Modelling and Analysis of a Coaxial Tiltrotor

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ABSTRACT
This paper proposes a new coaxial tiltrotor design and discusses the development of equations of motion for its analysis in airplane mode, helicopter mode, and transition mode flights. The proposed design is characterized by a coaxial prop-rotor system that is capable of converting from a lifter to a thruster between vertical and forward flight conditions. Half of the wing is also tilted along with the rotor, while the remaining out-board half remains in horizontal position at all times. The rotor dynamics is modeled using rigid blades with only flap degree of freedom. Inflow is estimated using Drees’ model. This paper is divided into two parts. First part of the paper compares and highlights the performance improvement of the proposed vehicle concept over that of the conventional coaxial helicopter. Second part analyses the problem of transition of the vehicle from helicopter mode to the fixed wing mode in a quasi-steady manner. Proposed tiltrotor configuration offers significant improvements over helicopter configuration with dramatic improvements in maximum cruise velocity, range, endurance and rate of climb. The quasi-steady transition analysis shows that the proposed design can transition from helicopter mode to aircraft mode successfully at wide range of forward speeds.

INTRODUCTION
Conventional helicopters continue to be significantly slower, noisier and suffer from higher vibration than their fixed wing counterparts. They also have lower range and endurance for similar all up weights. Fixed wing aircraft seems to be better in every possible aspect, except that they can’t hover or do Vertical Take-off and Landing (VTOL). As helicopters progressed past the World War II, researchers started running into expected speed limitations inherent to all rotorcraft that generate all of its lift and thrust from a rotor in edgewise flight. In addition to a push for higher cruise speeds there was requirement for greater range. There were several ways to achieve improved speed and range and they all have their merits. Therefore, in 1950s and 1960s there was a significant impetus on the development of novel hybrid air vehicles, such as tiltwings, compound helicopters etc., which could bridge the gap between aircrafts and helicopters. Most of these designs were complicated and could not be realized with the state of technology available during 1950s and 60s.

During 1970s work began on the XV-15 (Refs. 1, 2) which was a 13,000 lb (6000 kg) tiltrotor. Each rotor had three 12.5 foot radius blades mounted on a gimbaled hub. A large modeling effort was undertaken to accompany development and flight test of the aircraft. This modeling effort included development of a real-time flight simulation model for the XV-15 (Ref. 3). Eight subsequent simulation periods provided major contributions in the areas of control concepts, cockpit configuration, handling qualities, pilot workload, failure effects and recovery procedures, and flight boundary problems and recovery procedures. The fidelity of the simulation also made it a valuable pilot training aid, as well as a suitable tool for military and civil mission evaluations. Recent simulation periods have provided valuable design data for refinement of automatic flight control systems. Throughout the program, fidelity has been a prime issue and has resulted in unique data and methods for fidelity evaluation which are presented and discussed in (Ref. 4).

As discussed in (Ref. 5), even though the XV-3 was generally the best behaved, it had several notable issues. During low speed flight, weak lateral-directional dynamic stability and longitudinal and directional controllability were experienced (Ref. 6). These issues were attributed to rotor wash at low altitudes. During cruise, lateral-directional (dutch-roll) and longitudinal (short-period) damping was reduced as speed was increased (Ref. 7). This was found to be due to large in-plane rotor forces created from blade flapping resulting from aircraft angular rates. In spite of these issues, the tiltrotor configuration proved to be effective and transition between helicopter and airplane mode was shown to be safe, paving the way for additional research efforts for this configuration.

An in-depth NASA investigation examined several types of rotorcraft for large civil transport applications, and con-
cluded that the tiltrotor had the best potential to meet the desired technology goals (Ref. 8). Goals were included for hover and cruise efficiency, empty weight fraction, and noise. The tiltrotor also presented the lowest developmental risk of the configurations analysed. One of the four highest risk areas identified by the investigation was the need for broad spectrum active control, including flight control systems, rotor load limiting, and vibration and noise. Some general information on the stability and control of tiltrotors can be found in the Generic Tiltrotor Simulation (GTRS) documentation of the XV-15 modeling as documented by Sam Ferguson of Systems Technology, Inc. (Refs. 9, 10).

Even though a lot of research has been done in tiltrotors, mainly XV-15, a coaxial tiltrotor has never been realised and is still in experimental stage. Moreover, literature that describes the basics of coaxial tiltrotor aeromechanics using the basic Euler equations, flapping equations of motions, and basic helicopter and airplane theory is difficult to find. A mono tiltrotor design has been proposed by the Baldwin Technology Company as an innovative VTOL concept that integrates a tilting coaxial rotor, an aerodynamically deployed folding wing, and an efficient cargo handling system (Ref. 11). It was found that the MTR concept, if practically realized, offers unprecedented performance capability in terms of payload, range, and mission versatility.

This paper proposes and analyses a new design for coaxial tiltrotor which is different and possibly simpler than that studied in (Ref. 11). Current design with coaxial rotor promises to offer several advantages over conventional tandem rotor tiltrotor systems (Ref. 12):

- lower actuator forces and moments for tilting the rotors as coaxial rotors tend to have significantly lower gyroscopic loads compared to single rotors during transition.
- tilting of the wing portion below the rotor minimizes the hover losses due to aerodynamic download and interference of the rotor wake and the wing.
- while a fully tilting wing is ideal to avoid losses, a half tilting reduces actuator forces and moments compared to a full tilting.
- the remaining fixed half wings is expected to help during the transition by contributing to lift generation at low forward speeds. process.

The trim analysis for coaxial rotor system is first developed and validated with Harrington rotor’s test data. The trim analysis for coaxial rotor system is then refined to model coaxial tiltrotor configuration. The performance benefits in terms of range, endurance, rate of climb etc. of proposed coaxial tiltrotor design over corresponding coaxial helicopter are systematically studied. A quasi-steady analysis of transition of the tiltrotor from helicopter mode to aircraft mode is carried out.

This paper discusses the modelling, performance study and transition analysis of a hybrid tiltrotor/tiltwing vehicle, which consists of a tilting coaxial rotor system (see Fig. 1). In addition, the portion of the wing in the downwash of the rotors also tilt along with the rotor system. This arrangement allows the vehicle to take-off and land and transition into a fixed wing aircraft by tilting the rotor and half-wing combination.

The hover performance analysis is carried out using Blade Element Momentum Theory (BEMT). The forward flight analysis is performed using a coaxial rotor dynamics analysis in which the blades are modelled as rigid with hinge offset and root spring with only flap degree of freedom. Aerodynamic loads are estimated using Blade Element Theory (BET) coupled to Drees inflow model. The trim analysis in helicopter mode and airplane mode is carried out by Newton Raphson approach.

The transition analysis is carried out in a quasi-steady manner and the ability of the proposed design to perform successful transition from helicopter mode to aircraft mode is studied at different forward speeds. The objective is to develop the methodology for analysis to provide systematic performance comparison with an equivalent pure helicopter (coaxial configuration). Impact of the proposed design changes on the key performance parameters such as speed, range, endurance etc. are discussed in detail. The detailed derivation of the equations for hover, forward flight trim and performance analysis and transition analysis is presented in Appendix A.

**PERFORMANCE ANALYSIS IN HOVER**

Numerical studies are done by modifying the XV-15 aircraft parameters for a coaxial tiltrotor design proposed in this paper. The modified set of parameters were then used as input for a quasi-steady simulation. The complete set of data is presented in Appendix B and are taken from (Refs. 2, 3). Table 1 summarises the input parameters used for this simulation. The following items should be noted prior to this discussion:

- The center of gravity is assumed to be constant and does not move with the movement of the nacelle.
All the trims were run at 6000 kg gross weight, a 0° flight path angle and turn rate equal to 0°/s.

Current analysis does not include modelling of stall characteristics of either rotary wing or fixed wing.

The performance analysis provides results for the minimum power consumption of the rotors under a given set of conditions, and so provides a datum against which the efficiency of a real rotor can be measured.

<table>
<thead>
<tr>
<th>No. of blades per rotor</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of rotors</td>
<td>2</td>
</tr>
<tr>
<td>Rotor solidity</td>
<td>0.089</td>
</tr>
<tr>
<td>No. of engines</td>
<td>1</td>
</tr>
<tr>
<td>Max. take-off weight(kg)</td>
<td>6000</td>
</tr>
<tr>
<td>Rotor Diameter(m)</td>
<td>8</td>
</tr>
<tr>
<td>Wingspan(m)</td>
<td>10</td>
</tr>
<tr>
<td>Max. power available(kW)</td>
<td>1800</td>
</tr>
<tr>
<td>Fuel weight(kg)</td>
<td>676</td>
</tr>
<tr>
<td>Wing aspect ratio</td>
<td>6.25</td>
</tr>
<tr>
<td>Specific fuel consumption (kg/Watt-s)</td>
<td>$3.8 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Table 1. Key design inputs for coaxial tiltrotor simulation

Validation of Hover Analysis

The modelling process involved developing a trim routine for a generic coaxial rotor and then modifying the math model to incorporate the effects of a tilting mast. The hover performance prediction for a regular coaxial rotorcraft using the BEMT is validated against the experimental results obtained by Harrington for nominally full scale rotor systems (Ref. 13). The results comparing the current predictions with experimental data for Harrington Rotor 1 with 25ft diameter and solidity of 0.054 for coaxial rotor with untwisted blades and a taper ratio of approximately 3:1 are shown in Fig. 2. Predicted results show excellent correlation.

The coupled trim code for coaxial rotor has also been developed and validated and the combination of the two analysis would be used to carry out performance evaluation and transition analysis of the novel coaxial tiltrotor/tiltwing vehicle.

Hovering Performance Comparison

An important feature of the proposed coaxial tiltrotor is the half tilting design. It is normally assumed that the total thrust, T, required by the rotor system is equal to the weight of the helicopter, W. However, there is usually an extra increment in power required because of the download or vertical drag, $D_v$, on the helicopter fuselage that results from the action of the rotor slipstream velocity. Typically, the vertical download on the fuselage can be up to 5% of the gross takeoff weight, but it can be much higher for rotorcraft designs such as compounds or tilt-rotors that have large wings situated in the downwash field below the rotor. The download penalty due to fuselage is unavoidable but design adjustments can be made such as a tilting wing to avoid vertical drag due to wing.

In the simplest form the vertical drag can be accounted for by assuming an equivalent drag area $f_v A$ or a drag coefficient $C_{D_v}$ based on a reference area, say $S_{ref}$. This means that the extra rotor thrust to overcome this drag will be

$$\Delta T = D_v = \frac{1}{2} \rho \bar{v}^2 f_v A$$

(1)

where $\bar{v}$ is the average velocity of the rotor slipstream.

Using the Simple Momentum Theory, an expression for calculating the net rotor power requirement is given by (Ref. 14):

$$P = \frac{W}{1 - \frac{W}{\rho \bar{V}^2 / \rho}}$$

(2)

Using the above equation, the download penalty is calculated for three different configurations based on the area of wing that is tilted along with the rotor system and shown in Table 2. These three configurations are:

1. No wing area under tilt
2. Half wing area under tilt
3. Full wing area under tilt

It can be intuitively stated that the actuator force required to tilt a wing is directly proportional to the wing area under tilt. On the other hand Table 2 shows an inverse relation in power required to hover with area of wing under tilt. Thus, qualitatively, it can be stated that a half tilting would provide a trade-off between actuator forces and download penalty and is therefore incorporated in the current hybrid tiltrotor design.
### Table 2. Comparison of power required in hover for different tiltwing configurations

<table>
<thead>
<tr>
<th>Vehicle Configuration</th>
<th>Power Required in Hover</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Tiltwing</td>
<td>2245 kW</td>
</tr>
<tr>
<td>Half Tiltwing</td>
<td>1546 kW</td>
</tr>
<tr>
<td>Full Tiltwing</td>
<td>1165 kW</td>
</tr>
</tbody>
</table>

**COAXIAL TILTROTOR SIMULATION**

The simulation was designed to incorporate the elements of basic rotary wing dynamics along with reasonable assumptions while maintaining the accuracy of the model. The performance analysis involves running the simulation for operational extremities of the proposed vehicle and obtaining the trim results. A qualitative and quantitative analysis is then carried out to predict the performance advantage of one operational mode over another.

The most important factor in the development of vehicles that can operate in multiple configurations/orientations is the feasibility of transition. To investigate the proposed design’s transition capabilities, a quasi-static analysis is carried out for a prescribed transition corridor. The results of this analysis then throws light on the feasibility and nature of transition.

**Trim Analysis**

The two operational extremities of the proposed design are helicopter mode (w/o wings) and airplane mode (with wings) for which the performance analysis is carried out. The vehicle configuration transitions from helicopter mode (w/o wings) ($\beta_M = 0^\circ$) to airplane mode (with wings) ($\beta_M = 90^\circ$) where $\beta_M$ is the angle between the coaxial rotor mast with the vertical axis of the aircraft. The analysis was carried out for speeds from 0 m/s to 105 m/s (378 km/hr) and from 90 m/s (324 km/hr) to 200 m/s (720 km/hr).

Figures 3 and 4 shows trim results for non dimensional rotor inflow on upper and lower rotor. In helicopter mode (w/o wings), the upper rotor experiences a lesser inflow compared to the lower rotor. This can be attributed to the fact that the lower rotor operates in the *vena contracta* of the upper rotor. In airplane mode (with wings), the inflow velocities seen by both the rotors are almost equal for a given flight condition as the difference due to interference is very small compared to the high inflow velocities as it operates at higher speeds.

Figures 5 and 6 the control angle requirements to trim the vehicle in a given flight condition. In helicopter mode (w/o wings), the collective pitch requirement for lower rotor is slightly higher than the upper rotor which is again a consequence of the lower rotor operating under the influence of the wake of the upper rotor. At hover, the cyclic pitch requirements are zero but as the vehicle gains forward speed, the rotors tilts backwards as the longitudinal cyclic pitch requirement increases in the negative direction. Lateral cyclic pitch requirement variation is very small as the rotor mast tilts to counter the unbalanced side slip forces and roll moments.

In airplane mode (with wings), the cyclic pitch requirements are very small compared to the collective pitch requirements. This result seem intuitive as the vehicle is moving in the axial direction. The collective power requirement for both rotors are almost equal. The difference between the two collectives is negligible compared to the high requirements in forward flight.

The collective pitch requirement shown in the results may appear absurd as they range from about $0^\circ$ to $45^\circ$ which is impossible to achieve without encountering stall. A further investigation of local blade element reveals the effective angle of attack at the blade element. The results are shown in Figures 8, 7 for extremities i.e. helicopter mode (w/o wings) and airplane mode (with wings). It is clear from the plots that the blade root might be under stall but for most part, the effective angle of attack is within reasonable limits.

The attitude of the vehicle in helicopter mode (w/o wings) is shown in Fig. 9. Pitch requirements in helicopter mode (w/o wings) shows a decrease which means a nose down movement which when compared to the longitudinal cyclic
pitch requirement in helicopter mode depicts that the rotor mast tilts backward as the vehicle pitches in nose down direction. The roll requirement is very negligible in helicopter mode (w/o wings). In airplane mode (with wings), the roll and pitch requirements are shown in Fig. 10.

**Performance Prediction**

To analyse the performance of the coaxial tiltrotor, several performance parameters are evaluated and compared for helicopter mode (w/o wings) against the airplane mode (with wings). These performance parameters include calculation of Power Requirements, Rate of Climb, Endurance and Range. Further, the advantages of a half tiltwing design are investigated using Simple Momentum Theory.

With all the power transmitted to the main rotor system through the shaft, we have $P = \Omega Q$. It can be shown that the nondimensional power coefficient is equal to the nondimensional torque coefficient. The total power can be written as a combination of four components as:

$$P = P_i + P_c + P_p + P_0 \quad (3)$$

The induced power is generated due to the drag induced as a consequence of development of lift at the blade element. Climb Power is the power required to climb. Parasite Power is required to overcome the drag of the helicopter. Profile power is needed to turn the rotor in air. Since the vehicle is in forward flight, the rotor disk is always under climb and thus climb power contributes significantly to the total power requirement. Parasite power is accounted for using the parasite drag which is calculated in terms of equivalent flat plate area. For a tiltwing configuration, this flat plate area will be a function of the mast angle but is assumed to be constant for this analysis.

The power curves for both helicopter mode (w/o wings) and airplane mode (with wings) are shown in Fig. 11. The following inferences are drawn from the results obtained.

- The power requirements for helicopter mode (w/o wings) reaches the power available for much lower airspeed than when the vehicle is in the airplane mode (with wings). This is a reflection of the clear airspeed advantage that the airplane possesses over the conventional helicopter.
• The maximum cruise speed in helicopter mode is predicted to exceed 100 m/s (360 km/hr) whereas in airplane mode (with wings), the vehicle can theoretically achieve speeds exceeding 200 m/s (720 km/hr).

• Upto 75% reduction in power requirements is predicted while flying in airplane mode (with wings) as compared to helicopter mode (w/o wings). The maximum reduction in power requirements can be achieved while flying at airspeeds close to 100 m/s (360 km/hr) than in helicopter mode (w/o wings) at the same speed.

Endurance

Total time an airplane stays in the air on a full tank of fuel for a given flight condition is defined as the endurance of an aircraft. The generalised endurance parameters are specific fuel consumption, aircraft weight, engine power, and fuel weight. Endurance can be calculated using the following equation.

\[
E = - \int_{W_{TO}}^{W_{RO}-W_{f}} \frac{dW}{cP}
\]  

The total time an airplane can fly is the integral of this expression and is shown in Fig. 13 for both helicopter and airplane mode (with wings). The inferences drawn from the results are presented below.
The maximum endurance of the vehicle is predicted to exceed 4.5 hours.

- Upto 450% increase in endurance is predicted in airplane mode (with wings) than in helicopter mode (w/o wings). The maximum improvement can be achieved by flying at 100 m/s (360 km/hr).

- For the same endurance, the vehicle is now capable of flying two times faster in airplane mode (with wings) compared to helicopter mode (w/o wings).

The rate of climb(ROC) capability vs. airspeed is shown in Fig.14 for both flight modes. As a result of much lower power requirements in airplane cruise, the rate of climb capability far exceeds that of in helicopter mode (w/o wings). The power available can be judged by comparing it with the engine power installed in XV-15 which consists of two engines of about 1250 kW whereas for a coaxial rotor the power available is assumed to be 1800 kW. Following inferences can be drawn from the results for climb capabilities:

- The maximum rate of climb of the vehicle is predicted to exceed 20 m/s (72 km/hr).
- The vehicle can climb faster as higher airspeeds in airplane mode (with wings) as compared to helicopter mode (w/o wings).
- Upto 500% increase is predicted in rate of climb when the vehicle is flying in airplane mode (with wings) at an airspeed of 100 m/s (360 km/hr) than in helicopter mode (w/o wings).

During transition, the mast angle changes the aircraft configuration as it varies from 0° to 90° where \( \beta_M = 0° \) corresponds to the helicopter mode (w/o wings) when the rotor plane is parallel to the horizontal axis. The mast tilts forward with a step size of 2.5° until the rotor plane is perpendicular to the horizontal axis. The operating speed range for \( \beta_M = 0° \) is 0 m/s to 50 m/s with a step size of 2.5 m/s. With every subsequent step increment in mast angle, each velocity step gets incremented by 2.5 m/s. i.e., at \( \beta_M = 2.5° \), the velocities span from 2.5 m/s to 52.5 m/s. Likewise, for \( \beta_M = 90° \), the velocity range is 90 m/s to 140 m/s. This is the prescribed transition corridor for a quasi-steady transition analysis.

For better visualization, the transition results are plotted with a colour gradient. A range of colours are generated corresponding to trim results for every value of \( \beta_M \). The colour field is indicated using a colour bar.
Inflow Velocities

Figures 15 and 16 shows the variation of inflow on lower and upper rotor with respect to forward speed. Note that ‘inflow’ here refers to the flow velocity perpendicular to the rotor disk. As the rotor disk tilts forward, more and more of the resultant flow becomes perpendicular to the rotor disks or parallel to the mast.

For $\beta_M$ close to zero, the inflow ratio dips with increase in airspeed before rising again. This is consistent with conventional rotor system results. For mast angles greater than 20°, the inflow ratio increases with increase in airspeed. This seems intuitive as the rotor disk is now more aligned with the oncoming flow.

Figure 17 shows the variation of flow parallel to the rotor disks. For a conventional helicopter, flow parallel to the rotor disks is a measure of the forward velocity of the vehicle which is why this quantity is referred to as advance ratio, denoted by $\mu$. However in this analysis, advance ratio is the flow velocity parallel to the rotor disk in longitudinal direction. The results for flow parallel to rotor disk shows an increase for small mast angles. As the mast tilts further forward, the flow becomes more and more perpendicular to the rotor disk. From the Fig. 17, it can be seen that the flow lines starts decreasing for mast angles greater than 45°.

Trim Control Angles

The proposed vehicle design has four control parameters, two collective pitch control, one for each rotor and two cyclic pitch control, common for both rotors. These four control parameters are depicted in Figures 18, 19, 20, and 21. To control the vehicle, the pilot must be able to influence all the control parameters at the same time. Since, there are four control parameters, four control columns are required. Let us assume that the vehicle has two control sticks located in front of the pilot and two pedal controls activated by the pilot’s feet. Two control sticks can be assigned to the collective pitch requirement and would influence the throttle. Similarly the cyclic pitch controls can be assigned to the two pedals and would influence the orientation of the vehicle.

Figures 18 and 19 shows the collective pitch requirement for both the rotors. The collective pitch requirement shows an initial decrease which would mean a pull back of the collective stick control at lower mast angles for increasing speed. As the mast tilts forward, the control stick is pushed to gain airspeed. For higher mast angles, the collective pitch requirement decreases relatively.

Fig. 20, depicts the lateral cyclic pitch control angle. The lateral cyclic pitch required to trim the aircraft as the nacelle tilts forward decreases initially, followed by a sharp before evening out to nearly shows. This follows intuition as the mast would not require to tilt laterally to maintain flight in airplane mode (with wings). The lateral cyclic pitch control shows huge variations during transition to balance the roll moments and side forces generated.

Fig. 21, depicts the longitudinal cyclic pitch control angle. The longitudinal cyclic pitch required to trim the aircraft
as the nacelle tilts forward tends to be negative, which would imply a tilt opposite to the direction of motion. This results in apparent negative speed stability while converting (aft stick requirement for increasing airspeed). There is a sharp decrease in the rate of change of longitudinal cyclic pitch. The longitudinal cyclic pitch eventually tends to even out to nearly zero as the rotor mast tilts close to 90°. This follows intuition as the mast would not require to tilt laterally to maintain flight in airplane mode (with wings).

![Fig. 18. Collective pitch of upper rotor vs. true airspeed](image)

![Fig. 19. Collective pitch of lower rotor vs. true airspeed](image)

**Attitude Parameters**

The pitch requirement, shown in Fig. 22, shows an increasing pitch requirement in the early stages of the transition which later evens out. At all points of transition, the pitch attitude is decreasing with increasing airspeed. The increasing pitch requirements with increasing mast angle for smaller mast angles can be attributed to lesser contribution of lift in providing lift at lesser speeds. Thus, the vehicle pitches up to allow the rotors to continue providing lift. As the vehicle gains forward speed and the mast tilts further forward, the wing starts contributing to the lift and the rotors start acting as propellers to provide thrust in forward flight. Also, the pitch requirement decreases as the wing generates higher lift for smaller pitch angles at higher speeds. Thus, the sharp decrease in pitch.

![Fig. 20. Lateral cyclic pitch of both rotors vs. true airspeed](image)

![Fig. 21. Longitudinal cyclic pitch of both rotors vs. true airspeed](image)

**Power in Transition**

The power consumption over the transition range is shown in Fig. 24. It can be noticed that the power consumption decreases as the rotor mast starts tilting forward upto 15° denoted by the green region. The transition from green to cyan to denotes an increase in the rate of power consumption. This rate increases upto 40° followed by a decrease in the rate of power consumption denoted by the region in blue to black.

**CONCLUSIONS**

This paper discusses the development of the equations of motion for coaxial tiltrotor aircraft covering airplane mode, heli-
copter mode, and conversion mode flight. Coaxial rotor trim and performance analysis is developed and refined for tiltrotor analysis. Subsequent analysis addresses the performance and transition aspects of the coaxial tiltrotor aircraft from the perspective of a quasi-steady trim. Qualitative analysis demonstrated substantial benefit of coaxial tiltrotor design if it were to be technically realized. The coaxial tilt rotor was modelled using rigid blades with root spring and hinge offset. Rotor inflow was modelled using Drees’ model. Coupled rotor trim analysis was carried out using Newton-Raphson to perform performance and transition analysis. In general, the proposed coaxial tiltrotor design offers significantly improved performance in the aircraft mode when compared with a coaxial helicopter of same disc loading. The following specific conclusions can be drawn from this conceptual study:

1. The maximum cruise speed is predicted to be 100% more in airplane mode than in helicopter mode. Up to 75% reduction in power requirements is anticipated while flying in airplane mode compared to helicopter mode (w/o wings) at higher flight speeds.

2. The maximum range of the vehicle increases by 100% in airplane mode than in helicopter mode (w/o wings). Up to 400% increase is predicted in range when flying in airplane mode at 100 m/s (360 km/hr) as compared to flying in helicopter mode (w/o wings) at the same speed.

3. The maximum endurance of the vehicle is predicted to exceed 4.5 hours and the vehicle can fly up to two times faster for the same endurance. A maximum possible increase of 450% in endurance is predicted in airplane mode compared to the highest possible speed in helicopter mode (w/o wings).

4. A maximum of 500% increase is predicted rate of climb when the vehicle is flying in airplane mode at an airspeed of 100 m/s (360 km/hr) as compared to flying in helicopter mode (w/o wings) at the same speed.

5. The analysis demonstrates the capability of the coaxial tiltrotor of transitioning from helicopter mode to aircraft mode at wide range of speeds. The simulated vehicle could transition from helicopter mode to aircraft mode for all the airspeeds between 0 m/s to 90 m/s.

6. The proposed half tiltwing design is expected to require smaller actuators with reduced power requirements when compared to a full tiltwing while giving advantage of reduced download penalty.

APPENDIX A

**Governing Equations**

First, the equations of motion are developed. These equations are then modified for an identical counter-rotating coaxial rotor system. The two set of rotor equations provided the coaxial rotor loads which are then added to the airframe loads for trim analysis. The transition analysis is carried out in quasi-steady manner to demonstrate the feasibility of the vehicle to complete successful transition.
Axes Systems

In order to define and transfer forces, moments, and motions for all the moving parts of the vehicle, several sets of coordinate systems need to be defined. The coordinate systems and the corresponding transformation matrices are discussed in the following subsections.

Gravity Coordinate System

The gravity axes system is shown in Fig. 25. It is fixed to the aircraft centre of gravity when it is stationary on the ground. The z-axis points vertically downwards in the direction of gravity or the center of the Earth. The x-axis points North and the y-axis points East.

Body Coordinate System

The body axes system is defined with respect to the gravity axes system with the help of euler angles which determine the attitude of the aircraft. This system is fixed to the aircraft at the CG and rotates with it. As shown in Fig. 26 the x-axis is directed out of the nose of the aircraft and is parallel to the longitudinal axis, the y-axis runs out the right wing, and the z-axis is perpendicular to the x and y axes directed downward. The roll, pitch and yaw then refers to the rotation about x-axis, y-axis and z-axis, respectively. \( T_{G\rightarrow B} \) denotes the rotation matrix defined by the Euler Angles.

\[
\begin{bmatrix}
\hat{i}_B \\
\hat{j}_B \\
\hat{k}_B
\end{bmatrix} = \begin{bmatrix} T_{G\rightarrow B} \end{bmatrix} \begin{bmatrix}
\hat{i}_G \\
\hat{j}_G \\
\hat{k}_G
\end{bmatrix}
\]

where

\[
T_{G\rightarrow B} = \begin{bmatrix}
c \theta c \psi & c \theta s \psi & -s \theta \\
-\phi s \theta c \psi - \phi c \psi & \phi s \theta s \psi + \phi c \psi & \phi c \theta \\
\phi s \theta c \psi + \phi c \psi & -\phi s \theta s \psi + \phi c \psi & \phi c \theta
\end{bmatrix}
\]

Nacelle Coordinate System

The nacelle axes system shown in Fig. 27 is defined with respect to the body axes system with the help of mast angle (\( \beta_M \)). When \( \beta_M = 0^\circ \), the vehicle is in helicopter mode. In this configuration, the x-axis runs parallel to the x-axis of the body frame, the y-axis runs out the right wing, and the z-axis is perpendicular to the x and y axes directed downward. This system is centered at the nacelle axis of rotation. It is fixed to the nacelle and rotates with it about the y-axis or nacelle axis of rotation.

\[
\begin{bmatrix}
\hat{i}_B \\
\hat{j}_B \\
\hat{k}_B
\end{bmatrix} = \begin{bmatrix} T_{G\rightarrow B} \end{bmatrix} \begin{bmatrix}
\hat{i}_{NAC} \\
\hat{j}_{NAC} \\
\hat{k}_{NAC}
\end{bmatrix}
\]

Non-rotating Hub Coordinate System

The non-rotating hub axes system is centered at the rotor hub as shown in Fig. 28. In helicopter mode, i.e., \( \beta_M = 0^\circ \), the x-axis runs parallel to the x-axis of the body frame, however it is directed aft. The y-axis runs parallel to the body y-axis and the z-axis is directed upwards. The non-rotating hub axis system is fixed to the hub, therefore when the nacelle rotates, the axis rotates with it.
Fig. 28. Non-rotating hub-fixed coordinate system

\[
\begin{bmatrix}
\hat{i}_{\text{NA}C} \\
\hat{j}_{\text{NA}C} \\
\hat{k}_{\text{NA}C}
\end{bmatrix} =
\begin{bmatrix}
-1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1
\end{bmatrix}
\begin{bmatrix}
\hat{i}_{\text{NR}} \\
\hat{j}_{\text{NR}} \\
\hat{k}_{\text{NR}}
\end{bmatrix}
\]

(9)

Rotating Hub Coordinate System

The rotating hub axes system, shown in Fig. 29 is fixed to the center of the rotor and rotates with it. It is defined with respect to the non-rotating hub axes system with the help of the azimuthal angle of rotation of the \( k \)th blade (\( \psi_k \)). When \( \psi_k = 0 \), the rotor blade passes over the tail of the aircraft. At this point, the x-axis is parallel to the x-axis of the body however, it is directed aft. The y-axis runs parallel to the body y-axis. The z-axis is perpendicular to the x and y axes and is directed upward.

\[
\begin{bmatrix}
\hat{i}_{\text{NR}} \\
\hat{j}_{\text{NR}} \\
\hat{k}_{\text{NR}}
\end{bmatrix} =
\begin{bmatrix}
\cos \beta_k & -\sin \beta_k & 0 \\
\sin \beta_k & \cos \beta_k & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\hat{i}_{\text{ROT}} \\
\hat{j}_{\text{ROT}} \\
\hat{k}_{\text{ROT}}
\end{bmatrix}
\]

(10)

Blade Coordinate System

The blade axes system, denoted by a prime, flaps with the blade. It is centered at the blade hinge point and defined with respect to the rotating hub axes system by the flap angle of the \( k \)th blade (\( \beta_k \)). The x-axis runs parallel to the blade, directed out of the blade. When \( \beta_k = 0 \), the z-axis is directed downward and the y-axis is then perpendicular to the x and z axes, directed opposite to the direction of motion.

\[
\begin{bmatrix}
\hat{i}_{\text{ROT}} \\
\hat{j}_{\text{ROT}} \\
\hat{k}_{\text{ROT}}
\end{bmatrix} =
\begin{bmatrix}
\cos \beta_k & -\sin \beta_k & 0 \\
0 & 1 & 0 \\
\sin \beta_k & 0 & \cos \beta_k
\end{bmatrix}
\begin{bmatrix}
\hat{i} \\
\hat{j} \\
\hat{k}
\end{bmatrix}
\]

(13)

Momentum Theory for Coaxial Rotors

The Momentum Theory or Simple Momentum Theory provides the most parsimonious flow model to study the hovering performance of a coaxial rotor system. The main advantage of the Simple Momentum Theory, however, is that it provides results for the minimum power consumption of the rotors under a given set of conditions, and so provides a datum against which the efficiency of a real rotor can be measured.

There are four primary ways of modelling the inflow of coaxial rotors:

1. The two rotors rotating in the same plane (practically, they would be very near to each other with minimal rotor spacing) and operated at equal thrusts.

2. The two rotors in the same plane of rotation and operated at equal and opposite torques, i.e., at a torque balance.

3. The two rotors rotating at equal thrusts, with the lower rotor operating in the fully developed slipstream of the upper rotor.

4. Two rotors rotating at equal and opposite torques, with the lower rotor in the fully developed slipstream (i.e., the vena contracta) of the upper rotor.
These cases are discussed in detail along with derivations of the induced power requirements and thrust sharing in (Refs. 14, 15). Practically, there is always a finite spacing between the two rotors in a coaxial system to avoid blade collisions between the two rotors. Also, the two rotors will generally operate at equal and opposite torques to provide zero net torque to the helicopter when it is operating in steady flight conditions. Therefore, for all the cases mentioned above, Case 4 is of primary practical importance, and is employed in the present analysis to calculate inflow through coaxial rotors.

Figure 30 shows the flow field of a coaxial rotor with the lower rotor operating in the vena contracta of the upper rotor. Thrusts shared by upper and lower rotors are represented by \( T_u \) and \( T_\ell \), respectively. Because, the upper rotor is not directly affected by the wake of the lower rotor, it can be treated as a single rotor that is connected to the lower rotor only through the need for a torque balance.

For a vehicle operating at forward speed \( V \) at a certain flight path angle (\( \theta_{FP} = 0 \) in this case) from the horizontal plane in gravity axes system, the velocity can be written as

\[
\vec{V}_G = -V \cos \theta_{FP} \hat{\imath}_G + V \sin \theta_{FP} \hat{k}_G
\]  

Using equations 7 and 10, we can write the flow velocity on the rotor disk as

\[
\vec{V}_{NR} = [T_{NR \rightarrow B}]^{-1} [T_{G \rightarrow B}] \vec{V}_G
\]  

where

\[
T_{NR \rightarrow B} = \begin{bmatrix}
-\cos \beta_M & 0 & \sin \beta_M \\
0 & 1 & 0 \\
-\sin \beta_M & 0 & -\cos \beta_M
\end{bmatrix}
\]

The velocity vector \( V_{NR} \) can be written as

\[
V_{NR} = V_{NR, iNR} + V_{NR, jNR} + V_{NR, kNR}
\]

We now define the non dimensional forward speed or advance ratio on the rotor disk as

\[
\mu = \frac{V_{NR}}{\Omega R}
\]

**Inflow on Upper Rotor**

The inflow on the upper rotor consists of two velocities, i.e., inflow due to vehicle velocity and inflow due to induced velocity. Following the momentum theory in hover, let us assume that the induced velocity at the upper rotor disk is \( \nu_u \), and in the far wake, \( w_u = 2\nu_u \). The induced velocities are directed parallel and opposite to the direction of the thrust vector, \( T_u \). We define the non dimensional induced velocity on the upper rotor as

\[
\lambda_{iu} = \frac{\nu_u \Omega R}{\Omega R}
\]

Thus, the total non-dimensional inflow on the upper rotor can be written as

\[
\lambda_u = \frac{V_{NR}}{\Omega R} + \frac{\nu_u}{\Omega R}
\]

Mass flow rate through the upper rotor is given by

\[
\dot{m}_u = \rho A \sqrt{\lambda_{iu}^2 + \mu^2}
\]

By momentum conservation

\[
T_u = (m)u 2V_u = 2\rho A (\Omega R)^2 \lambda_{iu} \sqrt{\lambda_{iu}^2 + \mu^2}
\]

Non-dimensionalizing thrust on the upper rotor, the thrust coefficient is given by

\[
C_{Tu} = 2\lambda_{iu} \sqrt{\lambda_{iu}^2 + \mu^2}
\]

Rearranging equation 22, the induced inflow on the upper rotor can be written as,

\[
\lambda_{iu} = \frac{C_{Tu}}{2 \sqrt{\lambda_{iu}^2 + \mu^2}}
\]

Using equation 23 and equation 19, the inflow equation for the upper rotor can be written as

\[
\lambda_u = \frac{V_{NR}}{\Omega R} + \frac{C_{Tu}}{2 \sqrt{\lambda_{iu}^2 + \mu^2}}
\]
Inflow on Lower Rotor

The lower rotor is identical to the upper rotor rotating in the opposite direction. The wake from the upper rotor now strikes the lower rotor parallel and opposite to the thrust vector \( T_f \) with velocity \( w_u \). At infinity, \( w_u = 2v_u \) and the wake area contracts from \( A \) to \( \frac{A}{2} \). Since the lower rotor is not at infinity, we assume a contraction factor \( f_c = \frac{A}{A_c} \), where \( A_c \) is the area of the wake from the upper rotor that strikes the lower rotor. The inflow field on the lower rotor now consists of two regions, i.e., the inner inflow field of area \( A_c \) which experiences wake from the upper rotor and the outer inflow field which is unaffected by the wake of the upper rotor.

From continuity, the velocity of the wake of the upper rotor is \( f_c v_u \). We assume that the induced inflow on the lower rotor is \( v_t \).

The non dimensional inflow on the inner region can be written as

\[ \lambda_{\text{inner}} = \frac{V_{NR}}{\Omega R} + \frac{v_t}{\Omega R} + f_c v_u \]

and on the outer region can be written as

\[ \lambda_{\text{outer}} = \frac{V_{NR}}{\Omega R} + \frac{v_t}{\Omega R} \]

The non dimensional induced velocity on the lower rotor can be defined as

\[ \lambda_t = \frac{v_t}{\Omega R} \]

Total Mass flow through the lower rotor can then be written as

\[ \dot{m}_l = \rho A_c \sqrt{\left(\lambda_{\text{outer}} \Omega R\right)^2 + V_{NR}^2} + \rho (A - A_c) \sqrt{\left(\lambda_{\text{inner}} \Omega R\right)^2 + V_{NR}^2} \]

By conservation of momentum, the thrust of the lower rotor can be expressed as,

\[ T_l = m_l w_l - \rho A_c (f_c v_u)(f_c v_u) \]

where \( w_l \) is the velocity of the wake of the lower rotor far downstream. The work done by the lower rotor can be written as

\[ T_l (v_u + v_t) = \dot{m}_l \frac{w_l^2}{2} - \rho A_c (f_c v_u)(f_c v_u)^2 \]

Eliminating \( w_l \) from equation 29 and 30, and substituting \( m_l \) from eq. 28, we get the following expression for induced inflow on the lower rotor.

\[ \lambda_i = \frac{1}{2C_T} \left( f_c \left( \frac{(C_T + f_c \lambda^3_{\text{out}})^2}{\sqrt{\left( \frac{f_c \lambda^3_{\text{out}}}{\lambda_i} + f_c - 1 \lambda_{\text{out}} \right)^2 + \mu^2} - f_c^2 \lambda^3_{\text{out}} + \frac{f_c}{\sqrt{\left( \lambda_i - \lambda_{\text{out}} \right)^2 + \mu^2}} \right) \right) \]

Thus, the inflow equation on the lower rotor then becomes

\[ \lambda_i = \lambda_{\text{in}} + \lambda_{\text{out}} + \frac{V_{NR}}{\Omega R} \]

Blade Element Theory

A detailed and systematic derivation of blade element theory is discussed in (Refs. 14, 16). The rotor blade is idealized as a rigid beam element undergoing only flapping motion. The position vector of any point \( P \) on the \( k^{th} \) blade, of an \( N_b \) bladed rotor system, in the deformed state can be written as

\[ r_p = r_i = r \cos \beta_k i_{ROT} + r \sin \beta_k k_{ROT} \]

The angular velocity of the rotor blade is taken as \( \dot{\omega} = \Omega \dot{k}_{ROT} \). It is assumed that the angular velocity of the rotor is a constant, and hence, the angular acceleration \( \frac{d^2 \dot{\omega}}{dt^2} = 0 \).

The absolute velocity of point \( P \) has two components: one due to rotation and another due to the flapping motion in the rotation frame. The absolute velocity of point \( P \) is given by

\[ \dot{v}_p = \left( \frac{d\dot{r}_p}{dt} \right)_{rel} + \dot{\omega} \times \dot{r}_p \]

\[ \dot{v}_p = -r \sin \beta_k \frac{d\beta_k}{dt} i_{ROT} + \Omega r \cos \beta_k i_{ROT} + r \cos \beta_k \frac{d\beta_k}{dt} k_{ROT} \]

The absolute acceleration of point \( P \) is given by

\[ \ddot{a}_p = \left( \frac{d^2\dot{r}_p}{dt^2} \right)_{rel} + \frac{d\dot{\omega}}{dt} \times \dot{r}_p + 2 \dot{\omega} \times \left( \frac{d\dot{r}_p}{dt} \right)_{rel} + \dot{\omega} \times (\dot{\omega} \times \dot{r}_p) \]

\[ \ddot{a}_p = \left( -r \cos \beta_k \left( \frac{d\beta_k}{dt} \right)^2 \right) + \left( -r \sin \beta_k \frac{d^2\beta_k}{dt^2} - \Omega^2 r \cos \beta_k \right) i_{ROT} + \left( -2 \Omega r \sin \beta_k \frac{d\beta_k}{dt} \right) j_{ROT} + \left( -r \sin \beta_k \left( \frac{d\beta_k}{dt} \right)^2 + r \cos \beta_k \frac{d^2\beta_k}{dt^2} \right) k_{ROT} \]
Flapping Equation of Motion

The flapping equation of motion for a blade is derived by making the total moment about the flap hinge at the root equal to zero.

\[
(M_I)_{flap} + (M_{ext})_{flap} = 0 
\]  \hspace{1cm} (38)

The inertia moment about the root is given as

\[
M_I = \int_0^R r_p^2 \times (\rho dr) 
\]  \hspace{1cm} (39)

Integrating the term over the length of the blade, the inertia moment in flap motion can be expressed as

\[
(M_I)_{flap} = I_b \frac{d^2 \beta_k}{dt^2} + \left(1 + \frac{\rho}{I_b} \frac{k_\beta}{I_\beta \Omega^2}\right) \Omega^2 I_b \sin \beta \cos \beta 
\]  \hspace{1cm} (40)

where \( I_b = \int_0^R \rho r^2 dr \) is the mass moment of inertia of the blade about the flap hinge at the center of the hub.

Invoking small angle assumption for flap angle \( \beta_k \), the inertia moment in the flap can be simplified as

\[
(M_I)_{flap} = I_b \frac{d^2 \beta_k}{dt^2} + \left(1 + \frac{\rho}{I_b} \frac{k_\beta}{I_\beta \Omega^2}\right) \Omega^2 I_b \beta_k 
\]  \hspace{1cm} (41)

We define, \( \left(1 + \frac{\rho}{I_b} \frac{k_\beta}{I_\beta \Omega^2}\right) = v_\beta^2 \), which allows the inertia moment to be expressed as

\[
(M_I)_{flap} = I_b \frac{d^2 \beta_k}{dt^2} + v_\beta^2 \Omega^2 I_b \beta_k 
\]  \hspace{1cm} (42)

The reduced expression, assuming \( \beta_k \) to be small, for the velocity of point \( P \), in the primed blade axes system, can be written as

\[
\dot{v}_P = \Omega r \dot{\beta} + r \frac{\dot{d}_k}{dt} \dot{\beta} = \Omega r \dot{\beta} + r \dot{\beta} \dot{\beta} 
\]  \hspace{1cm} (43)

For the sake of consistency and convenience, the time derivative of flap \( \beta_k \) is non-dimensionalized as \( \dot{\beta}_k = \Omega \frac{d \beta_k}{d \Omega} \)

\[
\Omega \frac{d \beta_k}{d \Omega} = \Omega \dot{\beta}_k 
\]  \hspace{1cm} (44)

The normal component or inflow \( \lambda \). The relative air velocity at the blade section due to the motion of the helicopter and the total induced flow can be written as components along the \( k^{th} \) blade axes system as

\[
\begin{align*}
\dot{V}_h &= \mu \Omega R \cos \psi \dot{\beta}_k \\
&- \mu \Omega \sin \psi \dot{\beta}_k \\
&- \lambda \Omega \dot{\beta}_k 
\end{align*} 
\]  \hspace{1cm} (45)

Resolving these velocity components in the blade-fixed system, we have

\[
\begin{align*}
\dot{V}_h &= (\mu \Omega R \cos \psi \cos \beta_k - \lambda \Omega \sin \beta_k) \dot{\beta}_k \\
&- \mu \Omega R \sin \psi \dot{\beta}_k \\
&- (\mu \Omega \cos \psi \beta_k + \lambda \Omega R) \dot{\beta}_k
\end{align*} 
\]  \hspace{1cm} (46)

Invoking a small angle assumption, equation 45 can be written as

\[
\begin{align*}
\dot{V}_h &= (\mu \Omega R \cos \psi \dot{\beta}_k - \lambda \Omega R \dot{\beta}_k) \dot{\beta}_k \\
&- \mu \Omega \sin \psi \dot{\beta}_k \\
&- (\mu \Omega \cos \psi \dot{\beta}_k + \lambda \Omega R) \dot{\beta}_k
\end{align*} 
\]  \hspace{1cm} (47)

Substituting equations 43 and 46, we get

\[
\dot{V}_h = (\mu \Omega \cos \psi \dot{\beta}_k - \lambda \Omega R \dot{\beta}_k) \dot{\beta}_k \\
- (\mu \Omega \sin \psi \dot{\beta}_k + \lambda \Omega R) \dot{\beta}_k \\
- (\mu \Omega \cos \psi \dot{\beta}_k + \lambda \Omega R) \dot{\beta}_k
\]  \hspace{1cm} (48)

The velocity components can then be expressed as

Tangential velocity component at any radial location \( r \):

\[
U_T = \Omega r + \mu \Omega R \sin \psi_k = \Omega R \left( \frac{r}{R} + \mu \sin \psi_k \right) 
\]  \hspace{1cm} (49)

Radial velocity component along the blade:

\[
U_R = \mu \Omega R \cos \psi_k - \lambda \Omega R \dot{\beta}_k 
\]  \hspace{1cm} (50)

The normal velocity component at \( r \):

\[
U_p = \lambda \Omega R + \Omega \dot{\beta}_k + \beta_k \mu \Omega R \cos \psi_k 
\]  \hspace{1cm} (51)

The resultant velocity of the oncoming flow can be written, by assuming \( U_p < U_T \), as

\[
U = \sqrt{U_T^2 + U_p^2} \approx U_T 
\]  \hspace{1cm} (52)

The inflow angle is given by

\[
\tan \phi = \frac{U_p}{U_T} 
\]  \hspace{1cm} (53)
and for small angles
\[ \phi \approx \frac{U_p}{U_T} \]  

(54)

The sectional aerodynamic lift and drag acting on the airfoil are given as
\[ L = \frac{1}{2} \rho U^2 C_{L} \]  

(55)
\[ D = \frac{1}{2} \rho U^2 C_{D} \]  

(56)

Resolving these forces along the normal and in-plane directions of the blade fixed frame,
\[ F_i = L \cos \phi - D \sin \phi \]  

(57)
\[ F_j = -(L \sin \phi + D \cos \phi) \]  

(58)

The components of these distributed aerodynamic loads in the rotating blade frame, after neglecting radial drag effects, can be given as
\[ F_{i,\text{ROT}} = -F_i \sin \beta_k \]  

(59)
\[ F_{j,\text{ROT}} = F_j \]  

(60)
\[ F_{k,\text{ROT}} = F_i \cos \beta_k \]  

(61)

Invoking small angle approximation and assuming \( L > D \), the aerodynamic force components can be approximated as
\[ F_{i,\text{ROT}} \approx F_i \]  

(62)
\[ F_{j,\text{ROT}} \approx -(L \phi + D) \]  

(63)
\[ F_{k,\text{ROT}} \approx -F_i \beta_k \]  

(64)

The aerodynamic root moment can be obtained as
\[ M_A = \vec{r} \times \vec{F} = (r \cos \beta_k \hat{i}_{\text{ROT}} + r \sin \beta_k \hat{k}_{\text{ROT}}) \times \vec{F} \]  

(65)

In component form,
\[ M_A = -F_{i,\text{ROT}} r \sin \beta_k \hat{i}_{\text{ROT}} + (F_{i,\text{ROT}} r \sin \beta_k - r \cos \beta_k F_{k,\text{ROT}}) \hat{j}_{\text{ROT}} + r \cos \beta_k F_{j,\text{ROT}} \hat{k}_{\text{ROT}} \]  

(66)

The blade pitch angle consists of the pilot input and a linear geometric twist of the blade
\[ \theta = \theta_0 + \theta_1 \cos \psi_k + \theta_2 \sin \psi_k \]  

(67)

The velocity components can be written in non-dimensional form (from equations 49 and 51) as
\[ u_T = \frac{U_T}{\Omega R} = \frac{r}{R} + \mu \sin \psi_k \]  

(68)
\[ u_p = \frac{U_p}{\Omega R} = \lambda + \frac{r^2 \beta_k}{R} + \beta_k \mu \cos \psi_k \]  

(69)

Substituting \( C_L = a \alpha_c \) and \( \alpha_c = \theta - \phi = \theta - \frac{U_p}{U_T} \), in equations 52 and 55, the aerodynamic lift per unit span of the blade can be written as

The blade forces per unit span of the blade are
\[ F_{i,\text{ROT}} = -\frac{1}{2} \rho C a (\Omega R)^2 [u_T^2 \theta - u_{p,\text{ROT}}] \beta_k \]  

(70)
\[ F_{j,\text{ROT}} = \frac{1}{2} \rho C a (\Omega R)^2 [u_T^2 u_p - u_T^2 + C_d a u_T^2] \]  

(71)
\[ F_{k,\text{ROT}} = \frac{1}{2} \rho C a (\Omega R)^2 [u_T^2 \theta - u_{p,\text{ROT}}] \]  

(72)

The external flap moment due to the distributed aerodynamic lift acting on the blade can be written, using equations 57 and 65, as
\[ (M_{\text{ext}})_{\text{flap}} = \int_e^R (r \hat{r} \times \vec{F}_{i,\text{ROT}}) dr = \int_e^R (-\hat{r} + (r - e) F_i) dr \]  

(73)

Using equations 62, 38 and 41, we get the flap equation as
\[ I_b \left( \frac{d^2 \beta_k}{dt^2} + \nu_\beta^2 \Omega^2 \beta_k \right) = \int_e^R (r - e) F_{k,\text{ROT}} dr \]  

(74)

where \( \nu_\beta \) is the rotating natural frequency of flap dynamics. Non-dimensionalising the time derivative term on the LHS as \( \frac{d^2 \beta_k}{dt^2} = \Omega^2 \beta_k \) and the integral on the RHS with respect to the rotor radius \( R \) as \( x = \frac{r}{R} \). The flap equation can be written in symbolic form as
\[ \ddot{\beta}_k + \nu_\beta^2 \beta_k = \gamma M_{\text{flap}} \]  

(75)

Here, \( \gamma = \frac{\rho a CR^4}{I_b} \), the aerodynamic flap moment is then,
\[ M_{\text{flap}} = \frac{1}{\rho a C \Omega^2 R^4} \int_e^R (r - e) F_{k,\text{ROT}} dr = \frac{1}{2} \int_e^R \left( x - \frac{e}{R} \right) (u_T^2 \theta - u_{p,\text{ROT}}) dx \]  

(76)

This is a simplified flap equation for a blade with an hinge offset \( e \) and a flap spring of stiffness \( k_\beta \). This equation is solved numerically using Newmark’s Algorithm to obtain a steady-state solution while assuming that the aerodynamic loads lag the flap response. The solution obtained is then used to calculate rotor hub forces.
Drees Inflow

During the transition from hover into level forward flight, that is, within the range $0.0 \leq \mu \leq 0.1$, the induced velocity in the plane of the rotor is the most uniform, it being strongly affected by the presence of discrete tip vortices that sweep downstream near the rotor plane. Following Glauert’s result for longitudinal inflow in high speed forward flight and considering a longitudinal and a lateral variation in the inflow, the induced inflow ratio can be written as

$$\lambda_i = \lambda_0 (1 + k_x r \cos \psi + k_y r \sin \psi)$$  \hspace{1cm} (77)

Here $k_x$ and $k_y$ can be viewed as weighting factors and represent the deviation of the inflow from the uniform value predicted by the simple momentum theory. A parsimonious linear inflow model frequently employed in basic rotor analysis is attributed to Drees(1949). In this model, the coefficients of the linear part of the inflow are given by:

$$k_x = \frac{4}{3} \left(1 - \cos \chi - 1.8 \mu^2 \right) / \sin \chi \quad \text{and} \quad k_y = -2 \mu$$  \hspace{1cm} (78)

where the wake skew angle or $\chi = \tan^{-1} \left( \frac{\mu_k}{\mu_c + \lambda_i} \right)$.

Rotor Load Calculations

The distribution of aerodynamic and centrifugal forces along the span, and the structural dynamics of the blade in response to these forces create shear loads and bending loads at the blade root.

For a zero hinge offset, the blade root is at the center of rotation. For a non-zero hinge offset, it is at a distance $e$ outboard from the center of rotation. By ‘loads’ we mean ‘reaction’ forces generated by the net balance of all forces acting over the blade span. Let $s_x$, $s_y$, and $s_z$ be the three shear loads, in-plane, radial, and vertical. Let $n_x$, $n_y$, and $n_z$ be the bending loads, flap bending moment, torsion moment (positive for leading edge up), and chord bending moment (positive in lag direction). They occur at the blade root, rotate with the blade, and vary with the azimuth angle. Thus they are called the rotating root loads or root reactions. Since the forces and moments are calculated by integrating over a complete rotation, the centrifugal and trigonometric terms that integrate to zero over the limits 0 to $2\pi$ are ignored in the formulation. The root forces and moments are calculated in the non-dimensional form in the blade axes system are given as follows:

$$C_{s_x} = \frac{1}{\rho A (\Omega R)^2} \int_e^R r \frac{dF_{s_x}}{dr} \, dr$$

$$= \frac{aC}{2\pi R} \int_{x}^{R} (u_{p \theta} \theta - u_{p \psi} \sin \psi) \, dx$$  \hspace{1cm} (79)

$$C_{s_y} = \frac{1}{\rho A (\Omega R)^2} \int_e^R r \frac{dF_{s_y}}{dr} \, dr$$

$$= \frac{aC}{2\pi R} \int_{x}^{R} (u_{p \theta} \theta - u_{p \psi} \sin \psi) \, dx$$  \hspace{1cm} (80)

$$C_{s_z} = \frac{1}{\rho A (\Omega R)^2} \int_e^R r \frac{dF_{s_z}}{dr} \, dr$$

$$= \frac{aC}{2\pi R} \int_{x}^{R} (u_{p \theta} \theta - u_{p \psi} \sin \psi) \, dx$$  \hspace{1cm} (81)

$$C_{n_x} = \frac{1}{\rho A (\Omega R)^2} \int_e^R r \frac{dM_{n_x}}{dr} \, dr$$

$$= \frac{aC}{2\pi R} \int_{x}^{R} (u_{p \theta} \theta - u_{p \psi} \sin \psi) \, dx$$  \hspace{1cm} (82)

$$C_{n_y} = \frac{1}{\rho A (\Omega R)^2} \int_e^R r \frac{dM_{n_y}}{dr} \, dr$$

$$= \frac{aC}{2\pi R} \int_{x}^{R} (u_{p \theta} \theta - u_{p \psi} \sin \psi) \, dx$$  \hspace{1cm} (83)

$$C_{n_z} = \frac{1}{\rho A (\Omega R)^2} \int_e^R r \frac{dM_{n_z}}{dr} \, dr$$

$$= \frac{aC}{2\pi R} \int_{x}^{R} (u_{p \theta} \theta - u_{p \psi} \sin \psi) \, dx$$  \hspace{1cm} (84)

Root Loads in Rotating Hub Axes System The root loads in rotating hub axes system are obtained by simply transferring the root loads to the hub. For a non-zero offset, the rotating hub loads are:

$$C_{F_{s_x}} = C_{s_x} \quad \quad C_{M_{s_x}} = C_{n_x}$$  \hspace{1cm} (85)

$$C_{F_{s_y}} = C_{s_y} \quad \quad C_{M_{s_y}} = C_{n_y} + e C_{s_y}$$  \hspace{1cm} (86)

$$C_{F_{s_z}} = C_{s_z} \quad \quad C_{M_{s_z}} = -C_{s_z} - e C_{s_y}$$  \hspace{1cm} (87)

The rotating frame hub loads for a counter rotating rotor can be written as

$$C_{F_{s_x}} = C_{s_x} \quad \quad C_{M_{s_x}} = C_{n_x}$$  \hspace{1cm} (88)

$$C_{F_{s_y}} = -C_{s_y} \quad \quad C_{M_{s_y}} = C_{s_y} + e C_{s_y}$$  \hspace{1cm} (89)

$$C_{F_{s_z}} = C_{s_z} \quad \quad C_{M_{s_z}} = C_{s_z} + e C_{s_y}$$  \hspace{1cm} (90)

Loads in Non-rotating Hub Axes System Root loads in the non-rotating hub axes system can be obtained by simply resolving them in the NR frame for each blade and adding them together. Let $m = 1, 2, ..., N_b$ be the blade number, $\psi_m$ be the azimuthal location of each blade $m$. The non-dimensional loads can be expressed as,
ferred to the body axes system. The forces can be easily trans-
ferred using the rotation matrices in the following way.

\[
C_{F_{NR}} = \sum_{m=1}^{N_b} (C_{F_{ROT}} \cos \psi_m + C_{F_{ROT}} \sin \psi_m)
\]

\[
C_{F_{NR}} = \sum_{m=1}^{N_b} (C_{F_{ROT}} \sin \psi_m - C_{F_{ROT}} \cos \psi_m)
\]

\[
C_{F_{NR}} = \sum_{m=1}^{N_b} C_{F_{ROT}}
\]

\[
C_{M_{NR}} = \sum_{m=1}^{N_b} (C_{M_{ROT}} \cos \psi_m + C_{M_{ROT}} \sin \psi_m)
\]

\[
C_{M_{NR}} = \sum_{m=1}^{N_b} (C_{M_{ROT}} \sin \psi_m - C_{M_{ROT}} \cos \psi_m)
\]

\[
C_{M_{NR}} = \sum_{m=1}^{N_b} C_{M_{ROT}}
\]

The assumption here is that all blades have identical root
loads, only shifted in phase. In case the blades are similar,
the hub loads transmit all the harmonics. Such is the case of
damaged or dissimilar rotors. The periodically varying loads
over one complete rotation of the rotor blade are

The trim equations for the aircraft depend on the forces and
moments of each component. The proprotor contributions are
discussed earlier. The methodology used to calculate the air-
frame forces and moments is to determine the lift and drag of
each component, and then, using the relative position on the
aircraft, determine the associated body axis forces and mom-
ents. By definition:

- Dynamic Pressure: \( q = \frac{1}{2} \rho V^2 \)
- Lift: \( L = \frac{1}{2} \rho V^2 AC_L \)
- Drag: \( D = \frac{1}{2} \rho V^2 AC_D \)

The airframe components used for this analysis were the
wing, fuselage, horizontal tail, and vertical tail. For the pur-
pose of this analysis, the lift and drag from the nacelles were
ignored. Assumptions made to develop these equations are
listed below:

- \( C_L = \frac{L}{qA} \) and \( C_D = \frac{D}{qA} \).
- \( C_L \) is linear, therefore, \( C_\ell = C_{La} \alpha \).
- \( C_M \) is linear, therefore, \( C_M = C_{Ma} \alpha + C_{Mo} \).
- Drag Coefficient \( C_D = C_{Dv} + kC_\ell^2 \).
- Wing has a constant airfoil section.
- Proprotor effects on the airflow over the wing and other
aircraft components are negligible.
- The small angle approximation was made for the angle of
attack and sideslip angles.

### Wing

The equations for lift and drag developed for the wing are
as follows:

\[
L_w = \frac{1}{2} \rho V^2 A_w [C_{Lw}(\alpha_w - \alpha_{0w})]
\]

\[
D_w = \frac{1}{2} \rho V^2 A_w C_{Dw}
\]

where \( k_w = \frac{1}{\pi e_w AR_w} \)
The wing forces in gravity frame for an airplane at a given flight path angle can be written as:

\[
\vec{F}_{Gw} = \begin{bmatrix}
-D_w \cos \theta_{FP} - L_w \sin \theta_{FP} \\
0 \\
-L_w \cos \theta_{FP} + D_w \sin \theta_{FP}
\end{bmatrix}
\] (111)

And in the body frame as

\[
\vec{F}_{Bw} = [T_{G\rightarrow B}] \vec{F}_{Gw}
\] (112)

The moment due to wing forces can be evaluated as

\[
\vec{M}_{Bw} = \vec{r}_w \times \vec{F}_{Bw}
\] (113)

where \(\vec{r}_w\) is the position vector of the line of aerodynamic center of the wing from the aircraft CG.

Fuselage

The equations for lift and drag developed for the fuselage are as follows:

\[
L_f = \frac{1}{2} \rho V^2 A_f [C_{L_f}(\alpha_f - \alpha_0)]
\] (114)

\[
D_f = \frac{1}{2} \rho V^2 A_f C_{D_f}
\] (115)

where

\[
k_f = \frac{1}{\pi e_f AR_f}
\]

\[
C_{D_f} = C_{D_0} + k_f C_{L_f}^2
\]

The fuselage forces in gravity frame for an airplane at a given flight path angle can be written as:

\[
\vec{F}_{Gf} = \begin{bmatrix}
-D_f \cos \theta_{FP} - L_f \sin \theta_{FP} \\
0 \\
-L_f \cos \theta_{FP} + D_f \sin \theta_{FP}
\end{bmatrix}
\] (116)

And in the body frame as

\[
\vec{F}_{Bf} = [T_{G\rightarrow B}] \vec{F}_{Gf}
\] (117)

The moment due to fuselage forces can be evaluated as

\[
\vec{M}_{Bf} = \vec{r}_f \times \vec{F}_{Bf}
\] (118)

where \(\vec{r}_f\) is the position vector of the line of aerodynamic center of the fuselage from the aircraft CG.

Horizontal Tail

The equations for lift and drag developed for the horizontal tail are as follows:

\[
L_{HT} = \frac{1}{2} \rho V^2 A_{HT} [C_{L_{HT}}(\alpha_{HT} - \alpha_0)]
\] (119)

\[
D_{HT} = \frac{1}{2} \rho V^2 A_f C_{D_{HT}}
\] (120)

where

\[
k_{HT} = \frac{1}{\pi e_{HT} AR_{HT}}
\]

\[
C_{D_{HT}} = C_{D_0} + k_f C_{L_{HT}}^2
\]

The horizontal tail forces in gravity frame for an airplane at a given flight path angle can be written as:

\[
\vec{F}_{GHT} = \begin{bmatrix}
-D_{HT} \cos \theta_{FP} - L_{HT} \sin \theta_{FP} \\
0 \\
-L_{HT} \cos \theta_{FP} + D_{HT} \sin \theta_{FP}
\end{bmatrix}
\] (121)

And in the body frame as

\[
\vec{F}_{BHT} = [T_{G\rightarrow B}] \vec{F}_{GHT}
\] (122)

The moment due to horizontal tail forces can be evaluated as

\[
\vec{M}_{BHT} = \vec{r}_{HT} \times \vec{F}_{BHT}
\] (123)

where \(\vec{r}_{HT}\) is the position vector of the line of aerodynamic center of the horizontal tail from the aircraft CG.

Vertical Tail

The equations for lift and drag developed for the vertical tail are as follows:

\[
L_{VT} = \frac{1}{2} \rho V^2 A_{VT} [C_{L_{VT}}(\alpha_{VT} - \alpha_0)]
\] (124)

\[
D_{VT} = \frac{1}{2} \rho V^2 A_f C_{D_{VT}}
\] (125)

where

\[
k_{VT} = \frac{1}{\pi e_{VT} AR_{VT}}
\]

\[
C_{D_{VT}} = C_{D_0} + k_f C_{L_{VT}}^2
\]

The vertical tail forces in gravity frame for an airplane at a given flight path angle can be written as:

\[
\vec{F}_{GVT} = \begin{bmatrix}
-D_{VT} \cos \theta_{FP} - L_{VT} \sin \theta_{FP} \\
0 \\
-L_{VT} \cos \theta_{FP} + D_{VT} \sin \theta_{FP}
\end{bmatrix}
\] (126)

And in the body frame as

\[
\vec{F}_{BVT} = [T_{G\rightarrow B}] \vec{F}_{GVT}
\] (127)
The moment due to vertical tail forces can be evaluated as

\[ \vec{M}_{BV_T} = \vec{r}_{VT} \times \vec{F}_{BV_T} \]  

(128)

where \( \vec{r}_{VT} \) is the position vector of the line of aerodynamic center of the vertical tail from the aircraft CG.

**Equilibrium Equations**

The equations of equilibrium can be derived by simply adding all the forces and moments in the body frame. The resultant forces and moments in the body frame can be written as a sum of all the components of the aircraft.

\[ \vec{F}_B = \vec{F}_{Bu} + \vec{F}_{Bl} + \vec{F}_{Bw} + \vec{F}_{Bf} + \vec{F}_{BHT} + \vec{F}_{BV_T} + \vec{W} = 0 \]  

(129)

\[ \vec{M}_B = \vec{M}_{Bu} + \vec{M}_{Bl} + \vec{M}_{Bw} + \vec{M}_{Bf} + \vec{M}_{BHT} + \vec{M}_{BV_T} = 0 \]  

(130)

The components of forces and moments in body frame together are the six equilibrium equations which are used for trim analysis using standard Newton-Raphson technique.

**APPENDIX B**

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius, R</td>
<td>4.0</td>
<td>m</td>
</tr>
<tr>
<td>Rotor Speed, ( \Omega )</td>
<td>589</td>
<td>RPM</td>
</tr>
<tr>
<td>No. of blades, ( N_b )</td>
<td>3</td>
<td>–</td>
</tr>
<tr>
<td>Chord, C</td>
<td>0.4</td>
<td>m</td>
</tr>
<tr>
<td>Taper ratio, TR</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td>Lift Curve Slope, a</td>
<td>5.73</td>
<td>/rad</td>
</tr>
<tr>
<td>Profile Drag Coefficient, ( C_d )</td>
<td>0.01</td>
<td>–</td>
</tr>
<tr>
<td>Blade Flapping Inertia, ( I_p )</td>
<td>140</td>
<td>kg-m²</td>
</tr>
<tr>
<td>Flapping Spring Constant, ( K_{\beta} )</td>
<td>1800</td>
<td>kg-m/rad</td>
</tr>
<tr>
<td>Hinge Offset, ( e/R )</td>
<td>0.15</td>
<td>–</td>
</tr>
<tr>
<td>Twist, ( \theta_{tw} )</td>
<td>-41</td>
<td>deg</td>
</tr>
<tr>
<td>Twist at hub, ( \theta_{tw0} )</td>
<td>40</td>
<td>deg</td>
</tr>
<tr>
<td>Rotor(upper) height from CG, ( h_1 )</td>
<td>2</td>
<td>m</td>
</tr>
<tr>
<td>Rotor(lower) height from CG, ( h_2 )</td>
<td>1.5</td>
<td>m</td>
</tr>
<tr>
<td>X distance of mast from CG, ( X_{CG} )</td>
<td>0</td>
<td>m</td>
</tr>
<tr>
<td>Y distance of mast from CG, ( Y_{CG} )</td>
<td>0</td>
<td>m</td>
</tr>
<tr>
<td>Rate of Nacelle Movement, ( \beta_M )</td>
<td>0</td>
<td>rad/s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td>( A_{VT} )</td>
<td>2.4</td>
<td>m²</td>
</tr>
<tr>
<td>Span</td>
<td>( b_{VT} )</td>
<td>2.4</td>
<td>m</td>
</tr>
<tr>
<td>Aspect Ratio</td>
<td>( AR_{VT} )</td>
<td>2.4</td>
<td>–</td>
</tr>
<tr>
<td>Lift Curve Slope, ( a_{VT} )</td>
<td>3.06</td>
<td>/rad</td>
<td></td>
</tr>
<tr>
<td>Zero-Lift Angle of attack, ( \alpha_{0l,VT} )</td>
<td>0</td>
<td>rad</td>
<td></td>
</tr>
<tr>
<td>Profile Drag Coefficient, ( C_{d0,VT} )</td>
<td>0.017</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>Wing chord, ( c_w )</td>
<td>1.6</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>Wing efficiency factor, ( e_w )</td>
<td>0.9</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Taper Ratio, ( \lambda )</td>
<td>1</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>Moment coefficient vs. AOA, ( C_{M\alpha} )</td>
<td>0</td>
<td>rad</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td>( A_{HT} )</td>
<td>4.5</td>
<td>m²</td>
</tr>
<tr>
<td>Span</td>
<td>( b_{HT} )</td>
<td>4</td>
<td>m</td>
</tr>
<tr>
<td>Aspect Ratio</td>
<td>( AR )</td>
<td>3.55</td>
<td>–</td>
</tr>
<tr>
<td>Incidence angle, ( i_{HT} )</td>
<td>0</td>
<td>rad</td>
<td></td>
</tr>
<tr>
<td>Lift Curve Slope, ( a_{HT} )</td>
<td>4.03</td>
<td>/rad</td>
<td></td>
</tr>
<tr>
<td>Zero-Lift Angle of attack, ( \alpha_{0l,HT} )</td>
<td>0</td>
<td>rad</td>
<td></td>
</tr>
<tr>
<td>Profile Drag Coefficient, ( C_{d0,HT} )</td>
<td>0.0088</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>Efficiency factor, ( e_{HT} )</td>
<td>0.8</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Distance from aircraft CG</td>
<td>( X_{HT} )</td>
<td>14.2</td>
<td>m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
</table>
| Table 4. Fuselage Parameters
| Parameter Description | Value | Units |
| Flat Plate Drag, \( f \) | 1.0   | m²   |
| Lift Curve Slope, \( a_f \) | 0.286 | /rad |
| Zero lift angle of attack, \( \alpha_{0l,f} \) | -0.14 | rad |
| Zero AOA Moment Coefficient, \( C_{M0,f} \) | -0.0096 | kg-m |
| Moment Coefficient vs. AOA, \( C_{M\alpha,f} \) | 1.145 | /rad |

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wing Area, ( A_w )</td>
<td>16.0</td>
<td>m²</td>
</tr>
<tr>
<td>Span, ( b_w )</td>
<td>10</td>
<td>m</td>
</tr>
<tr>
<td>Aspect Ratio, ( AR_w )</td>
<td>6.25</td>
<td>–</td>
</tr>
<tr>
<td>Incidence angle, ( i_w )</td>
<td>0</td>
<td>rad</td>
</tr>
<tr>
<td>Lift Curve Slope, ( a_w )</td>
<td>5.31</td>
<td>/rad</td>
</tr>
<tr>
<td>Zero-Lift Angle of attack, ( \alpha_{0l,w} )</td>
<td>-0.07</td>
<td>rad</td>
</tr>
<tr>
<td>Profile Drag Coefficient, ( C_{d0,w} )</td>
<td>0.017</td>
<td>–</td>
</tr>
<tr>
<td>Wing chord, ( c_w )</td>
<td>1.6</td>
<td>m</td>
</tr>
<tr>
<td>Wing efficiency factor, ( e_w )</td>
<td>0.9</td>
<td>0</td>
</tr>
<tr>
<td>Taper Ratio, ( \lambda )</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td>Moment coefficient vs. AOA, ( C_{M\alpha,w} )</td>
<td>0</td>
<td>rad</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
</table>
| Table 5. Wing Parameters
| Parameter Description | Value | Units |
| Area                  | \( A_{w} \) | 16.0  | m² |
| Span                  | \( b_{w} \) | 10.0  | m  |
| Aspect Ratio          | \( AR_{w} \) | 6.0   | –  |
| Incidence angle, \( i_{w} \) | 0 | rad |
| Lift Curve Slope, \( a_{w} \) | 5.36  | /rad |
| Zero-Lift Angle of attack, \( \alpha_{0l,w} \) | -0.07 | rad |
| Profile Drag Coefficient, \( C_{d0,w} \) | 0.017 | – |
| Wing chord, \( c_{w} \) | 1.6   | m     |
| Wing efficiency factor, \( e_{w} \) | 0.9   | 0     |
| Taper Ratio, \( \lambda \) | 1     | –     |
| Moment coefficient vs. AOA, \( C_{M\alpha,w} \) | 0  | rad |

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
</table>
| Table 6. Vertical Tail Parameters
| Parameter Description | Value | Units |
| Area                  | \( A_{VT} \) | 2.4   | m²    |
| Span                  | \( b_{VT} \) | 2.4   | m    |
| Aspect Ratio          | \( AR_{VT} \) | 2.4   | –     |
| Lift Curve Slope, \( a_{VT} \) | 3.06  | /rad  |
| Zero-Lift Angle of attack, \( \alpha_{0l,VT} \) | 0 | rad |
| Profile Drag Coefficient, \( C_{d0,VT} \) | 0.0017 | – |
| Oswald’s efficiency factor | \( e_{VT} \) | 1 | – |
| Distance from aircraft CG | \( X_{VT} \) | 14.5  | m    |

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
</table>
| Table 7. Horizontal Tail Parameters
| Parameter Description | Value | Units |
| Area                  | \( A_{HT} \) | 4.5   | m²    |
| Span                  | \( b_{HT} \) | 4     | m    |
| Aspect Ratio          | \( AR \) | 3.55  | –     |
| Incidence angle, \( i_{HT} \) | 0 | rad |
| Lift Curve Slope, \( a_{HT} \) | 4.03  | /rad  |
| Zero-Lift Angle of attack, \( \alpha_{0l,HT} \) | 0 | rad |
| Profile Drag Coefficient, \( C_{d0,HT} \) | 0.0088 | – |
| Efficiency factor, \( e_{HT} \) | 0.8   | 0     |
| Distance from aircraft CG | \( X_{HT} \) | 14.2  | m    |

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
</table>
| Table 8. Fuselage Parameters
REFERENCES


