Color Image Compression Using 2-Dimensional Principal Component Analysis (2DPCA)

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Abstract: *Two dimensional principal component analyses (2DPCA) is recently proposed technique for face representation and recognition. The standard PCA works on 1-dimensional vectors which has inherent problem of dealing with high dimensional vector space data such as images, whereas 2DPCA directly works on matrices i.e. in 2DPCA, PCA technique is applied directly on original image without transforming into 1 dimensional vector. This feature of 2DPCA has advantage over standard PCA in terms of dealing with high dimensional vector space data. In this paper a working principle is proposed for color image compression using 2DPCA. Several other variants of 2DPCA are also applied and the proposed method effectively combines several 2DPCA based techniques. Method is tested on several standard test images and found that the quality of reconstructed image is better than standard PCA based image compression. The other performance measures, such as computational time, compression ratio are ameliorated. A comparative study is made for color image compression using 2DPCA.*

Keywords: Image Compression; PCA; 2DPCA; Alternative 2DPCA; $(2D)^2$ PCA; DiaPCA;

1. Introduction

Dimensionality reduction is one of the key techniques in data analysis, aimed at revealing meaningful structure and unexpected relationship in multivariate data. It assembles numerous methods, all striving to present high-dimensional data in low dimensional space, in a way that faithfully captures desired structural elements of the data. Dimensionality reduction is used for many purposes. For example, it is beneficial as a visualization tool to present multivariate data in a human accessible form, as a method of feature extraction, and as a preliminary transformation applied to the data prior to the use of other analysis tools like clustering and classification.

There are various methods for dimensionality reduction. Principal component analysis (PCA) also known as Karhunen- Loeve expansion, is one of the classical dimensionality reduction methods used for feature extraction which has been widely used in variety of areas such as signal processing, pattern recognition, data mining, computer vision and machine learning. The dimensionality reduction problem is directly related to Image compression. PCA has been widely applied in the area of image compression in various forms. PCA has been applied as standalone image compression technique as well as pre-processing or post-processing step in combination with several other techniques such as neural network, discrete wavelet transform based methods. In all PCA based methods when applied to images, the 2 dimensional image matrix must be previously transformed into 1 dimensional image vectors. The resulting image vectors of image usually lead to a high dimensional image vector space because generally image size taken for image compression application is of 128×128, 256×256 and more. The high dimensional image vector space further creates the problem for evaluating covariance matrix accurately and resulting covariance matrix has large size. Furthermore, computing the eigenvectors of a large size covariance matrix is very time consuming. Fortunately, the eigenvectors can be calculated efficiently using the Singular Value Decomposition (SVD) technique and process of generating the covariance matrix is actually avoided. However, this does not imply that the eigenvectors can be evaluated accurately in this way since eigenvectors are statistically determined by covariance matrix, no matter what method adapted for obtaining them [1].

In the area of face recognition, recently a new method, 2-Dimensional Principal Component Analysis (2DPCA) has been proposed by *Jian Yang et. a*l. [1]. As historical background, PCA has been applied in several areas and among others a most successful application is the human face recognition. Principal component analysis (PCA) was first efficiently employed by Kirby and Sirovich [2] ,[3] to represent human faces. They showed that any face image could be reconstructed by approximately as a weighted sum of small collection of images that define a facial basis (eigen-images), and a mean image of the face. Their work immediately led to the PCA "Eigenface" technique [4] by Turk and Pentland for face recognition in 1991. Since then, PCA has been widely investigated and has become one of the most successful approaches in face recognition [5], [6], [7], [8].

Generally another approach taken in image compression using PCA, is to divide image in 8×8 blocks and arrange the block data in vector form. To increase the compression, if too few eigenvectors of covariance matrix is selected as basis vectors for subspace transformation in PCA, it leads to patches in reconstructed image otherwise if more eigen vectors are selected as basis vectors for subspace transformation in PCA , compression will not be sufficient. Also with 8×8 blocks, the covariance matrix has the size 64×64, which is still large to calculate and time consuming. Another problem is that image data are not properly correlated in terms of 64 dimensions for 8×8 blocks cut across full image. To overcome the problem of large covariance matrix in PCA, 2DPCA directly computes the eigenvectors of the so-called image covariance matrix without matrix–to-vector conversion. As a result, 2DPCA has two important advantages over PCA. First, it is easier to evaluate the covariance matrix accurately. Second, computation of corresponding eigenvectors is more efficient and less time is required to determine the corresponding eigenvectors than PCA.

In this paper, a working principle is proposed for color image compression using 2DPCA. The paper is organized as follows: In section 2, the basic idea of 2DPCA and their variants are reviewed. In section 3, proposed working principle for color image compression is described. In section 4, results are presented. Section 5, contains the conclusion and discussion on topic.

2. Brief reviews of 2DPCA and variants

2.1 2DPCA

Let $\mathbf{X}$ denotes an n-dimensional unitary column vector called as projection vector. $\mathbf{A}$ is $m \times n$ random image matrix which is transformed into $\mathbf{Y}$ using $\mathbf{X}$ by following linear transformation:

$$\mathbf{Y} = \mathbf{AX} \tag{1}$$

Therefore to obtain an m-dimensional projected vector $\mathbf{Y}$, which is called the projected feature vector of image A, the total scatter of the projected samples $J(\mathbf{X})$ is used.

$$J(\mathbf{X}) = \mathrm{tr}\,(\mathbf{S_X}) \tag{2}$$

Where $\boldsymbol{S_X}$ is covariance matrix of projected feature vectors and $\mathrm{tr}(\mathbf{S_x})$ is trace of $\boldsymbol{S_X}$. By maximizing (2), projection direction $\mathbf{X}$ can evaluated on which all samples are projected, thus the total scatter of the resulting projected samples is maximized. Now,

$$\begin{aligned} J(\mathbf{X}) = \mathrm{tr}\,(\mathbf{S_X}) &= \mathrm{tr}\,\{E[(\mathbf{Y} - E\mathbf{Y})(\mathbf{Y} - E\mathbf{Y})^T]\} \\ &= \mathrm{tr}\{\, E[(\mathbf{AX} - E(\mathbf{AX}))(\mathbf{AX} - E(\mathbf{AX}))^T] \\ &= \mathrm{tr}\{\mathbf{X}^T E[(\mathbf{A} - E\mathbf{A})^T(\mathbf{A} - E\mathbf{A})]\mathbf{X}\}^T \end{aligned} \tag{3}$$

Therefore, define image covariance (scatter) matrix:

$$\mathbf{G} = E[(\mathbf{A} - E\mathbf{A})^T(\mathbf{A} - E\mathbf{A})] \tag{4}$$

This turns out to be a nonnegative definite matrix of size n x n. The average image $\mathbf{D}$, for available L number of images $\mathbf{A_k}, (\mathbf{k} = 1,2,.........\mathbf{L})$ each of size m x n is,

Therefore, $$\mathbf{D} = \frac{1}{\mathbf{L}}\sum_{\mathbf{k}=1}^{\mathbf{L}} \mathbf{A_k} \tag{5}$$

Then $$\mathbf{G} = \frac{1}{\mathbf{L}}\sum_{\mathbf{k}=1}^{\mathbf{L}}[(\mathbf{A_k} - \mathbf{D})^T(\mathbf{A_k} - \mathbf{D})] \tag{6}$$

It has been shown [1] that the optimal value for the projection axis $\mathbf{X}$, i.e. called optimal projection axis $\mathbf{X_{opt}}$, is the eigenvector of the $\mathbf{G}$, corresponding to the largest eigenvalues. The set of projection axes $\mathbf{X}_1, \mathbf{X}_2 ....\mathbf{X_d}$ are the eigenvectors corresponding the d largest eigenvalues of $\mathbf{G}$. These $\mathbf{X_i}, \mathbf{i} = 1,2,......\mathbf{d}$ are further used to extract the projected feature vectors $\mathbf{Y_i}, \mathbf{i} = 1,2,........\mathbf{d}$ of m dimension for an image $\mathbf{A_k}$.

$$\mathbf{Y_i} = \mathbf{A_k X_i},\ \mathbf{k} = 1,2,.........\mathbf{L} \text{ and } \mathbf{i} = 1,2,......\mathbf{d} \tag{7}$$

Thus feature matrix for an image $\mathbf{A_k}$, is formed as

$$\mathbf{F} = [\mathbf{Y}_1, \mathbf{Y}_2,...\mathbf{Y_i}....\mathbf{Y_d}]_{\mathbf{m \times d}} \tag{8}$$

The reduced size of feature matrix for an image is the key to image compression. The reconstructed image from its feature matrix can be formed as,

$$\overline{\mathbf{A}}_{\mathbf{k}} = [\mathbf{Y}_1, \mathbf{Y}_2.....\mathbf{Y_i}....\mathbf{Y_d}]_{\mathbf{m \times d}}[\mathbf{X}_1, \mathbf{X}_2,......\mathbf{X_d}]^T_{\mathbf{d \times n}} \tag{9}$$

2.2 Variants of 2DPCA

2.2.1 Alternative 2DPCA

It has been shown [9] that 2DPCA is effectively working in the row direction of image as if image matrix $\mathbf{A_k}$, $\mathbf{A_k} = [(\mathbf{A_{k1}})^T(\mathbf{A_{k2}})^T.........(\mathbf{A_{km}})^T]^T$, is arranged using row vectors $\mathbf{A_{kj}}$ i.e. $\mathbf{A_{kj}}$ is j^th row of image $\mathbf{A_k}$. Similarly average image $\mathbf{D} = [(\mathbf{D_{k1}})^T(\mathbf{D_{k2}})^T.......(\mathbf{D_{km}})^T]^T$ is arranged where $\mathbf{D_{kj}}$ is j^th row of average image. Then image covariance matrix (n x n) in the row direction is,

$$\mathbf{G_r} = \frac{1}{\mathbf{L}}\sum_{\mathbf{k}=1}^{\mathbf{L}}\sum_{\mathbf{j}=1}^{\mathbf{m}}(\mathbf{A_{kj}} - \mathbf{D_{kj}})^T(\mathbf{A_{kj}} - \mathbf{D_{kj}}) \tag{10}$$

The (10) is further extended in column direction and alternative $\mathbf{G_c}$ (m x m) is described [9] as:

$$\mathbf{G_c} = \frac{1}{\mathbf{L}}\sum_{\mathbf{k}=1}^{\mathbf{L}}\sum_{\mathbf{j}=1}^{\mathbf{n}}(\mathbf{A_{kj}} - \mathbf{D_{kj}})(\mathbf{A_{kj}} - \mathbf{D_{kj}})^T \tag{11}$$

of size. Image matrix $\mathbf{A_k} = [\mathbf{A_{k1}}\ \mathbf{A_{k2}}..........\mathbf{A_{kn}}]^T$ where $\mathbf{A_{kj}}$ is j^th column of image $\mathbf{A_k}$ and similarly average image $\mathbf{D_k} = [\mathbf{D_{k1}}, \mathbf{D_{k2}},.....\mathbf{D_{kn}}]^T$ where $\mathbf{D_{kj}}$ is j^th column of average image. Apart from description given in [9], (8) can be derived by interchanging rows and columns of image $\mathbf{A_k}$ and proceeding same way as in section 2.1. In this case $\mathbf{Z_p}, \mathbf{p} = 1,2,.......\mathbf{q}$ are the q eigen vectors of (m dimensional) column direction image covariance matrix $\mathbf{G_c}$ corresponding the q largest eigenvalues of $\mathbf{G_c}$. The projected feature matrix $\mathbf{F_c} = \mathbf{Z}^T\mathbf{A_k}$ is of dimension q x n.

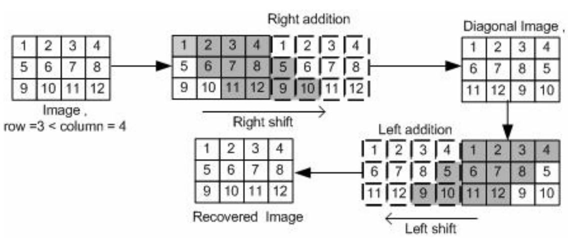
2.2.2 2-Directional 2-Dimensional PCA $(2D)^2$ PCA

2DPCA and alternative 2DPCA only works in the row and column direction of images respectively. That is, 2DPCA learns an optimal matrix $\mathbf{X}$ from a set of training images reflecting information between rows of images, and then projects an m x n image $\mathbf{A_k}$ onto $\mathbf{X}$, yielding an m x d row feature matrix $\mathbf{F_{rk}} = \mathbf{A_k X}$. Similarly, the alternative 2DPCA learns optimal matrix $\mathbf{Z}$ reflecting information between columns of images, and then projects $\mathbf{A_k}$ onto $\mathbf{Z}$, yielding a q x n matrix column feature matrix $\mathbf{F_{ck}} = \mathbf{Z}^T\mathbf{A_k}$ [9] . A way to simultaneously use the projection matrices $\mathbf{X}$ and $\mathbf{Z}$ is described as $\mathbf{F_{rck}} = \mathbf{Z}^T\mathbf{A_k X}$ of size q x d. The matrix

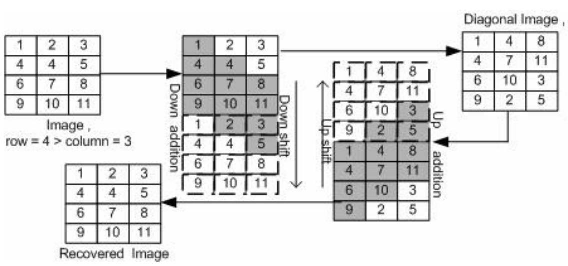
$\mathbf{F_{rck}}$ is also called the coefficient or row-column feature matrix in image representation, which can be used to reconstruct the original image $\mathbf{A_k}$, using $\overline{\overline{\mathbf{A_k}}} = \mathbf{Z F_{rck} X^T}$.

2.2.3 Diagonal Principal Component Analysis (DiaPCA)

In 2DPCA, the projective vectors only reflect variations between either rows or columns of image which implies all the essential information contained may not be covered [10]. To overcome this, another variant of 2DPCA, known as DiaPCA is recently proposed [10] which extract the diagonal image from image. If number of rows of an image is less than number of column then strategy in figure (1.a) is used for forming diagonal image from an image and backward. If number of column of an image is less than number of rows then strategy in figure (1.b) is used for forming diagonal image from an image and backward.



1(a)



1(b)

Figure 1: Constructing diagonal image using DiaPCA

3. Working principle of 2DPCA based color image compression

The block diagram corresponding to the proposed working method is shown in figure (2). First, color image is converted into YCbCr (one luminance Y and two chrominance Cb, Cr) format from RGB image knowing the fact that in YCbCr format, 3 components Y, Cb and Cr are mutually less correlated than in compare to three components R, G and B in RGB format. The resulting each component in matrix form, termed as frame, dealt separately. Each frame is further divided into sub-frames in raster scan to generate set of training images. Apart from blocking in standard PCA, here sub-frames are of larger size and hence capture the more variations of image scenes. Further, for each sub-frame, diagonal sub-frame figure (1) is calculated using DiaPCA. Using diagonal sub-frames for a frame, frame covariance matrix is calculated by applying both row variation (2DPCA) and column variation (Alternative 2DPCA). The corresponding row projection matrix X and column projection matrix Z is evaluated and combined using $(2D)^2$PCA which results a feature matrix $\mathbf{F}_{rck}$ for every diagonal sub-frame $\mathbf{A}_k$ for a frame. To reconstruct the image, feature matrix $\mathbf{F}_{rck}$ is first converted back to $\overline{\overline{\mathbf{A_k}}}$ (inverse diagonal sub-frame) and then use the procedure in figure (1) to reconstruct the sub-frame. The reconstructed sub-frames are arranged in raster scan to reconstruct the frame and combination of 3 reconstructed frames represents the reconstructed image in YCbCr format. When YCbCr format reconstructed image is converted back to RGB format, the reconstructed image is recovered.

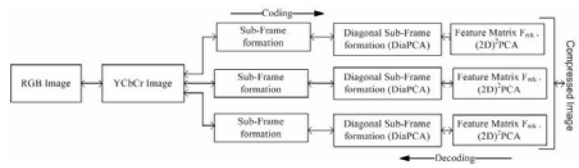


Figure 2: Block Diagram of color image compression using 2DPCA

The expression for calculating compression ration is used as:

$$CR = \frac{\alpha}{\beta.\mu + \theta + \phi}$$

Where, α = size of frame of an image (i.e. m x n)
β = size of feature matrix Frck (i.e. q x d)
μ = no. of sub-frames in a frame (i.e. L)
θ = size of row projection matrix **X** (i.e. n x d)
φ = size of column projection matrix **Z** (i.e. m xq)

Here every element of matrix $\mathbf{F}_{rck}$ takes on average 8 bits. If $\mathbf{F}_{rck}$ is further to be used with any other algorithm of compression, it can be assumed that every element of $\mathbf{F}_{rck}$ matrix takes on average 16 bits and bits per element can be decided at end stage of combined method.

4. Results

The working methodology is tested on standard test images, i.e., lena and mandril. Obtained results are shown in table 1,2, and 3. Overall three variations are carried out. First, by varying the number of sub-frames in frames of given image. Second, by varying number of eigenvectors for both $\mathbf{G}_r$ and $\mathbf{G}_c$ i.e., **X** and **Z** (same number of eigenvectors for both) in frame of a given size as in first variation. Third variation is done by varying the image size and repeating first and second variation. The compression ratio is calculated by assuming every element of resulting feature matrices take on average 8 bits for digital representation. Compression ratio shown in table 1 and 2 are less than the compression ratio shown in table 3. The quality of reconstructed image is good when more number of eigenvectors is considered but in that case compression ratio reduces.

Table 1: 2D PCA (Row- Wise)

(a)

Rs	Cs	Eigen vectors	PSNR	RMSE	CR
4	4	6	20.0006	20.6363	10.0392
4	4	8	23.5935	17.0140	7.5294
4	4	10	24.7595	14.8817	6.0235
6	6	6	24.1034	16.0120	7.0287
6	6	8	25.6243	13.4123	5.2716
6	6	10	26.8820	11.5895	4.2172
8	8	6	25.0234	14.4846	5.2513
8	8	8	26.9810	11.4587	3.9385
8	8	10	28.6876	9.3910	3.1508
(b)					
4	4	6	18.6449	29.8105	10.0392
4	4	8	19.2658	27.7533	7.5294
4	4	10	19.9987	25.5070	6.0235
6	6	6	19.6266	26.6230	7.0287
6	6	8	19.5643	26.8118	5.2716
6	6	10	20.4813	24.1271	4.2172
8	8	6	20.3950	24.3667	5.2513
8	8	8	21.4858	21.4924	3.9385
8	8	10	22.6147	18.8725	3.1508
(c)					
6	6	8	25.3518	13.9609	10.4191
6	6	10	26.5747	12.0952	8.3353
6	6	12	27.5920	10.7428	6.9461
8	8	8	26.5602	12.1208	7.8769
8	8	10	27.9670	10.2825	6.3015
8	8	12	29.0641	9.0450	5.2513
10	10	8	28.0816	10.1530	6.3615
10	10	10	29.5083	8.5992	5.0892
10	10	12	30.6546	7.5220	4.2410
(d)					
6	6	8	20.6595	23.8009	10.4191
6	6	10	21.210	22.3486	8.3353
6	6	12	21.7202	21.0843	6.9461
8	8	8	21.2802	22.1476	7.8769
8	8	10	21.9980	20.4216	6.3015
8	8	12	22.6108	19.0404	5.2513
10	10	8	21.9924	20.4332	6.3615
10	10	10	22.7486	18.7477	5.0892
10	10	12	23.4222	17.3643	4.2410

Table 3: (2D) * PCA (Both Row & Column-Wise)

(a)

Rs	Cs	Eigen vectors	PSNR	RMSE	CR
4	4	6	21.4918	21.8603	48.7619
4	4	8	23.0089	18.1937	32
4	4	10	24.1478	15.9543	22.7556
6	6	6	23.3588	17.4554	36.4089
6	6	8	24.8860	14.6000	22.0215
6	6	10	26.0870	12.6941	14.7604
8	8	6	24.4008	15.5292	24.3810
8	8	8	26.2298	12.4874	14.2222
8	8	10	27.7799	10.4225	9.3091
(b)					
4	4	6	17.5137	33.9717	48.7619
4	4	8	18.0964	31.7529	32
4	4	10	18.6435	29.8150	22.7556
6	6	6	18.3194	30.9447	36.4089
6	6	8	18.9257	28.8589	22.0215
6	6	10	19.4548	27.1523	14.7604
8	8	6	18.8807	29.0077	24.3810
8	8	8	19.5958	26.7155	14.2222
8	8	10	20.2993	24.6369	9.3091
(c)					
6	6	8	24.6453	15.1218	71.5459
6	6	10	25.7736	13.2481	49.4611
6	6	12	26.7226	11.8569	36.2879
8	8	8	25.8255	13.1707	51.2000
8	8	10	27.1020	11.3438	34.1333
8	8	12	28.1323	10.0572	24.3810
10	10	8	27.1780	11.2510	36.3282
10	10	10	28.4847	9.6617	23.7880
10	10	12	29.5827	8.5004	16.7783
(d)					
6	6	8	19.3742	27.5731	71.5459
6	6	10	19.6971	26.5657	49.4611
6	6	12	19.9807	25.7126	36.2879
8	8	8	19.6130	26.7818	51.2000
8	8	10	20.0185	25.5786	34.1333
8	8	12	20.3705	24.5561	24.3810
10	10	8	20.1063	25.3435	36.3282
10	10	10	20.5229	24.1558	23.7880
10	10	12	20.9081	23.1075	16.7783

Table 2: Alternative 2D PCA (2DPCA column wise)

(a)

Rs	Cs	Eigen vectors	PSNR	RMSE	CR
4	4	6	24.8866	14.5935	10.0392
4	4	8	26.2063	12.5091	7.5294
4	4	10	27.3496	10.9553	6.0235
6	6	6	26.4766	12.1289	7.0287
6	6	8	28.1185	10.0227	5.2716
6	6	10	29.4771	8.5666	4.2172
8	8	6	28.1160	10.0252	5.2513
8	8	8	29.8527	8.2035	3.9385
8	8	10	31.4363	6.8351	3.1508
(b)					
4	4	6	18.7091	29.5936	10.0392
4	4	8	19.4078	27.3010	7.5294
4	4	10	19.9215	25.7335	6.0235
6	6	6	19.5643	26.8118	7.0287
6	6	8	20.2829	24.6832	5.2716
6	6	10	20.8731	23.0615	4.2172
8	8	6	20.2701	24.7200	5.2513
8	8	8	21.0609	22.5684	3.9385
8	8	10	21.8056	20.7142	3.1508
(c)					
6	6	8	27.6186	10.6794	10.4191
6	6	10	28.7283	9.3854	8.3353
6	6	12	29.7385	8.3443	6.9461
8	8	8	29.0558	9.0382	7.8769
8	8	10	30.3000	7.8235	6.3015
8	8	12	31.5283	6.7832	5.2513
10	10	8	30.3013	7.8207	6.3615
10	10	10	31.7048	6.6445	5.0892
10	10	12	33.0888	5.6596	4.2410
(d)					
6	6	8	20.2210	24.9800	10.4191
6	6	10	20.5798	23.9620	8.3353
6	6	12	20.8959	23.1014	6.9461
8	8	8	20.5136	24.1252	7.8769
8	8	10	20.9257	23.0076	6.3015
8	8	12	21.3317	21.9473	5.2513
10	10	8	21.0548	22.6831	6.3615
10	10	10	21.5233	21.4859	5.0892
10	10	12	21.9771	20.3882	4.2410

Details of symbols used in tables:

a = 256 x 256 Lena color image
b = 256 x 256 Mandril color image.
c = 512 x 512 Lena color image.
d = 512 x 512 mandril color image.

If image size is m x n x 3 then image frame used for simulation has size
R_r = rows in frame is floor (m / Rs)
R_c = columns in frame is floor (n / Cs)

Floor(x) rounds the x to the nearest lower integer.

Rs is no. of rows division in R_r for sub-frame formation.
Cs is no. of column division in R_c for sub-frame formation.

Eigen Vectors are the no. of Eigen vectors taken from corresponding covariance matrix.

5. Conclusion

The results obtained by using the proposed working method for color image compression are found impressive on account of both compression ratio and quality of reconstructed image. This methodology can be further applied with other compression algorithm to increase the compression ratio. The potential of 2DPCA based techniques is such that they can be used in matrix form which has profound effect on image applications, video applications. The proposed working plan fully explores and utilizes 2DPCA based techniques.

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