_2 / V_T)]}
\]

\[
I_{c6} = \frac{I_{EE}}{[1 + \exp(-V_1 / V_T)][1 + \exp(V_2 / V_T)]}
\]
The differential output current is then given by

$$\Delta I = I_{c3-5} - I_{c4-6} = I_{c3} + I_{c5} - (I_{c4} + I_{c6}) = (I_{c3} - I_{c6}) - (I_{c4} - I_{c5}) = I_{EE} \tanh(V_1 / 2V_T) \tanh(V_2 / 2V_T)$$

The dc transfer characteristic, then, is the product of the hyperbolic tangent of the two input voltages. The are three main application of Gilbert cell depending of the V1 and V2 range:

- (1) If $V_1 < V_T$ and $V_2 < V_T$ then: $\tanh(V_{1,2} / 2V_T) \approx V_{1,2} / 2V_T$ and it woks as multiplier.

- (2) If one of the inputs of a signal that is large compared to $V_T$, this effectively multiplies the applied small signal by a square wave, and acts as a modulator.

- (3) If both inputs are large compared to $V_T$, and all six transistors in the circuit behave as nonsaturating switches. This is useful for the detection of phase differences between two amplitude-limited signals, as is required in phase-locked loops, and is sometimes called the phase-detector mode.
(1) If $V_1 < V_T$ and $V_2 < V_T$ then: $	anh(x) = x + x^3/3 + ... \approx x$

- Thus for small-amplitude signals, the circuit performs an analog multiplication. Unfortunately, the amplitudes of the input signals are often much larger than $V_T$.
- An alternate approach is to introduce a nonlinearity that predistorts the input signals to compensate for the hyperbolic tangent transfer characteristic of the basic cell.
- The required nonlinearity is an inverse hyperbolic tangent characteristic
We assume for the time being that the circuitry within the box develops a differential output current that is linearly related to the input voltage \( v_i \). Thus

\[
I_1 = I_{o1} + K_1V_1 \quad \text{and} \quad I_2 = I_{o1} - K_1V_1
\]

Here \( I_{o1} \) is the dc current that flows in each output lead if \( V_1 \) is equal to zero, and \( K_1 \) is the transconductance of the voltage-to-current converter.

The differential voltage developed across the two diode-connected transistors is

\[
\Delta V = V_T \ln \left( \frac{I_{o1} + K_1V_1}{I_s} \right) - V_T \ln \left( \frac{I_{o1} - K_1V_1}{I_s} \right) = V_T \ln \left( \frac{I_{o1} + K_1V_1}{I_{o1} - K_1V_1} \right)
\]

Using the identity:

\[
\tanh^{-1} x = \ln \left( \frac{1 + x}{1 - x} \right) / 2
\]

We get

\[
\Delta V = 2V_T \tanh^{-1} \left( \frac{K_1V_1}{I_{o1}} \right)
\]

And finally

\[
\Delta I = I_{EE} \left( \frac{K_1V_1}{I_{o1}} \right) \left( \frac{K_2V_2}{I_{o2}} \right)
\]
\[ V_{out} = I_{EE} K_3 \frac{K_1}{I_{o1}} \frac{K_2}{I_{o2}} V_1 V_2 = 0.1V_1 V_2 \]
Gilbert cell as a Balanced Modulator

- In communications systems, the need frequently arises for the multiplication of a continuously varying signal by a square wave.
- This is easily accomplished with the multiplier circuit by applying a sufficiently large signal directly to the cross-coupled pair.

\[ V_m(t) = V_m \cos \omega_m t \]

\[ V_c(t) = \sum_{n=1}^{\infty} A_n \cos n\omega_c t, \text{ where } A_n = \sin \left( \frac{n\pi}{2} \right) / \frac{n\pi}{4} \]

\[ V_o(t) = K[V_c(t)V_m(t)] = K\sum_{n=1}^{\infty} A_n V_m \cos \omega_m t \cos n\omega_c t = \]

\[ = K\sum_{n=1}^{\infty} \frac{A_n V_m}{2} \cos(n\omega_c t - \omega_n t)\cos(n\omega_c t + \omega_n t) \]
Spectra for balanced modulator

- The spectrum has components located at frequencies $\omega_m$ above and below each of the harmonics of $\omega_c$, but no component at the carrier frequency $\omega_c$ or its harmonics. The spectrum of the input signals and the resulting output signal is shown below.

- The lack of an output component at the carrier frequency is a very useful property of balanced modulators. The signal is usually filtered following the modulation process so that only the components near $\omega_c$ are retained.
Gilbert cell as a phase detector

- If unmodulated signals of identical frequency are applied to the two inputs, the circuit behaves as a *phase detector* and produces an output whose dc component is proportional to the phase difference between the two inputs.
The output waveform that results is shown in Fig. 10.16c and consists of a dc component and a component at twice the incoming frequency. The dc component is given by:

\[ V_{\text{average}} = \frac{1}{2\pi} \int_{0}^{2\pi} V_o(t) d(\omega_o t) = \frac{-1}{\pi} [A_1 - A_2] \]

where areas \( A_1 \) and \( A_2 \) are as indicated. Thus

\[ V_{\text{average}} = -\left[ I_{EE} R_C \frac{\pi - \phi}{\pi} - I_{EE} R_C \frac{\phi}{\pi} \right] = I_{EE} R_C \left( \frac{2\phi}{\pi} - 1 \right) \]

If input signals are comparable to or smaller than \( V_T \), the circuit still acts as a phase detector.

However, the output voltage then depends both on the phase difference \textit{and} on the \textit{amplitude} of the two input waveforms.