Sparse control and stabilization to consensus of collective behavior models.

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In the last years there have been many works on models of self-alignment and consensus dynamics, with the objective of modelling and explaining self-organization. However, the evidence that in practice self-organization does not necessarily occur leads to the natural question of whether it is possible to externally influence the dynamics in order to promote the formation of certain desired patterns.

Once this question is posed, one is also faced with the issue of defining the best way of achieving the result, seeking for the most "economical" way to realize a certain outcome.

In a series of works with M. Caponigro, M. Fornasier, B. Piccoli, and F. Rossi, we have addressed the issue of finding the sparsest control strategy in order to lead us optimally towards a given outcome, in this case the achievement of a state where the group will be able by self-organization to reach an alignment consensus.

As a consequence we provide a mathematical justification to the general principle according to which "sparse is better": in order to achieve group consensus, a policy maker not allowed to predict future developments should decide to control with stronger action the fewest possible leaders rather than trying to act on more agents with minor strength.

Our main model is the Cucker-Smale model, both in finite dimension and in infinite dimension (kinetic Cucker-Smale model, obtained by mean-field limit). We establish local and global sparse controllability properties to consensus. We also generalize our results to dissipative control-affine systems by developing a sparse version of the classical Jurdjevic-Quinn strategy.