Bubble pinch-off and scaling during liquid drop impact on liquid pool

Bahni Ray,1 Gautam Biswas,1,2 and Ashutosh Sharma3
1Department of Mechanical Engineering, Indian Institute of Technology, Kanpur, Uttar Pradesh 208016, India
2Central Mechanical Engineering Research Institute (CSIR), Durgapur, West Bengal 713209, India
3Department of Chemical Engineering, Indian Institute of Technology, Kanpur, Uttar Pradesh 208016, India

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Simulations are performed to show entrapment of air bubble accompanied by high speed upward and downward water jets when a water drop impacts a pool of water surface. A new bubble entrapment zone characterised by small bubble pinch-off and long thick jet is found. Depending on the bubble and jet behaviour, the bubble entrapment zone is subdivided into three sub-regimes. The entrapped bubble size and jet height depends on the crater shape and its maximum depth. During the bubble formation, bubble neck develops an almost singular shape as it pinches off. The final pinch-off shape and the power law governing the pinching, \( r_n \propto A(t_0 - t)^\alpha \) varies with the Weber number. Weber dependence of the function describing the radius of the bubble during the pinch-off only affects the coefficient \( A \) and not the power exponent \( \alpha \).

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I. INTRODUCTION

A drop or bubble breaks up by forming a neck that thins to microscopic dimensions and approaches singularity both in space and time where pressure grows infinitely large. For the pinch-off of droplet, the dynamics close to pinch-off exhibit self-similar behaviour. The formation and pinch-off of a low-viscosity liquid droplet in air is described by a balance between surface tension and inertia, resulting in a 2/3 scaling exponent.1 The inverse problem of bubble collapse has been extensively studied in the case of pinch-off of a bubble rising from a needle or nozzle,2–4 the breakup of gas bubble in straining flow,5,6 bubble entrapment when a disk quickly pulled through water surface,7,8 or bubble entrainment due to Faraday excitation.9 If the dynamics near pinching is solely governed by inertia,2–4 then the neck radius \( r_n \) is expressed in the time remaining until collapse \( (t_0 - t) \), where \( t_0 \) is the time instant of pinching, as \( r \propto (t_0 - t)^{1/2} \), or with a logarithmic correction as shown by Gordillo et al.5,6 as \( r_n (- \log r_n)^{1/4} \propto (t_0 - t)^{1/2} \). The collapse may be slowed down by viscosity3 as \( r_n \propto (t_0 - t) \) or by surface tension10 as \( r_n \propto (t_0 - t)^{2/3} \) or it may be accelerated by the inertia of the gas flowing inside the neck5 as \( r_n \propto (t_0 - t)^{1/3} \).

Another example of bubble pinch-off phenomena is air-bubble entrainment due to liquid drop impact on a liquid pool where the process of pinching occurs very fast. Bubble entrainment zone is within the transition regime between coalescence and splashing phenomena on drop impact onto a liquid pool.11–17 Pumphrey and Elmore15 showed that bubble entrainment had a lower and an upper boundary on the Weber number-Froude number plane which Oguz and Prosperetti12 determined as \( \text{We} = 41.3 \text{Fr}^{0.176} \) and \( \text{We} = 48.3 \text{Fr}^{0.247} \), respectively.

We solved the Navier-Stokes equations in axisymmetric coordinates along with the surface tension force as the body force in an incompressible flow. The coupled level-set and volume-of-fluid (CLSVOF) method has been used to track the time-evolution of the interface deformation. The CLSVOF method was discussed in detail by Sussman and Puckett.18 In the CLSVOF method the LS function19 is used only to compute the geometric properties at the interface, while the void fraction
is advected using the VOF approach.20 The method has been modified by Tomar et al.21 to simulate bubble growth in film boiling and Chakraborty et al.22,23 used it to simulate the dynamics of air bubble from submerged orifice. In our earlier paper (Ray et al.24) we used a similar method to capture the complete and partial coalescence phenomena when a liquid drop impacts on a liquid-liquid interface.

In the present work we extended our earlier study24 for water drop impact on air-water interface to investigate the bubble entrapment phenomena. The purpose here is to study the time evolution of the bubble pinch-off followed by jet ejection for different Weber numbers at high Froude number and Reynolds number. An attempt is made to describe the necking stage of the bubble entrapped by numerical simulation. The results are compared with various experimental and theoretical results in literature within the limits of our numerical mesh, i.e., times to pinch-off of about 1 μs and spatial resolutions of 100 μm/grid.

II. FORMULATION

A. Computational domain

Complete numerical simulation of the processes of bubble entrapment is performed for a two-dimensional incompressible flow which is described in axisymmetric coordinates (r, z) on a domain with dimensions 7D x 14D and pool depth 7.2D. The drop is placed at 0.02D from the flat interface, where D is the initial drop diameter. The pool surface is assumed to be flat and motionless and the drop is assumed to be spherical and travelling at the impact velocity, U. We verified carefully that our results are independent of mesh size and time stepping. A grid mesh of 350 x 700 is taken. With low spatial resolution of 200 μm/grid (175 x 350) the bubble pinch-off is not observed and for higher resolution of 50 μm/grid (700 x 1400), the crater depth, bubble size, and jet height are invariant with respect to the spatial resolution of 100 μm/grid (350 x 700). We chose the time step as Δt = 10⁻⁶s for all the simulations satisfying Δt ≤ min(Δt_{conv}, Δt_{cap}), where convective time step, Δt_{conv} ≤ (Δr_{max} + Δz_{max})⁻¹ and capillary time step, Δt_{cap} ≤ 1/2(ρ_0 + ρ_a)/πσ. The geometrical dimensions are nondimensionalized by drop diameter D and time is nondimensionalized by tc = DU, where U is the impact velocity of the drop. The dimensionless numbers deployed to describe the phenomena are: Weber number We = ρ_w U² D/σ, Froude number Fr = U²/(gD), and Reynolds number Re = ρ_w U D/μ_w, where ρ_w is the density of water, μ_w is the viscosity of water, σ is surface tension, and g is the acceleration due to gravity. The properties of air and water are kept constant as given in Table I.

B. Numerical approach

Using the level-set formulation due to Chang et al.,25 the mass and momentum equation for incompressible two-phase flow can be written as

\[ \nabla \cdot \vec{V} = 0, \]  

\[ \rho \left( \frac{\partial \vec{V}}{\partial t} + \nabla \cdot (\vec{V} \vec{V}) \right) = -\nabla P + \rho \frac{1}{Fr} \nabla \cdot \left( \mu \left( \nabla \vec{V} + (\nabla \vec{V})^T \right) \right) + \frac{1}{We} \kappa (\phi) \nabla H (\phi). \]
The level-set function \( \phi \) is maintained as the signed distance from the interface close to the interface. Hence, near the interface,

\[
\phi(\vec{r}, t) = \begin{cases} 
< -d & \text{in the air cells} \\
= 0 & \text{at the interface} \\
> +d & \text{in the water cells}
\end{cases},
\]

where \( d = d(\vec{r}) \) is the shortest distance of the interface from point \( \vec{r} \). The unit normal vector \( \vec{n} \) and the mean curvature \( \kappa \) of the interface are simply

\[
\vec{n} = \frac{\nabla \phi}{|\nabla \phi|} \quad \text{and} \quad \kappa = -\nabla \cdot \frac{\nabla \phi}{|\nabla \phi|}.
\]

Due to the motion of interface, the interface is captured by solving the advection for the level-set function \( \phi \) and for the volume fraction \( F \) in its conservative form,

\[
\frac{\partial \phi}{\partial t} + \nabla \cdot (\vec{V} \phi) = 0,
\]

\[
\frac{\partial F}{\partial t} + \nabla \cdot (\vec{V} F) = 0,
\]

Void fraction \( F \) is introduced as the fraction of the liquid inside a control volume (cell), where the void fraction takes the values 0 for air cell, 1 for water cell, and between 0 and 1 for a two-phase cell. The density and viscosity are derived from the level-set function as

\[
\rho(\phi) = \rho_w H(\phi) + \rho_d (1 - H(\phi)),
\]

\[
\mu(\phi) = \mu_w H(\phi) + \mu_d (1 - H(\phi)),
\]

where \( H(\phi) \) is the Heaviside function,

\[
H(\phi) = \begin{cases} 
1 & \phi > \varepsilon \\
\frac{1}{2} + \frac{1}{2} \sin \left( \frac{\pi \phi}{\varepsilon} \right) & |\phi| \leq \varepsilon \\
0 & \phi < -\varepsilon
\end{cases},
\]

where \( \varepsilon \) is the interface numerical thickness over which the phase properties are interpolated. The boundary conditions are symmetry or free slip condition at the left and right boundaries, outflow boundary conditions on the top surface, and no slip and impermeability conditions on the bottom surface of the domain. The solution algorithm has been described in earlier papers. \( ^{21-24} \)

III. RESULTS AND DISCUSSIONS

A. Bubble entrapment phenomenon

Figure 1(a) shows the high speed experimental images and the computed shape of the free surface profiles during bubble entrapment presented by Morton \textit{et al.}\( ^{14} \) and the predictions of the current study. Morton \textit{et al.}\( ^{14} \) used the volume-of-fluid (VOF) method as the interface tracking method, which showed good qualitative agreement with the experimental image. The only limitation was that after the pinch-off, the trapped bubble disappeared since the dynamics of gas-phase was neglected in their numerical model. Previous attempts to simulate drop impact were carried out by Harlow and Shannon\( ^{26} \) using SOLution Algorithm (SOLA-VOF) method and Oguz and Prosperetti\( ^{12} \) and Elmore \textit{et al.}\( ^{27} \) using Boundary Integral method. The numerical methods predicted the bubble entrapment process but could not predict the after effect of bubble pinching. Wang \textit{et al.}\( ^{28} \) used CLSVOF method to capture entrapped bubble along with the thin jet phenomena and their results matched quite well with Morton \textit{et al.}\( ^{14} \). In the present CLSVOF method the change in profiles along with the time sequences shows good agreement with the experiments of Morton \textit{et al.}\( ^{14} \).

The cavity depth and jet height also reveal good match with their experimental data at early and later stages. The oscillation in cavity depth shown by Wang \textit{et al.}\( ^{28} \) during later stage is avoided here. The inward and outward jets which follow after the bubble entrapment are nicely captured in our simulation. The bubble diameter (~0.18D) is also quite comparable with the experimental data (~0.2D).
FIG. 1. Experimental (left column in (a) and solid circles in (c)) and computational (middle column in (a) and triangles in (c)) representing the bubble entrapment phenomena, cavity depth, and jet height by Morton et al.12 and present numerical method (right column in (a) and solid line in (c)) for a 2.9 mm drop impacting from a height of 170 mm (Fr = 85, We = 96, Re = 4480). (b) The inward jet in entrapped bubble.
The process of bubble entrapment can be best understood from the vertical and horizontal flows during the free surface deformation. The process consists of three stages: expansion, retraction, and necking. After the drop impacts a wave-swell is created at the liquid surface (radial deformation) and a crater is formed (axial deformation). During the expansion stage, wave-swell height and crater-depth increase. The crater shape changes from spherical to U-shape (at $t = 6.2$ in Fig. 1(a)). At retraction stage the wave-swell height and crater-depth decrease and the crater transforms to flat V-shape (at $t = 8.6$ in Fig. 1(a)). The liquid from wave-swell converges toward the side walls of the crater (at $t = 10.7$ in Fig. 1(a)) leading to neck formation. Finally, the crater side wall collides and causes pinch-off (at $t = 10.9$ in Fig. 1(a)). There is a strong pressure drop below the crater base after pinch-off and a high speed jet is ejected upward (at $t = 12.9$ in Fig. 1(a)) and downward (Fig. 1(b)). These jets, known as Worthington jets, are also observed during solid body impact on water, drop impact on a hydrophobic surface, and breakup of bubbles in co-flowing liquid. The potential energy which is stored during the crater formation is converted to kinetic energy of the jet during bubble pinch-off. Due to Rayleigh-Plateau instability, the upward jet breaks up into numerous secondary droplets. Numerically, the downward jet inside the bubble during bubble entrapment phenomena is captured for the first time (Fig. 1(b)) in this paper. In this case the jet attempts to break the bubble into two but due to surface tension forces the bubbles retain its spherical shape. Experimental evidences by Elmore et al. show that the jet may break into drops inside the bubble or in some cases it can split the bubble into two. Gekle et al. and Gekle and Gordillo explained the formation of this high speed jet after solid object impact on a liquid surface or after the pinch-off of a gas bubble from an underwater nozzle. It is not due to a hyperbolic flow around the singular pinch-off point but due to continuous radial energy focusing along the entire wall of the cavity. The fluid is not accelerated upward continuously from the pinch-off singularity but instead acquires its large vertical momentum in a small zone located around the jet base.

Bubble entrapment characteristics as a function of Weber number are studied at negligible gravity ($Fr = 100$) and viscosity effect ($Re = O(10^3)$). Two different types of bubble entrapment processes are observed: large bubble with thin small jet (Fig. 2(a)) and small bubble with long thick jet (Fig. 2(b)) as the Weber number is increased. Figure 2(c) shows the difference in the depth of crater base along the symmetry axis. After initial drop impact both the phenomena behave quite similarly. During the expansion stage at $t = 8$, the crater depth gradually increases, after which the retraction begins from crater side walls. The depth stagnates during the period $t \approx 8–10$ until the crater obtains a flat V-shape indicated by A and B. Flow reversal of the crater base at the symmetry axis begins now. The retraction is stronger at the crater side wall just above the crater base than at other positions. A sub-crater is formed at the crater base (position C and D). The hydrostatic pressure from the side walls empowers the upward force trying to pull back the interface to initial shape. The crater base again moves downward before pinch-off to form a spherical bubble. The inset figure indicates that the change in crater base depth during retraction stage is slower for high We. Another notable difference is the upward jet emanating out during bubble pinch-off. For $We = 150$, a short, thin, high-speed jet appears which again pinches out numerous small secondary drops (Fig. 2(a)). Later the velocity of the jet decreases and the jet thickens, rises to a certain height, and finally collapses into the water surface. In the case of $We = 160$, a similar thin jet is formed which produces many secondary droplets and instead of collapsing quickly, it elongates to form a long, thick jet (Fig. 2(b)). The jet speed decreases and owing to capillary instability, it breaks up to yield one or two large secondary droplets.

This new small bubble entrapment zone along with the large bubble entrapment zone is shown in Weber-Froude diagram in Fig. 3. At low Froude numbers, the new zone lies above the upper limit of bubble entrapment and for higher Froude numbers this zone is within the bubble entrapment regime.

**B. Bubble entrapment sub-regimes**

The final crater shapes before pinch-off at different Weber numbers are shown in Fig. 4(a). It is seen that there is a competition between the retraction of crater side-wall and crater base. For $We > 150$, retraction of the base is faster than that of the side wall and the size of sub-crater and
FIG. 2. (a) and (b) Different crater and jet shapes during large and small bubble entrapment phenomena. The white arrows indicate the entrapped bubbles. (c) Comparing nondimensional crater depth for two cases, one with solid lines: large bubble: $Fr = 100$, $We = 150$, $Re = 4941$, and other with dashed lines: small bubble: $Fr = 100$, $We = 160$, $Re = 5294$. The crater depth is measured from the initial water surface position.

The final bubble is much smaller. Based on bubble diameter, maximum jet height and jet speed as a function of Weber number, the bubble entrapment phenomena are divided into three regimes in Fig. 4(b): capillary regime (Regime I), inertia-capillary regime (Regime II), and the inertial regime (Regime III). The maximum jet height is the distance of the jet tip from the initial liquid surface just before the last secondary drop is ejected and the jet falls back. The jet speed is the speed of the jet

tip when it is initially formed during pinch-off. In the capillary regime the bubble size increases as the surface tension force decreases. More is the surface tension force, lesser is the radial expansion of the sub-crater base and the crater tries to retain its initial shape faster (Fig. 4(a) left row). Further increase in surface tension force does not bring about pinch-off any more. In the inertia-capillary regime, the interplay of both inertia and surface tension force is seen. These forces accelerate the retraction phase and hence the bubble size reduces. The expansion of the sub-crater base continues as in Regime I, but the increased inertia force reduces the bubble size.

In Regime III, the inertia force is the dominating force and here the retraction from the crater base is faster than the side walls. This regime is not always reproducible experimentally as described by Elmore et al. They observed the event “E3” experimentally where a single bubble, multiple bubbles, or no bubble were produced from one impact to next impact unpredictably. According to Gekle and Gordillo the azimuthal distortions triggered by gas shear or geometrical asymmetries leads to this unpredictability in experimental results. The numerical result of Elmore et al. before pinch-off is similar to our observations. The jet height in these regimes shows that in Regime I, the maximum jet height is not influenced much, and as the Weber number increases in Regime II, the jet height gradually increases. In the inertia-regime, the jet increases to large height. The thin jet is ejected at high speed in Regimes I and II and as the bubble size decreases, the jet is thick, long, and the initial jet velocity is less (Regime III). The bubble diameter, maximum jet height, and initial jet velocity are also measured for finer spatial resolution shown in Table II.

C. Bubble pinch-off scaling

The equation describing bubble dynamics near the pinching region is a cylindrical version of the Rayleigh-Plesset equation, which describes the collapse of an infinite cylindrical cavity under
TABLE II. Bubble diameter, jet height, and jet speed at two different spatial resolutions.

<table>
<thead>
<tr>
<th>Parameters Grid</th>
<th>Bubble diameter</th>
<th>Jet height</th>
<th>Jet speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fr = 100, We = 130</td>
<td>350 × 700</td>
<td>0.2099</td>
<td>0.379</td>
</tr>
<tr>
<td>Fr = 100, We = 130</td>
<td>700 × 1400</td>
<td>0.210</td>
<td>0.377</td>
</tr>
<tr>
<td>Fr = 100, We = 190</td>
<td>350 × 700</td>
<td>0.0428</td>
<td>3.968</td>
</tr>
<tr>
<td>Fr = 100, We = 190</td>
<td>700 × 1400</td>
<td>0.0428</td>
<td>3.980</td>
</tr>
</tbody>
</table>

uniform pressure,

\[(\ddot{r}r + \dot{r}^2) \ln \frac{r}{r_{\infty}} + \frac{1}{2} \dot{r}^2 - \frac{\sigma}{\rho r} = g z, \tag{9}\]

where \(r\) is the radius of the bubble, \(r_{\infty}\) is the radius where the flow is quiescent (crater opening on the equilibrium liquid pool surface), \(z\) is the depth of the neck below the equilibrium surface, \(\sigma\) is the surface tension, and \(\rho\) is the density of the surrounding liquid. The equation indicates that the phenomenon of bubble pinch-off is driven by inertia of the collapsing flow (first two terms in Eq. (9)) and the surface tension force contributes to accelerate the liquid inward (third term in Eq. (9)). The right hand term is the constant hydrostatic force. For low-viscosity fluids, the inertia term is the dominating force; the asymptotic solution is obtained by neglecting other terms and Eq. (9) becomes

\[\ddot{r}r + \dot{r}^2 = 0. \tag{10}\]

Solving Eq. (10), the radius of the bubble is given by \(r_{\text{neck}} \propto A(t_0 - t)^{0.5}\). The time scale in the case of low-viscous fluids is the capillary time defined by \((\rho R^3/\sigma)^{1/2}\), where \(R\) is the drop radius. Thus, the coefficient \(A = (\sigma R^3/\rho)^{1/2}\) and for the parameters used in this paper \((R = D/2 = 0.0025 \text{ m}, \sigma = 72.8 \times 10^{-3} \text{ N/m}, \rho = 998.12 \text{ kg/m}^3)\) the value of \(A = 0.02 \text{ m/s}^{1/2}\). Figure 5 shows the behaviour of \(r_{\text{neck}}\) with \((t_0 - t)\) for different Weber numbers varying as \(r_{\text{neck}} = A(t_0 - t)^{\alpha}\).

For the three regimes, the characteristic exponent is \(\alpha = 0.5 \pm 0.02\) and the coefficient \(A = 0.02 \pm 0.006 \text{ m/s}^{1/2}\) (Figs. 6(a) and 6(b)). Thus, the value of the coefficient \(A\) varies more than the power exponent \(\alpha\) with the Weber number. The exponent describing the pinch-off process of bubbles is quite sensitive to the spatial resolution.\(^{34}\) Burton et al.\(^{3}\) experiments on bubble pinch-off in liquids of different viscosities showed three different modes of pinch-off. For lower values of the external viscosity, the axial curvature is more hyperbolic and the neck radius undergoes a sudden rupture at \(r_{\text{neck}} \approx 25 \mu\text{m}\) at a time scale of 10 \(\mu\text{s}\). They showed that bubbles in low viscosity
 fluids follow \((t_0 - t)^{0.5}\) power law until the shear between the interior and exterior flows causes an instability that leads to the rupture of the bubble. According to Keim et al.\(^{35}\) the pinch-off appears to be cylindrically symmetric and proceeds, without rupture, to scales below their camera resolution (4 \(\mu m\)) and derived the power law exponent \(\alpha = 0.56 \pm 0.03\). Thoroddsen et al.\(^{36}\) observed a continuous necking down to the pixel-resolution of 10 \(\mu m\) at largest frame rates with slope of \(\alpha = 0.57 \pm 0.03\). The experiment\(^7, 8\) of violent collapse of the void created at a fluid surface by the impact of an object is purely governed by liquid inertia. In the limiting case of large Froude numbers the inertial scaling \((t_0 - t) \propto r_{\text{neck}}^2 \sqrt{-\ln r_{\text{neck}}^2}\) is obtained also shown for bubble pinch-off at high Re by Gordillo et al.\(^{5, 6}\) The value of power law exponent thus can be expressed as \(\alpha = 0.5 \times \lfloor 1 + 0.5/[- \ln (t_0 - t)] \rfloor\). Analytical work of Eggers et al.\(^{33}\) where the surface tension, gas density, and viscosity are neglected, \(\alpha\) is time dependent expressed as \(\alpha = 0.5 \times \lfloor 1 + 0.5/\sqrt{-\ln (t_0 - t)} \rfloor\). Gekle et al.\(^{34}\) used boundary integral simulations to show the universal behaviour as Eggers et al.\(^{33}\) with different onset time for this universal pinch-off stage.

Taking into account surface tension, gas density, and viscosity, Gordillo\(^{37}\) derived a system of two-dimensional Rayleigh-like equation which showed good agreement with the numerical and experimental\(^{36, 38, 39}\) results. Neglecting surface tension, gas density, and viscosity, Gordillo\(^{37}\) derived the equations

\[
\ln (R_0 r_1) \frac{d \ln (R_0 R_0)}{d (-\ln R_0)} - 1 = 0, \tag{11}
\]

\[
\ln (R_0 r_1) \frac{d \ln (R_0 R_1)}{d (-\ln R_0)} - 1 = 0. \tag{12}
\]

The theoretical prediction of Gordillo\(^{37}\) reduces to Eggers\(^{33}\) theoretical model, \(\alpha = 0.5 \times \lfloor 1 + 0.5/\sqrt{-\ln (t_0 - t)} \rfloor\). We have compared our numerical results with Gordillo’s\(^{37}\) model and the model showed good agreement. Figure 5 shows how the numerical evolution matches the asymptotic behaviour \((t_0 - t) \propto r_{\text{neck}}^2 \sqrt{-\ln r_{\text{neck}}^2}\) (by Gordillo et al.\(^{5, 6}\)) for \((t_0 - t) < 3 \times 10^{-5}s\), which slightly differs from the \((t_0 - t) \propto r_{\text{neck}}^2\) power law and the analyses of Gordillo\(^{37}\) and Eggers et al.\(^{33}\) Here, the comparison with analytical results is shown down to scales of the order of \(r_{\text{neck}} \approx 60 \mu m\). Our numerical data are in good agreement with the theoretical and experimental data although our simulations are unable to capture the exact instants of bubble pinch-off due to the limitation of computational resources.

![FIG. 6. (a) Plotting fitting exponent \(\alpha\) versus We, (b) plotting coefficient \(A\) versus We. Solid line in (a) and (b) is the theoretical value. Different symbols represent different regimes.](image)
It is notable that in Regime II, the pinching for \( \text{We} = 130 \) and \( 140 \) is similar. As the bubble size reduces at \( \text{We} = 150 \), the pinching is similar to Regime I (\( \text{We} = 120 \)). At Regime III (\( \text{We} = 160 - 190 \)), the pinching time is small and with increase of \( \text{We} \) from 160 to 180, the value of the coefficient increases and shows similar pinching characteristics for \( \text{We} = 180 \) and 190.

In inviscid conditions, the pinch-off dynamics of droplets is governed by a local balance between surface tension forces and inertia so the near field structure of drop pinch-off is self-similar and universal.\(^3\) Bergmann \textit{et al.}\(^7\) showed that the self-similar behaviour appears to hold only in the asymptotic regime of very high impact velocities at limit of infinite Froude number, where the influence of gravity becomes negligible and the collapse is truly inertially driven. During bubble entrapment, the power behaviour is again reflected by plotting the scaled axial and radial coordinates during necking.\(^4\) Figure 7 shows the variation of computed scaled crater profiles \( r/r_{\text{neck}} \) as a function of the scaled axial coordinate \( (z - z_{\text{neck}})/r_{\text{neck}} \), where \( z_{\text{neck}} \) is the axial coordinate at \( r = r_{\text{neck}} \). As \( (t_0 - t) \to 0 \), the scaled profiles show self-similarity and the extent of axial coordinate over which the similarity exists increases as the neck thins.

**IV. CONCLUSIONS**

In conclusion, the paper shows that the predictions due to the numerical method match well with the experimental results as well as confirm theoretical analysis of pinching dynamics. Simulations showed the bubble entrapment, growth of Worthington jets (thin and thick), and subsequent formation of secondary drops. Important phenomena such as downward jet during bubble pinch-off and small bubble entrapment above the large bubble entrapment zone of Oguz and Prosperetti\(^1\) have been presented. Three regimes based on bubble diameter versus Weber number have been defined and the scaling theory in each regime has been established.

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